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Control uncertain continuous-time chaotic dynamical system*

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Abstract: The new chaos control method presented in this paper is useful for taking advantage of chaos. Based on sliding mode control theory, this paper provides a switching manifold controlling strategy of chaotic system, and also gives a kind of adaptive parameters estimated method to estimate the unknown systems' parameters by which chaotic dynamical system can be synchronized. Taking the Lorenz system as example, and with the help of this controlling strategy, we can synchronize chaotic systems with unknown parameters and different initial conditions.

Key Words: Chaotic dynamical system, Sliding mode control, Synchronization
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INTRODUCTION

Chaotic motion is a complex nonlinear motion, whose orbits trajectory in the phase plane is very complex but not stochastic. Chaos plays an important role in dynamical systems and is applied in many fields such as physics, chemistry, economics, and so on (He *et al.*, 2002; Tong *et al.*, 2002). The dynamical characters of chaotic motion have been proved to be useful in describing and diagnosing nonlinear dynamical systems. Research on applications of chaos in communications has been motivated by the observation that chaotic systems can be synchronized, with basic idea that information can be converted to a remote receiver by means of a wideband chaotic system. Nowadays, chaotic synchronization is widely applied in secure communication systems, biomedical science, chemical reactor, and so on. People gradually bring out some theories and methods on chaotic synchronization, such as error-feedback synchronization strategy (Leung *et al.*, 1993; Lai *et al.*, 1993), Pecora-Carroll methods (Pecora *et al.*, 1991), and so on. Recently, Kolumban G. proved the reality and superiority of chaotic synchronization in secrecy communication at the theory and experiment level, which makes research on chaotic synchronization more practical in application (Kolumban *et al.*, 1998).

The switching manifold method (Fang *et al.*, 1999) was based on chaotic synchronization. Taking coupling Lorenz systems as example and using nonlinear control strategy, they researched synchronization control problems under different initial conditions of chaotic dynamics system, whose parameters are known. In this paper, we apply adaptive parameters estimated method and switching manifold method to synchronization systems, and achieve control of chaotic systems whose parameters are unknown.

DEFINITION OF CHAOTIC SYNCHRONIZATION

Consider two continuous-time chaotic systems $\sum$

$$\dot{\boldsymbol{X}} = f(\boldsymbol{X}, \{c_i\}), \quad i = 1, 2, \cdots, n \qquad (1)$$

$$\dot{\boldsymbol{Y}} = g(\boldsymbol{Y}, \{c_i'\}) + \boldsymbol{B}u, \quad i = 1, 2, \cdots, n \qquad (2)$$

where $\boldsymbol{X} = (x_1, x_2, \cdots, x_n)$ and $\boldsymbol{Y} = (y_1, y_2, \cdots, y_n)$ are n-dimensional state variables whose evolution are described by the function $f = (f_1,$

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$f_2, \cdots, f_n$) and $g = (g_1, g_2, \cdots, g_n)$; $\{c_i\}$ are the parameters of the system (1) and $\{c_i'\}$ are the system parameters which need to be estimated; $\boldsymbol{B}$ is the control matrix; u is the control input.

The systems are said to synchronize if

$$\lim_{t \to \infty} \| y(t) - x(t) \| = 0 \tag{3}$$

for any combination of initial states $x(0)$ and $y(0)$.

UNCERTAIN PARAMETERS ESTIMATION

If the system state variables $\boldsymbol{X} = (x_1, x_2, \cdots, x_n)$ and $\boldsymbol{Y} = (y_1, y_2, \cdots, y_n)$ are known, we estimate the system's unknown parameters in terms of the variable errors between two systems. Define error e_i as follows

$$e_i = y_i - x_i, \quad i = 1, 2, \cdots, n \tag{4}$$

With the aid of adaptive control theory (Maybhate *et al*., 1999), we use the descend method, introduce the time series signal x_i, y_i to identify the system's unknown parameters.

Theorem 1 Suppose function $f(x)$ and $g(x)$ of system $\sum$ is continuous and can be of differential, and suppose the Jacobian matrix is of bound and continuous, the systems' uncertain parameters can be estimated in the following form

$$\dot{c}' = -\delta(y(t) - x(t))\frac{\partial g}{\partial c'} \tag{5}$$

where c' is the system's unknown parameter and δ is the control factor of the system.

Proof: Define the dynamical error as

$$E(c', t) = (y(t) - x(t))^2 = e^2 \tag{6}$$

We note that if c' takes precisely the value of c, then the systems are synchronizing. We have $y(t) \to x(t)$ and $E \to 0$, as time $t \to \infty$. The error E and system's unknown parameters has the following relationship

$$\dot{c}' \propto -\frac{\partial E(c', t)}{\partial c'} \tag{7}$$

Input error E into the above equation

$$\frac{\partial E(c', t)}{\partial c'} = 2e \cdot \dot{e} = 2(y(t) - x(t)) \cdot \frac{\partial \dot{y}_i(t)}{\partial c'} \tag{8}$$

Further, to the lowest order in $\mathrm{d}t$, $\Delta y_i = \left(\frac{\partial \dot{y}_i(t)}{\partial c'}\right) \cdot \Delta c' \mathrm{d}t$. Eq. (7) can be written as

$$\dot{c}' \propto -(y(t) - x(t))\frac{\partial g}{\partial c'} \tag{9}$$

So, we have

$$\dot{c}' = -\delta \cdot (y(t) - x(t)) \cdot \frac{\partial g}{\partial c'} \tag{10}$$

where δ is a proportionality constant.

CHAOTIC SYNCHRONIZATION SCHEMES

Chaotic steady-state solutions are characterized by sensitive dependence on initial condition: trajectories of two identical autonomous continuous-time dynamical systems starting from slightly different initial conditions quickly become uncorrelated. It is nevertheless possible to synchronize these systems in the sense that a trajectory of one asymptotically approaches that of the others.

For synchronization purpose, the design of controller $u(t)$ is very important. Based on sliding mode control theory, the first step is to find a suitable switching manifold, then to design an effective nonlinear control to drive the error state to move towards this stable manifold. So the system's state error $\{e_i, i = 1, 2, \cdots, n\}$ approximate to zero, and we can realize the system synchronization.

Select the switching manifold to be in the form

$$w = f(e) \tag{11}$$

In order to achieve the control aim, this form needs three implied conditions (A, B, C):

A. $e = 0$;

B. $w \cdot \dot{w} < 0$;

C. $\frac{\partial \dot{w}}{\partial u} \neq 0$, almost everywhere (near $e = 0$).

In order to minimize the system error and let the controller be effective near $e = 0$, Condition (C) is put into use. The chaotic dynamical system can be synchronized if the controller can satisfy the above conditions.

CONVERGENCE CONDITION

Combining system error ($y_1 - x_1, y_2 - x_2, \cdots, y_n - x_n$) with uncertain parameters errors ($c' - c$), we can form a new variable

$$z = (z_1, z_2, \cdots, z_n, z_{n+1}) = (y_1 - x_1, y_2 - x_2, \cdots, y_n - x_n, c' - c)$$

Therefore

$$\dot{z} = \zeta(z) \quad (12)$$

In order to discuss the stabilization of the system at fixed point $z_e = 0$, *nonlinear function* $\zeta(z)$ can be evolved as Taylor progression near fixed point $z_e = 0$.

$$\dot{z} = \boldsymbol{J} \cdot (z - z_e) + R(z)$$

where $R(z)$ is the high rank derivative item of the evolving expression, $\boldsymbol{J}$ is the Jacobin matrix.

According to nonlinear control theory (Maybhate *et al.*, 1999), the nonlinear system can be stabilized if all of the eigenvalues of Jacobin matrix $\boldsymbol{J}$ have negative real parts.

In order to stabilize the chaotic synchronization system, we can solve the eigenvalues of Jacobin matrix $\boldsymbol{J}$ at fixed point $z_e = 0$, so

$$\lambda_i = \lambda_i(\delta, x, y) \quad i = 1, 2, \cdots, n, n+1 \quad (13)$$

where λ_i, $i = 1, 2, \cdots, n, n+1$ are the eigenvalues of Jacobin matrix $\boldsymbol{J}$, δ is control factor.

Let all of the real parts of eigenvalues less than zero

$$\mathscr{Re}\,\{\lambda_i(\delta, x, y)\} < 0 \quad i = 1, 2, \cdots, n, n+1 \quad (14)$$

Therefore, we can get the convergence condition.

SIMULATIONS AND DISCUSSION

The Lorenz system is taken as examples to illustrate the proposed control method

$$\begin{aligned} \dot{x}_1 &= -\sigma(x_1 - x_2) \\ \dot{x}_2 &= \rho x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 &= x_1 x_2 - b x_3 \end{aligned} \quad (15)$$

where (x_1, x_2, x_3) form the state space and (σ, ρ, b) form the three-dimensional parameter space. When $\sigma = 10$, $\rho = 28$, $b = \frac{8}{3}$, the Lorenz system is chaos.

Add a controller into the right-hand side of the first equation of the Lorenz system, so

$$\begin{aligned} \dot{y}_1 &= -\alpha(y_1 - y_2) + u \\ \dot{y}_2 &= -\beta y_1 - y_2 - y_1 y_3 u \\ \dot{y}_3 &= y_1 y_2 - \gamma y_3 \end{aligned} \quad (16)$$

where (y_1, y_2, y_3) are the new state variables and (α, β, γ) are the unknown parameters. u is the controller.

According to Theorem 1, the equation for evolution of the (α, β, γ) can have the forms

$$\begin{aligned} \dot{\alpha} &= -\delta(y_1(t) - x_1(t))(x_1(t) - x_2(t)) \\ \dot{\beta} &= -\delta(y_2(t) - x_2(t))y_1(t) \\ \dot{\gamma} &= -\delta(y_3(t) - x_3(t))(-y_3(t)) \end{aligned} \quad (17)$$

Under the control of u, we can choose properly δ to realize $\alpha \to \sigma, \beta \to \rho, \gamma \to b$, as time $t \to \infty$. To Lorenz system, choose

$$w(e) = be_3 - e_1^2 = 0 \quad (18)$$

Obviously, the above equation implies $e = 0$. From Eqs. (15) and (16), we have

$$\begin{aligned} \dot{e}_1 &= \dot{y}_1 - \dot{x}_1 = \alpha(e_2 - e_1) + \\ &\quad (\sigma - \alpha)(x_1 - x_2) + u \\ \dot{e}_2 &= \dot{y}_2 - \dot{x}_2 = e_1(\beta - x_3) + \\ &\quad (\beta - e_3)x_1 - e_2 - e_1 e_3 \\ \dot{e}_3 &= \dot{y}_3 - \dot{x}_3 = x_1 e_2 - \gamma e_3 - \\ &\quad (\gamma - b)x_3 + e_1(e_2 + x_2) \end{aligned} \quad (19)$$

From Eq. (19), we have

$$\dot{w} = b\dot{e}_3 - 2e_1\dot{e}_1 = (b - 2\alpha)e_1 e_2 + (2\alpha - \gamma)e_1^2 - b(\gamma - b)x_3 + 2e_1(\alpha - \sigma)(x_1 - x_2) - w\gamma + b(x_1 e_2 + e_1 x_2) - 2e_1 u \quad (20)$$

From the above equation, we have $\frac{\partial \dot{w}}{\partial u} = -2e_1$, which satisfy the Condition (C).

The aim of design controller $u(t)$ is to make w in Eq. (20) tend to be zero, so the design must follow this principle: the driving variable error $\{e_i, i = 1, 2, 3\}$ gradually tends to be stable switching manifold. We choose controller in the form

$$u = (\gamma/2 - \alpha)(e_2 - e_1) \quad (21)$$

where $\gamma > 0$. When $t \to \infty$, $e_1 \to 0$, $e_2 \to 0$, $\alpha \to$

$\sigma, \gamma \to b$, at this time

$$b(e_1 x_2 + x_1 e_2) + 2(\alpha - \sigma) e_1 (x_1 - x_2) - b(\gamma - b) x_3 \to 0$$

So

$$\dot{w} = b(e_1 x_2 + x_1 e_2) + 2(\alpha - \sigma) e_1 (x_1 - x_2) - bw - b(\gamma - b) x_3 \Rightarrow \dot{w} = -bw \quad (22)$$

From the above equation, we choose the controller to be of the form in Eq. (21), when $w = 0$, $\dot{w} = -bw = 0$, Condition (B) can be satisfied.

To demonstrate the effectiveness of the developed control method, we have taken Eq. (21) as the controller, and studied numerical simulation for synchronization of the coupling Lorenz system. To solute the differential equation, we use 4-order step-varying Runge-Kutta method. The initial conditions of Eq. (15) is (0.01, −0.01, 0.05), the initial conditions of Eq. (16) is (2.2, 1.01, 2); the control factor $\delta = 3.3$. In Eq. (16), we choose the initial values of the relevant parameters as $\alpha = 6$、$\beta = 13$、$\gamma = 0.2$. The simulating results are shown in Fig. 1. Fig. 1a are the curves of system variale errors e_1, e_2, e_3. Fig. 1b are the curves of estimated parameters errors $\alpha - \sigma, \beta - \rho, \gamma - b$. Fig. 1a show that we can make the error $\{e_i, i = 1, 2, 3\}$ be zero rapidly under the control of $u(t)$; and that the transition process is very short.

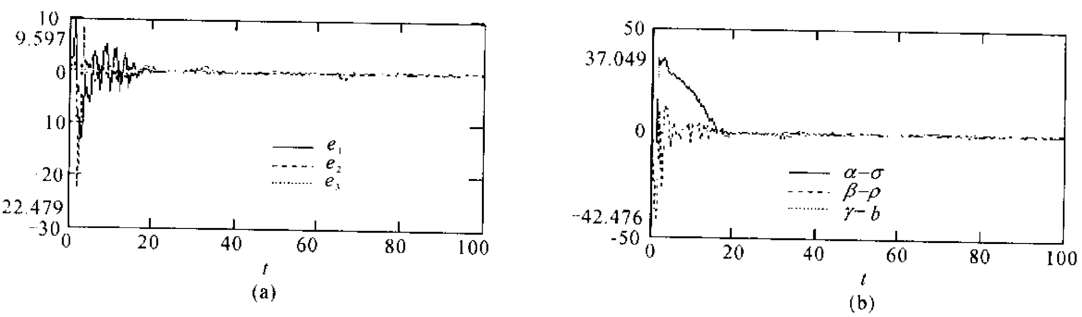

Fig. 1 The outputs of the synchronization system

(a) System Variable errors e_1, e_2, e_3; (b) Parameters errors

CONCLUSION

It is clear from Fig. 1 that these numerical simulations indeed have verified that the above theoretical analysis is correct and our nonlinear control strategy is effective.

In this paper, we settle following two problems:

(1) Adopting the adaptive parameter estimation, we realize the parameter c_i' in synchronization system gradually tending to be parameter c_i in the former system. To the Lorenz system, make $\sigma \to \sigma, \beta \to \rho, \gamma \to b$.

(2) When the control factor of the system satisfies $\delta \in (3.1, 6.4)$, under the controller $u(t)$, the system can be synchronized in different initial conditions. At the same time, the system's transition processing is short.

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