Foundation of three-dimensional mathematical models for glass furnace regenerator

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Received Sept.8，2001；revision accepted Feb.27，2002

Abstract: This paper presents a practical three-dimensional mathematical model of circulation and heat transfer in generator of glass melting furnaces. The model was based on the heat transfer between the smoke flow and the lattice units，and between the air flow and the lattice units. This model not only bypassed the difficulty of complicated computation of the heat transfer process in the regenerator of glass furnaces，but also avoided the irrationality of fixing the temperature distribution on the surfaces.

Use of the model yielded very important data and also the method for the design of the regenerator of glass furnaces in practical production.

Key words: Regenerator，Mathematical model，Gas flow，Temperature field
Document code: A **CLC number:**

INTRODUCTION

In recent years，mathematical simulation modeling has gained much attention because of its accuracy and general applicability and has been widely used in the glass melting industry to calculate the temperature field of the regenerator，etc. (Hu，1995；Sun et al.，1988；Wu et al.，1987). As for the gas field，the physical model is usually adopted(Qu，1990；1994). There is no serious research completed on the gas flow and temperature field of the regenerator up to now to simulate well the performance of the real regenerator. So we will present a reasonable method for estabilishing a three-dimensional mathematical model for the gas flow and temperature field. It must first be assumed that the regenerator gas is distributed homogeneously or distributed according to certain governing rules for each horizontal section. We are usually limited by the model and the medium materials when the gas flow field is set up by a physical model，which results in imperfect simulation for the regenerator.

After the construction of the regenerator，a three-dimensional mathematical model of the gas flow field in the regenerator was established to research the gas flow field. Another three-dimensional model of temperature field was then established.

MODELING FOR GAS FLOW FIELD IN REGENERATOR

In the regenerator，cheek bricks are regularly piled into cheek bodies to serve as the heat transfer medium. The smoke and air，with different period，run through the holes among the cheek bodies and exchange heat with the cheek bricks. In each period of time，the temperature in the regenerator changes rapidly in unsteady state. On the other hand，the changes of the smoke distribution and of the combustion-supporting air are very small except during their reversing direction，and so can be considered as steady state. To do the general fluid calculation by the boundary layer and wind-flow theory，the shape of every cheek brick need to be divided into meticulous grids and it is difficult for a microcomputer to calculate enormous arrays.

To set up this paper's model，we regarded the cheek bricks as resistance regularly distributed in the cheek bodies and opposite in direction to that of the gas flow in the three-dimensional directions. We considered the existence of cheek bricks by using the factor of porosity in the mathematical equations (Fan et al, 1992). Thus the calculation area can be taken as single gas phase spheres dividable evenly into grids without considering the real geometric shape of each cheek brick. The model disregards the gas leakage and the chemical reaction between gas and cheek bodies. To simplify the research, the assumptions for the gas flow in the regenerator are:

1. The gas flows steadily in the regenerator, i.e. the gas flow and temperature at inlet and outlet are stable. The transient gas flow field formed during gas reversing-direction may be disregarded.

2. The cheek bricks can be regarded as the resistance whose value is determined by empirical resistance coefficients and the gas velocities.

3. The empirical and evaluated temperature field can be used to calculate the gas heat physical parameters. The gas temperatures are homogeneous both in the upper and bottom part of the regenerator, while the gas temperatures in the cheek body area are linearly distributed.

4. The gas physical parameters are given by empirical equations or real-time measurement.

5. The gas is evenly distributed in the single calculating grid.

6. The gas flow is viscous and turbulent.

Thus, the mathematical model can be described by the following equations.

Continuity equation:

$$\frac{\partial}{\partial x}(\rho u P_1)+\frac{\partial}{\partial y}(\rho v P_2)+\frac{\partial}{\partial z}(\rho w P_3)=0 \tag{1}$$

where P_1, P_2, P_3 are the area porosity in three directions. In the Siemens cheek body area, they are 0.559, 0.559 and 0.717 respectively. In the upper and bottom pat of the regenerator, they are all 1.0.

Momentum conservation equation:

$$\begin{aligned}
\frac{\partial}{\partial x_j}(\rho v_j u P_j) &= \frac{\partial}{\partial x_j}\left(\mu_e \frac{\partial u}{\partial x_j} P_j\right)+ \\
&\left(-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x_j}\left(\mu_e \frac{\partial v_j}{\partial x}\right)+S_{f,u}\right)P_{\sigma v} \\
\frac{\partial}{\partial x_j}(\rho v_j v P_j) &= \frac{\partial}{\partial x_j}\left(\mu_e \frac{\partial v}{\partial x_j} P_j\right)+ \\
&\left(-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x_j}\left(\mu_e \frac{\partial v_j}{\partial x}\right)+S_{f,v}\right)P_{\sigma v} \\
\frac{\partial}{\partial x_j}(\rho v_j w P_j) &= \frac{\partial}{\partial x_j}\left(\mu_e \frac{\partial w}{\partial x_j} P_j\right)+ \\
&\left(-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x_j}\left(\mu_e \frac{\partial v_j}{\partial x}\right)+S_{f,w}\right)P_{\sigma v}
\end{aligned} \tag{2}$$

where j is the subscript 1, 2 or 3. V_1, V_2, V_3 are the velocity components of u, v, w and x_1, x_2, x_3 are the coordinate components of x, y and z. μ_e is effective turbulent viscosity coefficient.

$$\mu_e=\mu+\mu_t, \quad \mu_t=0.09\rho k^2/\varepsilon$$

μ is layer flow viscosity coefficient. $P_{\sigma v}$ is volume porosity. $S_{f,v}$, $S_{f,u}$, $S_{f,w}$ are the source terms which include the resistance.

Turbulent k-ε equation with buoyancy correction:

$$\begin{aligned}
\frac{\partial(\rho v_j K P_j)}{\partial x_j} &= \frac{\partial}{\partial x_j}\left(\frac{\mu_e}{\sigma_k}\frac{\partial k}{\partial x_j}P_j\right)+S_k P_{\sigma v} \\
\frac{\partial(\rho v_j \varepsilon P_j)}{\partial x_j} &= \frac{\partial}{\partial x_j}\left(\frac{\mu_e}{\sigma_k}\frac{\partial \varepsilon}{\partial x_j}P_j\right)+S_\varepsilon P_{\sigma v}
\end{aligned} \tag{3}$$

Where k is turbulent kinetic energy, ε is turbulent kinetic energy dissipation rate, δ_k is the effective P_r of k, δ_ε is the effective P_r of ε, Sk and S_ε are buoyancy correction equation source terms.

Compared with the general fluid flowing calculation equation, porosity terms as coefficients are put in all these equations and the resistance terms are added into the source terms of the momentum equation.

Fig. 1 can be used to explain the necessity and reasonability of the additional porosity. In the figure, the dotted frame is a side of a three-dimensional control body, the shade is the area of cheek bricks and the blank is the area where gas flows. The value of area 7 porosity is the ratio of blank area to dotted frame area, Consider the continuity equation in which porosity is excluded:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \tag{4}$$

Fig. 1 Calculation schematic

where u, v, w represent only the gas flow velocities through all the sides of a control body, not the velocities through the holes among cheek bricks. In the calculating grids of the cheek body area, the gas flow areas of all sides are different. It is erroneous to calculate by Equation (4) as it is. So porosity was also added into the other equations. Volume porosity, referring to gas flow area in unit grid, plus the filling ratio of cheek brick in unit volume of cheek body makes up 1.

The cheek body resists the gas flow, which can be considered by viscosity coefficients of gas or, in this paper, by adding a resistance term into the momentum source equation.

The resistance forces in three directions of the cheek body are different and different kinds of cheek bodies have different resistance forces. An example that uses not-overlapping Siemens cheek body is shown below.

Being similar to the condition that gas flows across the batch pile, the velocity of the cheek body gas in horizontal direction is very low. From the empirical data, we assume that the dynamic head decreases to 1 Pa when the gas flows 1 meter. In the perpendicular direction, the gas flow velocity is high and the influence of it on the resistance is great. So the gas flow resistance coefficient in perpendicular direction is:

$$\xi = 1.14H/d_e^{0.25} \qquad (5)$$

where ξ is synthetic resistance coefficient, H is the height(m) of cheek body, d_e is the equivalent diameter(m) of the cheek hole. The gas flow with velocity w, flowing through the cheek body with height dH and equivalent hole diameter d_e, will overcome the following resistance

$$f = \frac{1.14}{2} w^2 \rho \mathrm{d}H/d_e^{0.25} \qquad (6)$$

where ρ is the average density of gas flow through the height dH.

The convection heat transfer includes buoyancy existing in the flowing system with uneven fluid density usually caused by difference of temperatures or ingredients. The buoyancy correction source term must be used in momentum equation and turbulent k-e equation.

MODELING FOR TEMPERATURE FIELD IN REGENERATOR

Before modeling, we assume that:

1. The influence of the remaining gas from the last period in the regenerator can be ignored.

2. The volume of flow and the temperature of smoke and combustion-supporting gas are constant. The gas temperatures in the upper and lower part of the regenerator distribute homogeneously.

3. The mass flow of gas at every section of the regenerator remains constant. The velocity is determined by both mass flow and temperature.

4. In the cheek body area, the temperature calculation grids are divided, where both the gas velocity and temperature are even. The gas which flows through a given grid transfers the heat only with the cheek bricks in this grid. Only the outermost grids transfer heat with the outer environment.

5. In the perpendicular direction, the temperature of the gas which flows out from the preceding grid is equal to the temperature of that gas which flows into the next grid.

6. The outer wall temperature of regenerator distributes linearly along the height direction and keeps steady.

7. The convection, conduction and radiation are combined into an integrated heat transfer coefficient. Based on the theory of thermal balance, when smoke flows through the grid with height dH, we have the following equation:

Thermal decrement of smoke = heat transfer of smoke to cheek bricks + dissipation heat to external environment, i.e.:

$$W_{gy} C_y \Delta T_y \mathrm{d}\theta = \alpha_{y-s} A (T_y - t_s) \mathrm{d}\theta + Q_s \mathrm{d}\theta \qquad (7)$$

where W_{gy} is the mass flow of smoke(kg/s), C_y is the capacity of smoke which can be calculated by empirical equation (J/kg·℃), ΔT_y is the temperature change of smoke when it flows through this grid, α_{y-s} is the integrated coefficient of heat transfer between smoke and cheek bricks(J/m^2·℃·s), A is the heat transfer area of the grid(m^2), T_y is the smoke average temperature of the grid(℃), t_s is the cheek brick average temperature of the grid(℃), Q_s is the dissipation heat quantity of the grid during unit time(J/s).

The heat transfer between smoke and cheek bricks = the heat-absorbing of check bricks when their temperature rises, i.e.:

$$\alpha_{y-s}A(T_y - t_s)\mathrm{d}\theta = M_sC_s\frac{\partial t_s}{\partial\theta}\mathrm{d}\theta \qquad (8)$$

where M_s is the mass of cheek bricks in the grid (kg), C_s is the heat absorbing capacity of cheek bricks(J/kg·℃).

During the passing combustion-supporting air circulating, when the air flows through the grid with height $\mathrm{d}H$, we have the following equations:

1. Thermal increment of combustion-supporting air = its heat transfer to cheek bricks - the dissipation of heat to external environment, i.e.:

$$W_{ga}C_a\Delta T_a\mathrm{d}\theta = \alpha_{a-s}A(T_a - t_s)\mathrm{d}\theta + Q_s\mathrm{d}\theta \qquad (9)$$

where W_{ga} is the mass flow of combustion-supporting air(kg/s), C_a is the air's combustion-supporting capacity calculatble by empirical equation(J/kg.℃), ΔT_a is the temperature change of the combustion-supporting air flowing through this grid, α_{a-s} is the integrated coefficient of heat transfer between combustion-supporting air and cheek bricks(J/m^2·℃·S), T_a is the average temperature of combustion-supporting air in this grid(℃).

2. The heat transfer between combustion-supporting air and cheek bricks = the heat released by the cheek bricks with the temperature decreasing, i.e.:

$$\alpha_{a-s}A(T_a - t_s)\mathrm{d}\theta = M_sC_s\frac{\partial t_s}{\partial\theta}\mathrm{d}\theta \qquad (10)$$

The regenerator's temperature field can be calculated by the above four heat balance equations.

CONCLUSIONS

The real conditions of the regenerator cheek bodies are fully studied in the above mathematical model of regenerator gas flow field. The way of modeling is similar to the analysis method of combustion bed. The porosity coefficient and the source terms are corrected by adding resistance and buoyancy into the relevant equations for calculating the three dimensional mathematical distribution of the gas flowing in the regenerator. Three dimensional gas distribution data can be used in establishing the three-dimensional temperature field model for simulation of three dimensional heat transfer.

References

Fan W.H., Wan, Y.P., 1992. The Model and Calculation of Fluid Flow and Combustion. Publishing House of Chinese Science and Technology University, Hefei (in Chinese).

Hu, C. 1995. Computer simulation for raising regenerator efficiency. *Glass*, **22**(120): 1 (in Chinese).

Qu, Z.Y., 1994. The influence of bottom construction of regenerator on gas flow distrubution. *Glass*, **21**(112): 8 (in Chinese).

Qu, Z.Y., 1990. Study on gas flow distribution of siemens cheek body regenerator. *Glass*, **17**(88): 1 (in Chinese).

Sun, C.X., Le, J., 1988. Study on heat transfer mathematical simulation method of regenerator. *Journal of Silicate*, **16**: 206 (in Chinese).

Wu, Y.G., Sun, C.X., 1987. Computer simulation and application of glass furnace regenerator. *Glass*, **14**(72): 1(in Chinese).