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Transient response of piles-bridge under horizontal excitation*

ZHU Bin(朱 斌)†, CHEN Ren-peng(陈仁朋), CHEN Yun-min(陈云敏)

(*Geotechnical Engineering Institute, Zhejiang University, Hangzhou 310027, China*)

†E-mail: zhubin@ccea.zju.edu.cn
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Abstract: Moving ships and other objects drifting on water often impact a bridge's pile foundations. The mechanical model of the piles-bridge structure under horizontal forcing was established, and a time-domain approach based on Finite-difference Method was developed for analyzing the dynamic response of the piles-bridge structure. For a single pile, good agreement between two computed results validated the present approach. The slenderness ratio of the pile, the pile-soil stiffness ratio and the type of the structure influence the dynamic response of the piles-bridge structure. The computed results showed that the stiffness of the structure determines the dynamic response of the piles-bridge structure under horizontal forcing.

Key words: Piles-bridge, Horizontal excitation, Dynamic response
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INTRODUCTION

The impact on piles-bridge structures by moving ships or other drifting objects often destroys the bridge, so it is important to study the transient dynamic characteristics of the bridge under horizontal forcing. Commonly, dynamic analysis of the piles-bridge structure must consider not only the pile-soil interaction but also the interaction of the structure's individual parts, so the analysis is complicated, and it is not rational that the present bridge design code (HL-DICM, 1989) simplifies the problem as a static one.

The pile-soil interaction is complicated. There are many studies on the transverse vibration of the pile foundation under seismic load, or harmonic load (Flores-Berrones et al., 1982; Markris, 1994; Han, 1989). If the latter external load is transient, the problem was often solved by Transfer Matrix Method (Nogami et al., 1988), Finite Element Method (Zhao et al., 1997), Finite-difference Method (Chen et al., 2001) and Boundary Element Method (Mamoon et al., 1992; Cheung et al., 1992). Many studies had been carried out to analyze the dynamic response of a bridge under seismic excitation (Mylonakis et al., 1992). The structure analysis programs can even strictly analyze the complex behavior of intricate structures, but they are time and labor consuming, and can't simulate the pile-soil interaction very well. Nogami and Konagai (1988) developed an approximate approach based on the plane strain wave propagation model (Nogami et al., 1986; Novak et al., 1978) to analyze the transverse vibration of the single pile with time-domain transfer matrices. The model approach is suitable for analyzing the dynamic response of the pile under different kinds of forcings, but it can't be used to analyze complicated structures. In this paper, a simple approach based on Nogami's pile-soil interaction model is provided to analyze the dynamic response of the piles-bridge structure under horizontal transient excitation.

MECHANICAL MODEL

1. Impact load

A moving mass M with speed V moves towards a piles-bridge structure. When the colli-

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sion takes place, the piles-bridge structure begins to deform under the impact force $P(t)$. It is assumed that the speed of the object reaches zero after the collision. The interaction between the pile and water is neglected. According to the Theorem of Momentum, the impulse of the moving object is expressed as

$$MV = \int_0^{t_0} p(t)\mathrm{d}t \tag{1}$$

where $P(t)$ is the impact force, and t_0 is the duration of the impact force (the design code of China stipulates that $t_0 = 1\mathrm{s}$ if there is no information from practical test).

Assuming that the impact force has half-sine form as shown in Fig. 1a, then it can be expressed as

$$P(t) = P_{\max}\sin(\omega t) \quad 0 \leqslant t < t_0, \ \omega = \pi/t_0 \tag{2}$$

Substituting Eq. 2 into Eq. 1, yields the maximum impact force

$$P_{\max} = \pi MV/(2t_0) \tag{3}$$

2. Model of pile-soil interaction

The analysis model of the piles-bridge structure is shown in Fig. 1b. N vertical piles with uniform cross section are connected by a rigid top beam at the pile top and a rigid coupling beam above the ground level. The structure deforms elastically under the impact force. Nogami (1988) adopted a soil model with lumped dashpots and springs, developed with the theory of plane strain wave propagation, to evaluate the transverse vibration of single piles under horizontal forcing. The soil model is depicted in Fig. 1c, and the coefficients of damping and stiffness of springs are expressed as

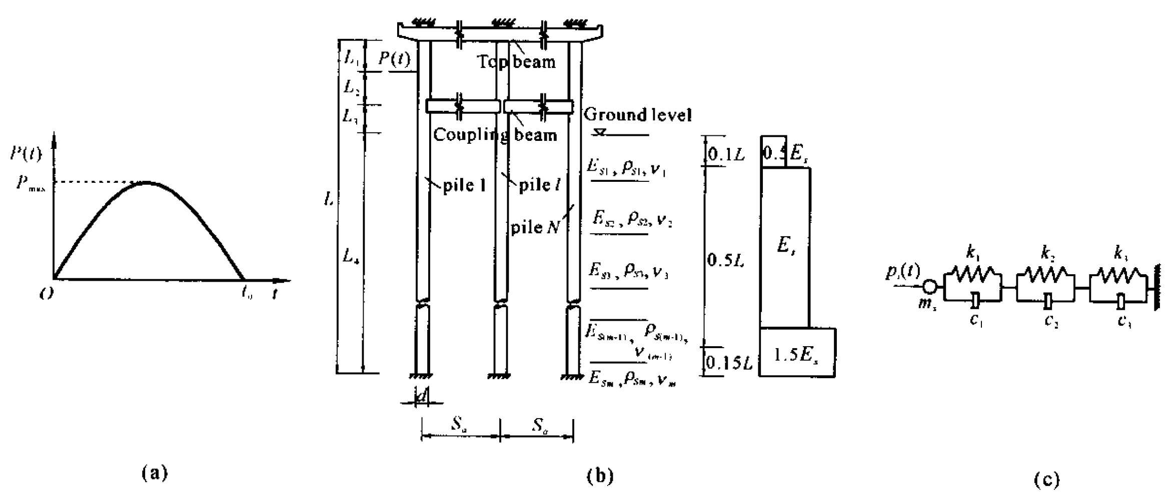

Fig. 1 Analysis model of the piles-bridge
(a) time history of the impact force; (b) model of the piles-bridge structure; (c) Winkler soil model for lateral pile shaft response)

$$k_n = \xi_k(\upsilon)G_s\begin{cases}3.518, & n = 1\\ 3.581, & n = 2\\ 5.529, & n = 3\end{cases} \tag{4a}$$

$$c_n = \xi_k(\upsilon)\frac{G_s r_0}{V_s}\begin{cases}113.097, & n = 1\\ 25.133, & n = 2\\ 9.362, & n = 3\end{cases} \tag{4b}$$

$$m_s = \zeta_m(\upsilon)\rho_s\pi r_0^2 \tag{4c}$$

where n is the number of springs and dashpots. ξ_k and ξ_m are coefficients depending only on Poisson's ratio of soil. m_s is the equivalent mass of the soil element and r_0 is the radius of the pile. v, ρ_s, G_s and V_s are the Poisson's ratio, density, shear modulus and shear wave velocity of the soil, respectively. Assuming the time interval as s, the interaction force p_i at t_i ($t_i = s \cdot i$) is

$$p_i = q_i + m_s\left(\frac{\partial^2 y}{\partial t^2}\right)_i \quad \left(i = 0 - \frac{t_0}{s}\right) \tag{5}$$

where $q_i = ky_i + d_i$, y_i is the flexural displacement of the pile shaft. The expressions of k and d_i are given by Nogami (1988) as

$$k = \left[\sum_{n=1}^{3} I_n(s) \right]^{-1} \quad (6)$$

$$d_i = -k \sum_{n=1}^{3} w_{i-1,n} e^{-\delta_n s} - kp_{i-1} \sum_{n=1}^{3} H_n(s) \quad (7)$$

where $\delta_n = \frac{k_n}{c_n}$, $H_n(s) = \frac{1}{k_n}$.
$\left[\frac{1}{\delta_n s} e^{\delta_n s} - \left(1 + \frac{1}{\delta_n s}\right) \right] e^{-\delta_n t_i}$, $I_n(s) = \frac{1}{k_n}$.
$\left[\left(1 - \frac{1}{\delta_n s}\right) e^{\delta_n s} + \frac{1}{\delta_n s} \right] e^{-\delta_n t_i}$, $w_{i,n} = w_{i-1,n} e^{-\delta_n s} + H_n(s) p_{i-1} + I_n(s) p_i$.

TRANSIENT RESPONSE OF A SINGLE PILE

Assuming the single pile as an Euler-Bernoulli beam attached on a Winkler soil medium, then its flexural vibration equation at t_i is (Clough et al., 1975)

$$E_p I \frac{\partial^4 y}{\partial x^4} + N_0 \frac{\partial^2 y}{\partial x^2} + m_p \frac{\partial^2 y}{\partial t^2} + p_i = 0 \quad (8)$$

where $E_p I$ is the bending stiffness of the pile shaft, m_p is the mass per unit length of the pile and N_0 is the vertical static force applied on the pile top. As for the pile segment above the ground surface, $p_i = 0$. The partial differential components $\left(\frac{\partial y}{\partial t}\right)_i$ and $\left(\frac{\partial^2 y}{\partial t^2}\right)_i$ can be discretized by the Newmark Beta method (Clough et al., 1975) as follows

$$\left(\frac{\partial^2 y}{\partial t^2}\right)_i = \frac{1}{\alpha \cdot s^2} \cdot y_i - \frac{1}{\alpha \cdot s^2} \cdot y_{i-1} - \frac{1}{\alpha \cdot s} \cdot \left(\frac{\partial y}{\partial t}\right)_{i-1} - \left(\frac{1}{2\alpha} - 1\right) \cdot \left(\frac{\partial^2 y}{\partial t^2}\right)_{i-1} \quad (9)$$

$$\left(\frac{\partial y}{\partial t}\right)_i = \left(1 - \frac{\delta}{\alpha}\right) \cdot \left(\frac{\partial y}{\partial t}\right)_{i-1} + \left(1 - \frac{\delta}{2\alpha}\right) \cdot s \cdot \left(\frac{\partial^2 y}{\partial t^2}\right)_{i-1} \quad (10)$$

where $\alpha = 0.25$ and $\delta = 0.5$. Substituting Eq. (5) into Eq. (8) and discretizing the partial differential components $\frac{\partial^2 y}{\partial x^2}$ and $\frac{\partial^4 y}{\partial x^4}$ by the center-difference method, Eq. 8 can be rewritten as follows

$$Ay_{i,j-2} + By_{i,j-1} + C_i y_{i,j} + By_{i,j+1} + Ay_{i,j+1} = \gamma_{i,j} \quad \left(i = 0 - \frac{t_0}{s},\ j = 0 - m\right) \quad (11)$$

where $A = \frac{E_p I}{h^4}$, $B = -\frac{4E_p I}{h^4} + \frac{N_0}{h_2}$, $C_i = \frac{6E_p I}{h^4} + k - \frac{2N_0}{h^2} + (m_p + m_s) \cdot \frac{1}{\alpha \cdot s^2}$, $\gamma_{i,j} = -d_i + (m_p + m_s)\left[\frac{1}{\alpha \cdot s^2} y_{i-1,j} - \left(1 - \frac{\delta}{\alpha}\right) \cdot \left(\frac{\partial y}{\partial t}\right)_{i-1,j} - \left(1 - \frac{\delta}{2\alpha}\right) \cdot s \cdot \left(\frac{\partial^2 y}{\partial t^2}\right)_{i-1,j} \right]$, h is the step length of difference and m is the number of pile segments. If the pile segment is above the ground surface, $k = 0$, $m_s = 0$ and $d_i = 0$ in Eq. (11).

Combining Eq. (9) with the boundary conditions at the pile top and the pile bottom, the following matrix equation can be formed

$$[K]\{Y\} = \{M\} \quad (12)$$

where $[K]$ is the coefficient matrix, $\{Y\}$ is the matrix of displacements at the center of every pile segment, $\{M\}$ is the constant matrix. Thus the transverse dynamic response of all pile segments of the single pile can be obtained. Fig. 2 describes the displacement response at the top of a rod fixed on the ground with a horizontal transient load applied at its top. Comparison with the results obtained by the ANSYS program of FEM (Finite Element Method) and the closed-form solution (Zhang et al., 1991) showed good agreement among the three computed results.

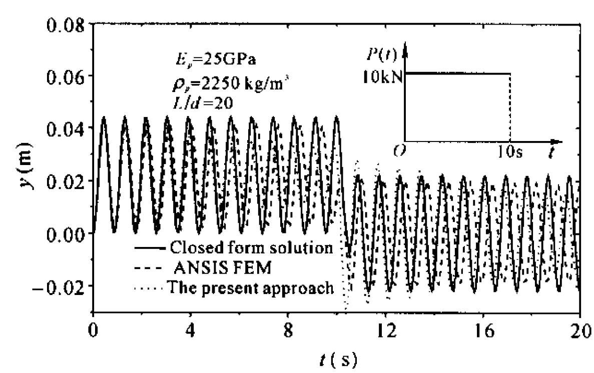

Fig. 2 Displacement response of the pile top

DYNAMIC RESPONSE OF THE PILES-BRIDGE STRUCTURE

The pile 1 shown in Fig. 1 is discretized in Fig. 3. For the rigidity of the top beam, the boundary conditions at the pile top of the piles-bridge structure are: Fig.3a there is no rotation, Fig.3b the horizontal displacement of each pile is identical, Fig.3c the total shear force of piles is equal to the inertial force of the top beam. The boundary conditions at the pile top are expressed as

$$\left(\frac{\partial y}{\partial x}\right)_{i,0,1} = \cdots = \left(\frac{\partial y}{\partial x}\right)_{i,0,l} = \cdots = \left(\frac{\partial y}{\partial x}\right)_{i,0,N} = 0 \quad \left(i = 0 \sim \frac{t_0}{s},\ l = 2 \sim N-1\right) \tag{13a}$$

$$y_{i,0,1} = \cdots = y_{i,0,l} = \cdots = y_{i,0,N} \quad \left(i = 0 \sim \frac{t_0}{s},\ l = 2 \sim N-1\right) \tag{13b}$$

$$\sum_{l=1}^{N} V_{i,0,l} + M_1\left(\frac{\partial^2 y}{\partial t^2}\right)_{i,0,1} = 0 \quad \left(i = 0 \sim \frac{t_0}{s}\right) \tag{13c}$$

where $y_{i,0,l}$, $V_{i,0,l}$ are the displacement and the shear at pile top of the pile l at the ith time step, respectively and M_1 is the mass of the top beam.

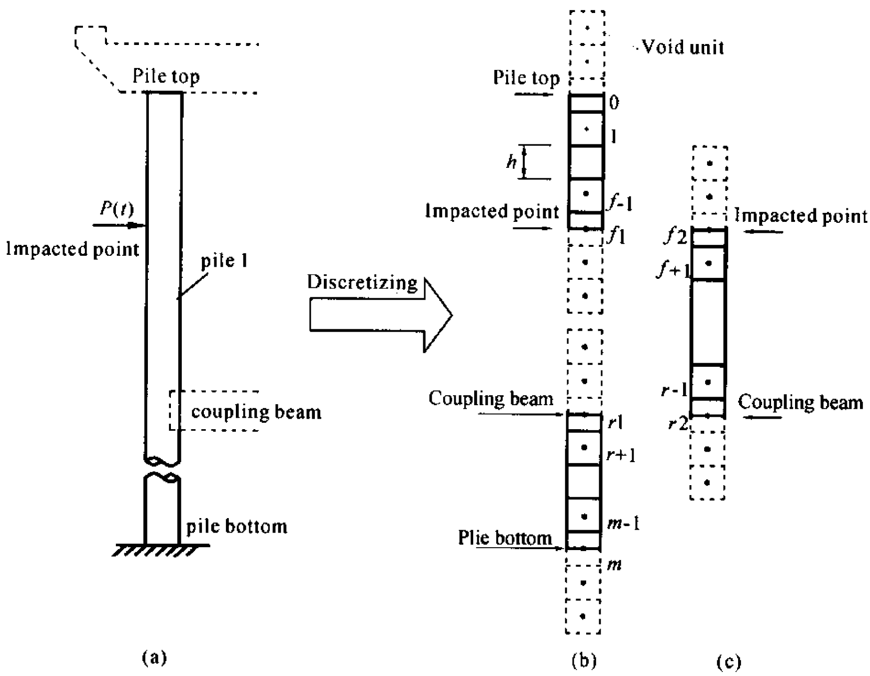

Fig.3 Discretizing model of pile 1

(a) model of pile 1 before discretizing; (b) pile segments above the impacted point and under the coupling beam; (c) pile segments between the impacted point and the coupling beam

For the fixed-end of the piles, the boundary conditions can be expressed as

$$y_{i,m,1} = \cdots = y_{i,m,l} = \cdots = y_{i,m,N} = 0 \quad \left(i = 0 \sim \frac{t_0}{s},\ l = 2 \sim N-1\right) \tag{14a}$$

$$\left(\frac{\partial y}{\partial x}\right)_{i,m,1} = \cdots = \left(\frac{\partial y}{\partial x}\right)_{i,m,l} = \cdots = \left(\frac{\partial y}{\partial x}\right)_{i,m,N} = 0 \quad \left(i = 0 \sim \frac{t_0}{s},\ l = 2 \sim N-1\right) \tag{14b}$$

where $y_{i,m,l}$ is the displacement at pile top of the pile; at the i th time step.

Assume that the impacting force is applied at the center of the fth pile segment of pile 1, and the coupling beam is at the rth pile segment. The fth pile segment is divided into the $f1$ pile segment and the $f2$ pile segment, and the rth pile segment is divided into the $r1$ pile segment and the $r2$ pile segment. The balance conditions of the force are written as

$$\begin{cases} M_{i,f1,1} + M_{i,f2,1} = 0 \\ V_{i,f1,1} + V_{i,f2,1} = P(t) \end{cases}$$

and

$$\sum_{i=1}^{N}(V_{i,r1,l} + V_{i,r2,l}) + M_r\left(\frac{\partial^2 y}{\partial t^2}\right)_{i,r,1} = 0.$$

The relevant conditions of the displacement are

$$\begin{cases} y_{i,f1,1} = y_{i,f2,1} \\ \left(\frac{\partial y}{\partial x}\right)_{i,f1,1} = \left(\frac{\partial y}{\partial x}\right)_{i,f2,1} \end{cases}$$

and

$$\begin{cases} y_{i,r1,1} = y_{i,r2,1} = \cdots = y_{i,r1,N} = y_{i,r2,N} \\ \left(\frac{\partial y}{\partial x}\right)_{i,r1,1} = \left(\frac{\partial y}{\partial x}\right)_{i,r2,1} = 0 \end{cases}$$

where $M_{i,j,l}$ is the moment at the center of jth pile segment of pile l at the ith time step, M_r is the mass of the coupling beam.

After substituting all expressions of moments and shears with center-difference expressions and the bending stiffness of the pile, according to Eq.9, the following matrix equation can be obtained

$$[K'] \{Y'\} = \{M'\} \quad (15)$$

where $[K']$ is the coefficient matrix, $\{Y'\}$ is the matrix of displacements at the center of every pile segment, $\{M'\}$ is the constant matrix. Thus, the transverse dynamic response of all pile segments of the piles-bridge structure can be obtained.

CASE STUDY

The computing parameters are: the Young's modulus of piles $E_p = 2.5 \times 10^7$ kN/m^2, the Young's modulus of the soil $E_s = 4000$ kN/m^2 (the pile-soil stiffness ratio $E_p/E_s = 6250$), the density of the pile material $\rho_p = 2500$ kg/m^3, the density of the soil material $\rho_s = 1700$ kg/m^3 ($\rho_p/\rho_s = 1.47$), the length of piles $L = 20$ m, the diameter of piles $d = 1$ m (the slenderness ratio of the pile $L/d = 20$), the distance between two piles near to each other $S_a = 6$ m, the soil Poisson's ratio $\upsilon = 0.35$. The soil layers are shown in Fig. 1b, the cross section of the top beam is 1.2 m × 1.3 m, the cross section of the coupling beam is 0.7 m × 1 m, the mass of the ship $M = 37\,500$ kg, and its speed $V = 10$ m/s, the duration of the impact force $t_0 = 1$ s, the vertical static force applied on the pile top $N_0 = 100$ kN. $L/L_1 = 10$, $L/L_2 = 10$, $L/L_3 = 20$. We assume that the transient load is applied on the top of the single pile, the pile is embedded in soil completely, the pile top is free and the pile bottom is fixed. The displacement response of the pile top is shown in Fig. 4. The good agreement between the results obtained by the present approach and the computing results of Nogami & Konagai (1988)'s approach validated the present approach.

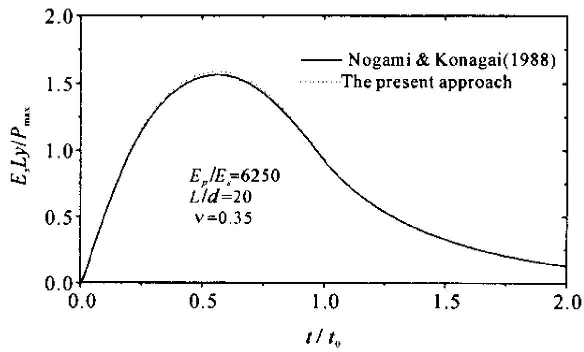

Fig.4 Two methods' comparison of the displacement response of the pile top

Three kinds of structures' displacement response at the impacted point is shown in Fig. 5. The three kinds of structures are respectively as follows: the first kind of piles-bridge structure (structure A) was composed of two piles and the top beam; the second kind of piles-bridge structure (structure B) was composed of three piles and the top beam; the third kind of piles-bridge structure (structure C) was composed of three piles, the top beam and the coupling beam. The computing results showed that the more complicated the structure was, the smaller its displacement response was.

Further analysis of structure B is given below. Fig.6 on the influence of the pile-soil stiffness ratio and the slenderness ratio of the pile on

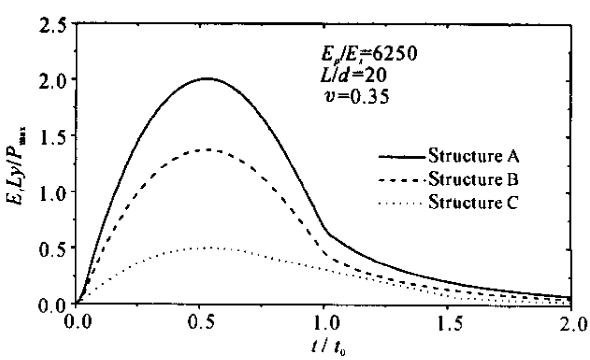

Fig.5 Comparison of the displacement response of three kinds of structures

the displacement response at the impacted point of structure B shows that the influence of the slenderness ratio of the pile is greater than that of the pile-soil stiffness ratio. The transverse displacement response and the moment response along the pile 1's shaft at different time are shown in Fig.7. Since the Euler-Bernoulli beam acts as a dispersive medium with respect to the flexural wave propagation, the disturbances (Fig. 7) are not as pronounced as propagating along the pile shaft. Fig.7 shows that the internal force at the impacted point and a certain depth under the ground surface is larger than the internal force at other places of the pile, so more attention must be paid to these places during designing. Compared with the computing results of the method specified in the design code (HLDI-CM, 1989), the displacement response and the moment response along the pile shafts at $t = 0.5\ t_0$ are shown in Fig. 8. The Criterion Method is actually a draft static method in which the dynamic stiffness of the soil is replaced by the static stiffness, the viscous damping of the soil and the dynamic inertia force of the structure are neglected, and so, it can't reflect the dynamic characteristics of the structure under horizontal transient forcing. In this paper, the static stiffness of the soil is expressed as follows (Bardenm 1963a; 1963b) during computing by the method specified in the design code

$$K = \frac{0.65 E_s}{d(1 - \upsilon^2)} \tag{16}$$

CONCLUSIONS

Based on the pile-soil interaction model proposed by Nogami (1988), this paper provides a time-domain analysis approach to evaluate the transverse dynamic response of the piles-bridge structure. Comparing with other methods showed

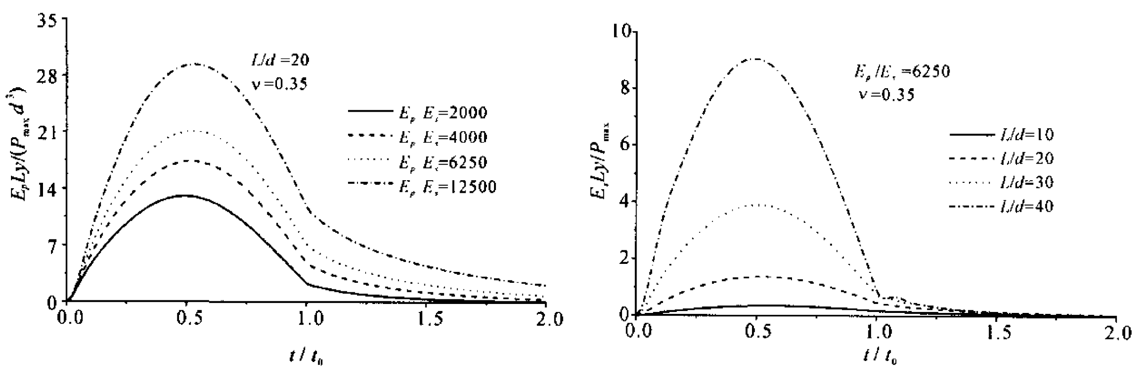

Fig.6 Influence of E_p/ E_s and L/d on the displacement response at the impacted point

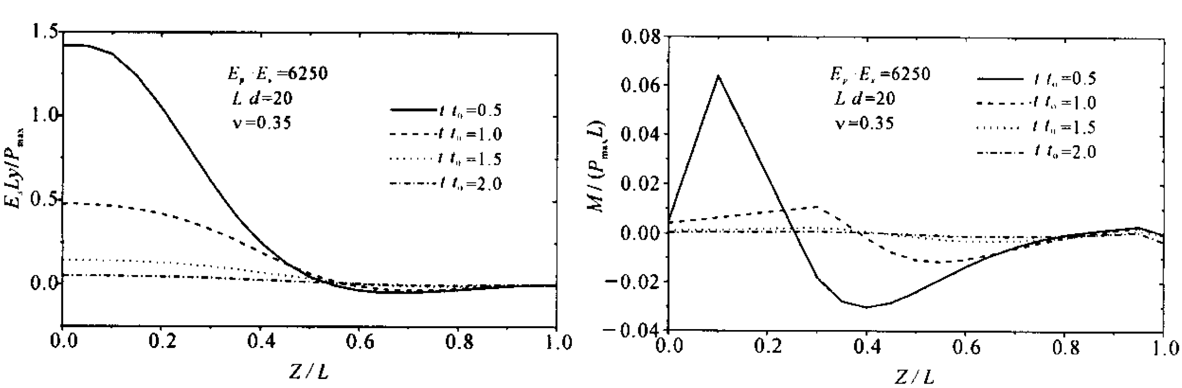

Fig.7 Displacement and moment response along pile shaft at different time

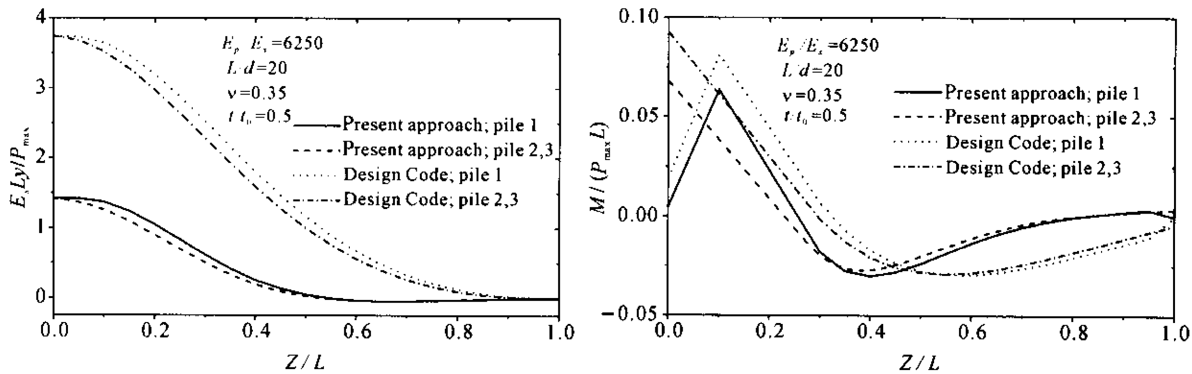

Fig.8 Displacement and moment response along pile shafts

that the analysis by the present approach is much more precise and stable. The approach can be used to analyze the transverse dynamic response of complicated structures embedded in the soil, and is valid for a wide frequency range. The computed results showed that the type of the structure and the slenderness ratio of the pile influence the transverse vibration of the structure more than the pile-soil stiffness ratio. It is important to increase the stiffness of the structure for the design of the bridge. It was found that the joints, the impacted point and a certain length of the pile under the ground surface of the structure have relatively large internal force, so attention should be paid to these positions during designing. The method specified in the design code has many disadvantages because it substitutes the dynamic stiffness of the soil with the static stiffness. Furthermore, it neglects the viscous damping of the soil and the dynamic inertia force of the structure. The present approach overcomes these disadvantages.

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