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A predictor-corrector interior-point algorithm for monotone variational inequality problems*

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Abstract: Mehrotra's recent suggestion of a predictor-corrector variant of primal-dual interior-point method for linear programming is currently the interior-point method of choice for linear programming. In this work the authors give a predictor-corrector interior-point algorithm for monotone variational inequality problems. The algorithm was proved to be equivalent to a level-1 perturbed composite Newton method. Computations in the algorithm do not require the initial iteration to be feasible. Numerical results of experiments are presented.

Key words: Variational inequality problems(VIP)，Predictor-corrector interior-point algorithm，Numerical experiments
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INTRODUCTION

Let $\boldsymbol{K}^n$ be a nonempty closed convex subset of $\boldsymbol{R}^n$ and let $\boldsymbol{F}:\boldsymbol{R}^n\rightarrow\boldsymbol{R}^n$ be a continuous mapping. A variational inequality problem (VIP) is

Find $\boldsymbol{x}^*\in\boldsymbol{K}$ such that $(\boldsymbol{x}-\boldsymbol{x}^*)\boldsymbol{F}(\boldsymbol{x})^*\geqslant 0$ for all $\boldsymbol{x}\in\boldsymbol{K}$.

This problem widely appeared in the study of various equilibrium models in economics，operations research and transportation. There are many available iterative methods for VIP，such as projection methods，nonlinear Jacobian methods，successive overrelaxation methods and generalized gradient methods (Harker et al.，1990). Several recently proposed interior-point algorithms for VIP include the primal scaling path-following algorithm (Tseng，1992)，the nonpolynomial long-step primal path-following algorithm (Wu，1993)，the entropy-like interior- proximal algorithm (Auslender and Haddou，1995)，the modified path-following algorithm (Wu，1997)，the long-step interior-point algorithm (Sun et al.，1998) and the potential reduction interior-point algorithm (Liang et al.，2000). Mehrotra introduced a remarkable predictor-corrector algorithm for linear programming (Mehrotra，1990). The predictor step was a damped Newton step for solving the KKT conditions and producing a new strictly feasible iteration. The subsequent corrector step was a centered Newton step，in which the choice of centering parameter was based on the predictor step. Numerical results indicated that this method represented a significant computation advance for linear programming，and is to date the most computationally efficient method for solving large-scale linear programming.

For nonlinear mapping $\boldsymbol{F}:\boldsymbol{R}^n\rightarrow\boldsymbol{R}^n$，the level-$m$ perturbed composite Newton method for $\boldsymbol{F}(\boldsymbol{x})=0$ takes m simplified Newton steps between every two Newton steps and has a Q-convergence rate of $m+2$ under the standard Newton method assumptions (Ortega and Rheinboldt，1970). This method cov-

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ers the middle ground between the extremes of the Newton method and simplified Newton method, and is of value when n is large and the mapping F can be evaluated cheaply. In this paper, we will give a predictor-corrector interior-point algorithm for the monotone variational inequality problem $VIP(\boldsymbol{K}, \boldsymbol{F})$ under the predictor-corrector motivation, where $\boldsymbol{K} = \{x \in \boldsymbol{R}^n \mid \boldsymbol{Ax} \leqslant \boldsymbol{b}, \boldsymbol{x} \geqslant 0\}$ is a polyhedral, $\boldsymbol{A}$ is a given $m \times n$ matrix and b is a given vector in $\boldsymbol{R}^n$. It is proved that the algorithm is equivalent to a level-1 perturbed composite Newton method. Computations in the algorithm do not require that the initial iteration be feasible. Some computational results are presented.

PREDICTOR-CORRECTOR INTERIOR-POINT ALGORITHM

In this section, we derive the predictor-corrector interior-point algorithm for the variational inequality problem $VIP(\boldsymbol{K}, \boldsymbol{F})$, where the mapping $\boldsymbol{F}: \boldsymbol{R}^n \rightarrow \boldsymbol{R}^n$ is assumed to be continuously differentiable and a monotone on $\boldsymbol{K}$. The mapping $\boldsymbol{F}$ is a monotone on $\boldsymbol{K}$, i.e., $(\boldsymbol{x} - \boldsymbol{y})^{\mathrm{T}}(\boldsymbol{F}(\boldsymbol{x}) - \boldsymbol{F}(\boldsymbol{y})) \geqslant 0$ for all $\boldsymbol{x}$, $\boldsymbol{y} \in \boldsymbol{K}$ if and only if $\nabla \boldsymbol{F}(\boldsymbol{x})$ is positive semidefinite on $\boldsymbol{K}$ (Ortega and Rheinboldt, 1970). It is easy to prove that the solution of $VIP(\boldsymbol{K}, \boldsymbol{F})$ is the same as that of the following constrained system of nonlinear equations:

$$\begin{cases} \boldsymbol{G}(\boldsymbol{z}) = \begin{pmatrix} \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{y} - \boldsymbol{u} \\ \boldsymbol{b} - \boldsymbol{Ax} - \boldsymbol{v} \\ \boldsymbol{XUe} \\ \boldsymbol{YVe} \end{pmatrix} = 0, \\ \boldsymbol{z} = (\boldsymbol{x}^{\mathrm{T}}, \boldsymbol{y}^{\mathrm{T}}, \boldsymbol{u}^{\mathrm{T}}, \boldsymbol{v}^{\mathrm{T}})^{\mathrm{T}} \geqslant \boldsymbol{0} \end{cases} \quad (1)$$

where $\boldsymbol{e}$ is the vector of all ones, $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{U}$, and $\boldsymbol{V}$ are diagonal matrices corresponding to $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{u}$ and $\boldsymbol{v}$, respectively, $\boldsymbol{u} \in \boldsymbol{R}^n$, $\boldsymbol{v} \in \boldsymbol{R}^m$ are auxiliary variables. For complementarity equation $\boldsymbol{XUe} = \boldsymbol{0}$, the Newton-type method deals with the linearized formula $\boldsymbol{X}\Delta\boldsymbol{u} + \boldsymbol{U}\Delta\boldsymbol{x} = -\boldsymbol{XUe}$ with a serious flaw. It forces the iterations to stick to the boundary of the feasible region once they approach that boundary. That is, if a component $[\boldsymbol{x}_k]_i$ of a current iteration becomes zero and $[\boldsymbol{u}_k]_i > 0$, then from the linearized formula, we see that $[\boldsymbol{x}_l]_i$ for all $l > k$; i.e., this component will remain zero in all future iterations. The analogous situation is also true for variables $\boldsymbol{u}$, $\boldsymbol{y}$, $\boldsymbol{v}$. Such an undesirable attribute clearly precludes the global convergence of the algorithm. An obvious correction is to modify the Newton formula so that zero variables can become nonzero in subsequent iterations. This can be accomplished by replacing the equation $\boldsymbol{XUe} = \boldsymbol{0}$ with the perturbed equation $\boldsymbol{XUe} = \boldsymbol{\mu e}$, $\mu > \boldsymbol{0}$. Of course, this is exactly the introduction of the notion of adherence to the central path. It is known that such adherence tends to keep the iterations away from the boundary and promote the global convergence of the Newton interior-point method.

Instead of Eq. (1), we consider the following constrained system of nonlinear equations:

$$\boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{y} - \boldsymbol{u} = 0, \boldsymbol{b} - \boldsymbol{Ax} - \boldsymbol{v} = \boldsymbol{0},$$
$$\boldsymbol{XUe} = \boldsymbol{\mu e}, \boldsymbol{YVe} = \boldsymbol{\mu e}, \ \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{v} \geqslant \boldsymbol{0} \quad (2)$$

Rather than applying the Newton method to Eq. (2) to generate correction terms to the current estimate, we substitute the new point $(\boldsymbol{x} + \Delta\boldsymbol{x}, \boldsymbol{y} + \Delta\boldsymbol{y}, \boldsymbol{u} + \Delta\boldsymbol{u}, \boldsymbol{v} + \Delta\boldsymbol{v})$ into Eq. (2) directly and obtain the equivalent system

$$\nabla \boldsymbol{F}(\boldsymbol{x})\Delta\boldsymbol{x} + \boldsymbol{A}^{\mathrm{T}}\Delta\boldsymbol{y} - \Delta\boldsymbol{u} = \boldsymbol{u} - \boldsymbol{A}^{\mathrm{T}}\boldsymbol{y} - \boldsymbol{F}(\boldsymbol{x} + \Delta\boldsymbol{x}) + \nabla \boldsymbol{F}(\boldsymbol{x})\Delta\boldsymbol{x},$$
$$-\boldsymbol{A}\Delta\boldsymbol{x} - \Delta\boldsymbol{v} = \boldsymbol{v} + \boldsymbol{Ax} - \boldsymbol{b},$$
$$\boldsymbol{X}\Delta\boldsymbol{u} + \boldsymbol{U}\Delta\boldsymbol{x} = \boldsymbol{\mu e} - \boldsymbol{Xu} - \Delta\boldsymbol{X}\Delta\boldsymbol{u},$$
$$\Delta\boldsymbol{v} + \boldsymbol{V}\Delta\boldsymbol{y} = \mu\boldsymbol{e} - \boldsymbol{Yv} - \Delta\boldsymbol{Y}\Delta\boldsymbol{v} \quad (3)$$

where $\Delta\boldsymbol{X}$ and $\Delta\boldsymbol{Y}$ are diagonal matrices corresponding to $\Delta\boldsymbol{x}$ and $\Delta\boldsymbol{y}$, respectively. To determine the correction approximately satisfying Eq. (3), we first solve the equations:

$$\nabla \boldsymbol{F}(\boldsymbol{x})\Delta\hat{\boldsymbol{x}} + \boldsymbol{A}^{\mathrm{T}}\Delta\hat{\boldsymbol{y}} - \Delta\hat{\boldsymbol{u}} = \boldsymbol{u} - \boldsymbol{A}^{\mathrm{T}}\boldsymbol{y} - \boldsymbol{F}(\boldsymbol{x}),$$
$$-\boldsymbol{A}\Delta\hat{\boldsymbol{x}} - \Delta\hat{\boldsymbol{v}} = \boldsymbol{v} + \boldsymbol{Ax} - \boldsymbol{b},$$
$$\boldsymbol{X}\Delta\hat{\boldsymbol{u}} + \boldsymbol{U}\Delta\hat{\boldsymbol{x}} = -\boldsymbol{Xu},$$
$$\boldsymbol{Y}\Delta\hat{\boldsymbol{v}} + \boldsymbol{V}\Delta\hat{\boldsymbol{y}} = -\boldsymbol{Yv}$$

for the affine direction $(\Delta\hat{\boldsymbol{x}}, \Delta\hat{\boldsymbol{y}}, \Delta\hat{\boldsymbol{u}}, \Delta\hat{\boldsymbol{v}})$, then use it to approximate the nonlinear terms on the right-hand side of Eq. (3) and estimate the centering parameter μ. To estimate μ, we first define

$$\alpha = \sigma \min\left\{\frac{\boldsymbol{x}_j}{-\Delta\hat{\boldsymbol{x}}_j} : \Delta\hat{\boldsymbol{x}}_j < 0; \ \frac{\boldsymbol{y}_j}{-\Delta\hat{\boldsymbol{y}}_j} : \Delta\hat{\boldsymbol{y}}_j < 0;\right.$$

$$\left.\frac{u_j}{-\Delta \hat{u}_j}:\Delta \hat{u}_j<0;\ \frac{v_j}{-\Delta \hat{v}_j},\Delta \hat{v}_j<0\right\} \quad (4)$$

where $\sigma \in (0,1)$, and compute the new complementarity gap

$\hat{g} = (x + \alpha\Delta\hat{x})^{\mathrm{T}}(u + \alpha\Delta\hat{u}) + (y + \alpha\Delta\hat{y})^{\mathrm{T}}(v + \alpha\Delta\hat{v})$, then choose the centering parameter μ by the following formula:

$$\mu = \begin{cases} \left(\dfrac{\hat{g}}{x^{\mathrm{T}}u + y^{\mathrm{T}}v}\right)^2\left(\dfrac{\hat{g}}{n}\right) & \text{if } |x^{\mathrm{T}}u + y^{\mathrm{T}}v| \geqslant 1 \\ \dfrac{x^{\mathrm{T}}u + y^{\mathrm{T}}v}{\varnothing(n)} & \text{if } |x^{\mathrm{T}}u + y^{\mathrm{T}}v| < 1 \end{cases},$$

$$\varnothing(n) = \begin{cases} n^2, & n \leqslant 5000, \\ n^{3/2}, & n > 5000. \end{cases} \quad (5)$$

It is easy to verify that, for the affine direction $(\Delta\hat{x}, \Delta\hat{y}, \Delta\hat{u}, \Delta\hat{v})$ and the centering parameter μ, the solution $(\Delta x, \Delta y, \Delta u, \Delta v)$ to the following equation satisfies Eq. (3) approximately,

$$\begin{aligned} &\nabla F(x)\Delta x + A^{\mathrm{T}}\Delta y - \Delta u = u - A^{\mathrm{T}}y - F(x + \Delta\hat{x}) + \nabla F(x)\Delta\hat{x}, \\ &-A\Delta x - \Delta v = v + Ax - b, \\ &X\Delta u + U\Delta x = \mu e - Xu - \Delta\hat{X}\Delta\hat{u}, \\ &Y\Delta v + V\Delta y = \mu e - Yv - \Delta\hat{Y}\Delta\hat{v} \end{aligned}$$

Finally, we determine the actual step length α from Eq. (4) using $\Delta x, \Delta y, \Delta u, \Delta v$ to replace $\Delta\hat{x}$, $\Delta\hat{y}$, $\Delta\hat{u}$, $\Delta\hat{v}$ and define the new point by $x_+ = x + \alpha\Delta x$, $y_+ = y + \alpha\Delta y$, $u_+ = u + \alpha\Delta u$, $v_+ = v + \alpha\Delta v$. The algorithm continues until $\|G(z)\| < \varepsilon$, where ε is a predetermined tolerance.

Now we summarize the proposed predictor-corrector interior-point algorithm for *VIP* (K, F). For the sake of simplicity, we let $\hat{e}$ denote the vector $(0, \cdots, 0, 1, \cdots, 1)$ whose numbers of zeros and ones are all $n + m$ and write $\Delta z = (\Delta z^{\mathrm{T}}, \Delta y^{\mathrm{T}}, \Delta u^{\mathrm{T}}, \Delta v^{\mathrm{T}})^{\mathrm{T}}$.

Algorithm 1 (Predictor-corrector interior-point method)

Given $z_0 > 0$, for $k = 0, 1, \cdots$,

1) Solve $\nabla G(z_k)\Delta z = -G(z_k)$ for Δz_p;

2) Solve $\nabla G(z_k)\Delta z = -$

$$\begin{pmatrix} F(x_k + \Delta x_p) - F(x_k) - \nabla F(x_k)\Delta x_p \\ 0 \\ \Delta X_p \Delta u_p \\ \Delta Y_p \Delta v_p \end{pmatrix} \text{ for } \Delta z_m;$$

3) Choose $\mu_k > 0$ and solve $\nabla G(z_k)\Delta z = \mu_k\hat{e}$ for Δz_c;

4) Set $\Delta z = \Delta z_p + \Delta z_m + \Delta z_c$;

5) Choose $\sigma_k \in (0,1)$ and set $\alpha_k = \min\{1, \sigma_k\hat{\alpha}_k\}$ where

$$\hat{\alpha}_k = \min\left\{\frac{x_j}{-\Delta x_j}: \Delta x_j < 0;\ \frac{y_j}{-\Delta y_j}: \Delta y_j < 0;\ \frac{u_j}{-\Delta u_j}: \Delta u_j < 0;\ \frac{v_j}{-\Delta v_j}: \Delta v_j < 0\right\};$$

6) Set $z_{k+1} = z_k + \alpha_k\Delta z$.

In this section we are not concerned with the specific choice of the initial iteration z_0 or the algorithmic parameter σ_k, we notice that Mehrotra' s choices allow us to obtain very impressive numerical results in the numerical experiments.

EQUIVALENCE

In this section we prove the equivalence between the predictor-corrector interior-point method and the level-1 perturbed composite Newton method. At first we present the perturbed composite Newton method for Eq. (1). Its idea is to replace the Newton component in the primal-dual interior-point algorithm with a composite Newton component.

Algorithm 2 (Level-*m* perturbed composite Newton interior-point method)

Given $z_0 > 0$, for $k = 0, 1, \cdots$,

1) Solve $\nabla G(z_k)\Delta z = -G(z_k)$ for Δz_0;

2) For $i = 1, \cdots, m$, solve $\nabla G(z_k)\Delta z = -G(z_k + \sum_{j=0}^{i-1}\Delta z_j)$ for Δz_i;

3) Choose $\mu_k > 0$ and solve $\nabla G(z_k)\Delta z = \mu_k\hat{e}$ for Δz_c;

4) Set $\Delta z = \sum_{i=0}^{m}\Delta z_j + \Delta z_c$;

5) Choose $\sigma_k \in (0,1)$ and set $\alpha_k = \min\{1, \sigma_k\hat{\alpha}_k\}$ where

$$\hat{\alpha}_k = \min\left\{\frac{x_j}{-\Delta x_j}: \Delta x_j < 0;\ \frac{y_j}{-\Delta y_j}: \Delta y_j < 0;\ \frac{u_j}{-\Delta u_j}: \Delta u_j < 0;\ \frac{v_j}{-\Delta v_j}: \Delta v_j < 0\right\}$$

6) Set $z_{k+1} = z_k + \alpha_k\Delta z$.

We say that two algorithms are equivalent if given a current iteration they produce the same subsequent iteration for the same choice of common algorithmic parameters.

Theorem 1 The predictor-corrector interior-point method (Algorithm 1) is equivalent to the level-1 perturbed composite Newton interior-point method (Algorithm 2).

Proof Let z be the current iteration and let Δz_p be the solution in predictor step for $VIP(\boldsymbol{K}, \boldsymbol{F})$. Comparing Algorithm 1 with Algorithm 2 ($m = 1$), we see that it is only needed to show that

$$\boldsymbol{G}(z + \Delta z_p) = \begin{pmatrix} \boldsymbol{F}(\boldsymbol{x}_k + \Delta \boldsymbol{x}_p) - \boldsymbol{F}(\boldsymbol{x}) - \nabla \boldsymbol{F}(\boldsymbol{x})\Delta \boldsymbol{x}_p \\ \boldsymbol{0} \\ \Delta \boldsymbol{X}_p \Delta \boldsymbol{u}_p \\ \Delta \boldsymbol{Y}_p \Delta \boldsymbol{v}_p \end{pmatrix}$$

By the assumption, Δz_p satisfies $\nabla \boldsymbol{G}(z_k) \Delta z_p = -\boldsymbol{G}(z_k)$. Simple algebra indicates that the above formula holds and we have established the equivalence.

COMPUTATIONAL RESULTS

In this section, we present the computational results in the implementation of the predictor-corrector interior-point method (Algorithm 1) for problem (VIP). Twenty-six standard test problems with linear constraints (Hock and Schittkowski, 1981) are solved. Because the nonlinear programming $\min_{x \in \boldsymbol{K}} f(\boldsymbol{x})$ is equivalent to $VIP(\boldsymbol{K}, \boldsymbol{F})$ with $\boldsymbol{F}(\boldsymbol{x}) = \nabla f(\boldsymbol{x})$, we reformulate the nonlinear programming problem as variational inequality problem. When a variable $[\boldsymbol{x}]_i$ is unconstrained or nonpositive constrained, we take the precaution of including the simple bounds: $[\boldsymbol{x}]_i \geqslant -100$ or $-100 \leqslant [\boldsymbol{x}]_i \leqslant 0$, respectively. In the experiments we had not verified whether the assumptions on $\boldsymbol{F}$ were valid, nevertheless, the runs on all the test problems terminated satisfactorily under the prescribed stopping rules.

The solution of system $\boldsymbol{G}(z)\Delta z = \boldsymbol{a}$ in steps 1, 2 and 3 was obtained in two steps: first solving $\boldsymbol{C}\Delta \boldsymbol{x} = \boldsymbol{d}$ for $\Delta \boldsymbol{x}$, where $\boldsymbol{a} = (\boldsymbol{a}_1^{\mathrm{T}}, \boldsymbol{a}_2^{\mathrm{T}}, \boldsymbol{a}_3^{\mathrm{T}}, \boldsymbol{a}_4^{\mathrm{T}})^{\mathrm{T}}$, $\boldsymbol{C} = \boldsymbol{X}^{-1}\boldsymbol{U} + \nabla \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{V}^{-1}\boldsymbol{Y}\boldsymbol{A}$, $\boldsymbol{d} = \boldsymbol{a}_1 + \boldsymbol{X}^{-1}\boldsymbol{a}_3 - \boldsymbol{A}^{\mathrm{T}}\boldsymbol{V}^{-1}(\boldsymbol{a}_4 + \boldsymbol{Y}\boldsymbol{a}_2)$then calculating $\Delta \boldsymbol{y}$, $\Delta \boldsymbol{u}$ and $\Delta \boldsymbol{v}$ from the expressions $\Delta \boldsymbol{v} = -\boldsymbol{A}\Delta \boldsymbol{x} - \boldsymbol{a}_2$, $\Delta \boldsymbol{u} = \boldsymbol{X}^{-1}(\boldsymbol{a}_3 - \boldsymbol{U}\Delta \boldsymbol{x})$ and $\Delta \boldsymbol{y} = \boldsymbol{V}^{-1}(\boldsymbol{a}_4 - \boldsymbol{Y}\Delta \boldsymbol{v})$. In the experiments of Algorithm 1, the initial iteration z_0 was set to be $10e$; the parameter μ_k was estimated according to Eq.(5) and $\sigma_0 = 0.5$, $\sigma_k = 1 - (1 - \sigma_{k-1})/2$, $k = 1, 2, \cdots$, the user-predetermined tolerance ε = 10e-6. The computer codes were written in Matlab 5.1. We ran the experiments on an AMD K6-2 computer. The results obtained are given in Table 1, where the number of variables n, the number of constraints m, and the

Table 1 Numerical results on Algorithm 1

Prob.	n	m	NI	f^*	NORM	Prob.	n	m	NI	f^*	NORM
HS1	2	1	6	0.00001	6.579E-8	HS2	2	1	7	0.05043	9.860E-7
HS3	2	1	7	0.00003	1.435E-7	HS4	2	1	5	2.66667	9.981E-8
HS5	2	2	6	−1.9132	1.302E-6	HS9	2	2	6	−0.5000	2.499E-6
HS21	2	3	7	−99.960	5.315E-7	HS28	3	2	7	0.00001	6.184E-6
HS35	3	1	7	0.11111	2.760E-7	HS36	3	4	8	−3300.0	3.685E-8
HS37	3	5	9	−3456.0	2.949E-7	HS38	4	4	7	0.00001	5.962E-8
HS41	4	6	7	1.92593	5.239E-6	HS44	4	6	8	−15.000	1.150E-6
HS45	5	5	7	0.99999	1.620E-7	HS48	5	4	9	0.00004	1.201E-7
HS49	5	4	7	0.00007	4.401E-6	HS50	5	6	7	0.00003	5.361E-6
HS51	5	6	8	0.00002	2.307E-6	HS52	5	6	7	5.32660	5.266E06
HS53	5	11	7	4.09300	7.787E-7	HS55	6	14	6	6.33333	1.852E-7
HS76	4	3	7	−4.6818	3.914E-6	HS86	5	10	11	−32.349	2.154E-6
HS110	10	10	8	−45.778	1.354E-6	HS118	15	44	14	664.820	8.398E-6

number of iterations NI, the final objective value f^* and the norm of $\boldsymbol{G}(\boldsymbol{z}^*)$ NORM achieved by the algorithm are given. From the structure of Algorithm 1, we know that the number of mapping $\boldsymbol{F}$ evaluations is double NI and that the number of Jacobian matrix $\nabla \boldsymbol{F}(\boldsymbol{x})$ evaluations is the same as NI.

We summarize the results of our numerical experimentation in the following comments:

1. The algorithm implemented does not require that the initial iteration be feasible and solves all the problems tested to the given tolerance.

2. The total number of iterations is small and the quadratic rate of convergence is observed in problems where second-order sufficiency is satisfied.

3. The proposed predictor-corrector interior-point algorithm is stable and robust for solving monotone variational inequality problems.

CONCLUSIONS

The level-1 perturbed composite Newton method for nonlinear equation has at least quadratic convergent rate under some conditions. The predictor-corrector interior-point method represents a significant computation advance and is the most computationally efficient method for large-scale linear programming. Motivated by the two methods, we have proposed a predictor-corrector interior-point algorithm for the monotone variational inequality problem on a polyhedral set in this paper. It is proved that the algorithm is equivalent to a level-1 perturbed composite Newton method. Computations in the algorithm do not require any feasibility of the initial iteration. Some computational results showed that the algorithm is stable and robust for solving monotone variational inequality problems. However, the convergent rate, convergence and the choice of parameters of the algorithm must be studied further.

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