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An electricity price model with consideration to load and gas price effects

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Abstract: Some characteristics of the electricity load and prices are studied, and the relationship between electricity prices and gas (fuel) prices is analyzed in this paper. Because electricity prices are strongly dependent on load and gas prices, the authors constructed a model for electricity prices based on the effects of these two factors; and used the Geometric Mean Reversion Brownian Motion (GMRBM) model to describe the electricity load process, and a Geometric Brownian Motion (GBM) model to describe the gas prices; deduced the price stochastic process model based on the above load model and gas price model. This paper also presents methods for parameters estimation, and proposes some methods to solve the model.

Key words: Electricity market, Stochastic process, Electricity price, Gas
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INTRODUCTION

In an electricity market, the electricity prices have important impact on such diverse issues as asset pricing, contracting, planning, and choice of operation policies for generation and transmission of electricity. Analysis of the historical electricity prices suggested that it is very difficult to model the behaviour of electricity prices because they are highly volatile. From the curve of the historical price we can see that there are many price "spikes" (price behaviour defined as suddenly upward or downward movement). Considering that the number of price spikes in one year is not negligible, we must take them into consideration when we model the price behaviours. Deng (2000) described three types of mean-reversion jump-diffusion models for modeling energy commodity spot prices with jumps and spikes; Barz and Johnson (1999) described brownian motion, mean reversion, geometric brownian motion and geometric mean reversion models, and also detailedly analysed the behaviour of prices in different regions and time. Monte-Carlo simulation is used for electricity derivative pricing in the jump-diffusion price models.

Most of the electricity price models reported in the literature are only one-parameter or few-parameters models based on the general Black-Scholes model and only considered the effects of historical prices and used historical price data to construct the electricity price process model in a method like that for constructing the stock market price model. Because the non-storability and transmission limitation of electricty, the electricity market displays different behaviors from that of the commodities market. There are many factors which significantly affect the electricity price behaviour, such as the load behaviour, different time periods (hour, day, week and year), different regions, the gas price behaviour and the generation and transmission capacities.

The electricity market deregulation processes had been carried out for a short time only and due to shortage of historical electricty prices, there are few papers reporting on results of analysis on the electricity price behaviours based on the affecting factors and the relationship between the electricity prices behaviours and load behaviour, gas prices behaviours, generation and transmission capacity, temprature, etc. Allen (1998) dealt with correlations among electricity prices and time, load, and temperature. But the models (Brownian Motion or Wiener Process, and AR(1) process) are very simple and the analysis can only be taken as an introduction.

Kreuzberg (1999) took the generation, transmission, efficiencies of power plants, and startup costs, into consideration, but did not analyze the correlations between them and the electricity prices; he took these factors into the model directly. In fact some parameters can be ignored because the correlations are so small. This paper's analysis results showed that the effect of the gas price and load behaviour is very important. Considering the difficulty of including all the factors in the electricity price model, we took the most important factors, load and gas price behaviour, as the variables to construct the model in the Part III of this paper. Simulation results demonstraded that this model is feasible.

CHARACTERISTICS OF THE PRICE AND LOAD BEHAVIOUR

Because of its non-storablility and limitations of transmission, electricity should be treated as a special commodity. The limitations on the possibilities of transmitting electricity across time and space strongly affect the behaviour of electricity spot and derivative prices as compared to other commodities, especially the behaviour of the price "spikes". The non-storability of electricity requires that electricity must be used at the same time it is generated. Limitations in transmission capacity and transmission losses are important considerations for deciding if transmission among some zones is economical. These limitations make electricity prices depend highly on the regions, which means that the prices depend on the local level of supply and demand, weather, and the fuel or gas prices.

The following sections will show the price behaviours in different time period, regions, temperature, load and gas price behaviour. The correlation between electricity prices and load, gas (fuel) prices will also be described.

1. Characteristics of load behaviour

From the following anaysis we can see that electricity prices are strongly load- and time-dependent. So it is important for us to incorporate the factors of the behaviour of the load and time into the electricity price model. The correlation factor between load and time, 0.44, is also very high. This shows that load is also strongly time-denpendent (Table 1).

Table 1 The correlation matrix (CM) in PJM in 1999

	Load	Price	Time
Load	1.00	0.68	0.44
Price	0.68	1.00	0.23
Time	0.44	0.23	1.00

Generally speaking, the load has the following characteristics:

1. Strongly time-dependent (time mean reversion and seasonal). Load changes according to different hours everyday, and everyday there are two load peaks, one at about 9:00 AM to 13:00 PM, the other from 16:00 PM to 19:00 PM. In one year, most load curves have also two peaks, one in summer and the other in winter. The height of the load peaks in different areas are different. For example, in the USA the peak in summer is higher than the one in winter, but in Germany the opposite is true.

2. Region dependent. In different areas, even at the same time, the electricity loads are different. It is caused by the different consumption structure at each region. The characteristics of electricity load seasonality are also different. For example, in California or PJM the peak in summer is higher than that in winter, but in Germany the opposite is true.

3. Strongly temperature dependent. Allen (1998) described in detail the relationship between temperature and electricity load for PJM where in summer the temperature is very high, so that the electricity demand will also be very high because a great part of the electricity is consumed to turn on fans or other cooling electrical equipment. In winter, when it is very cold, the electricity demand will also increase, because much electricity will be used for heating. And the relationships between temperature and electricity load are also greatly different in each region.

Fig. 1 shows the price and load behaviour in 1st and 2nd of April and August in CalPX (California Power Exchange) in 1999. It is obvious that the price and load behaviours are periodic and strongly time-dependent. Fig. 2 shows the behaviour of on peak price and on peak load from 1999 to 2000 in CalPX. It demonstrates the seasonal effects of the log price and log load.

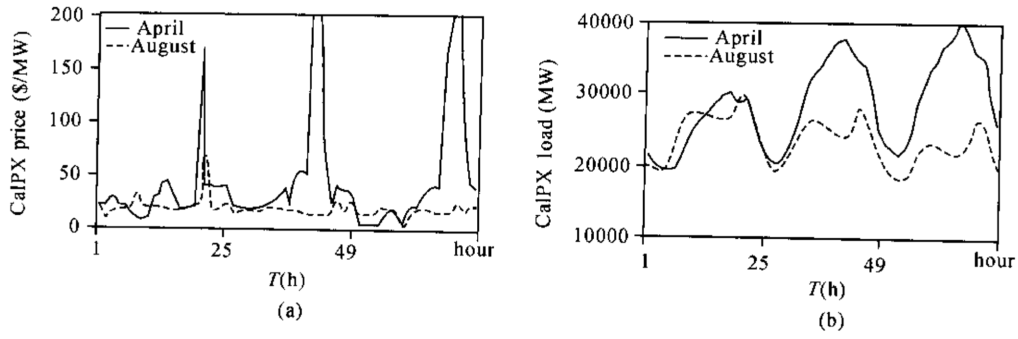

Fig.1 Price and load of the 1st and 2nd day of April and August CalPX in 1999
(a) 1st day; (b) 2nd day

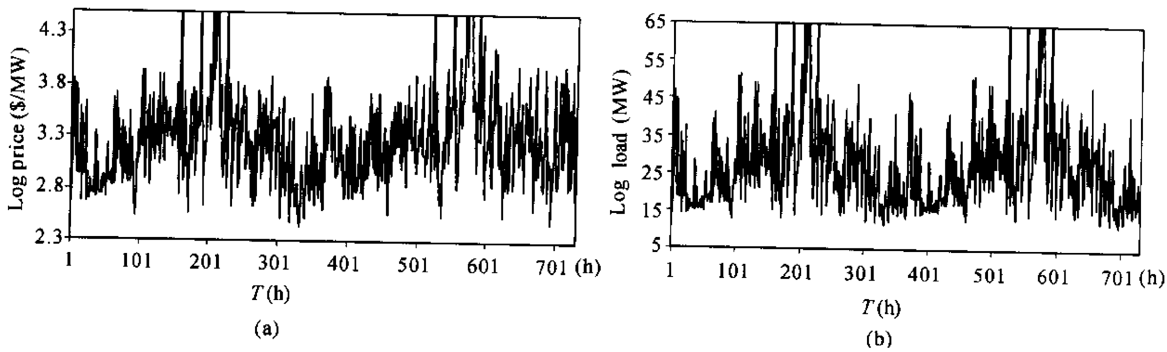

Fig.2 Relatioship between log price and log load 1999 – 2000 in CalPX
(a) Log on-peak price 1999 – 2000 in CalPX; (b) Log on-peak load 1999 – 2000 in CalPX

2. Characteristics of electricity price

The most important characteristics of the electricity price are:

1) High load-dependence;

2) Mean Reversion and Seasonal dependence;

3) High volatility and occasional price "spikes".

Fig.2 shows mean reversion and seasonal effects. Price "spikes" can be defined as abrupt up or down movements of electricity prices. Price "spikes" can be regarded as the result of the non-storable nature of electricity. But price "spikes" do not happen in all electricity markets. For example, the price curves in EEX (European Energy Exchange) are relatively flat; it means that there are no price "spikes". In the CalPX, there are such price "spikes". So we should construct different electricity price models for each market. The model used in this paper is based on EEX without the consideration of price "spikes" effects.

3. Correlation of price with load and gas (fuel) price

It is well known that price will go up with increase of electricity demand, and down with decrease of electricity demand. The relationship between price and demand for electricity is the same as that of some other commodities. Fig.1 and Fig.2 show the relation between electricity prices and load. If there are lots of gas generators and most of them are marginal units, then the behaviour of gas prices will also affect electricity prices significantly, because the cost of electricity produced by gas-powered generators depends on gas prices. Then it is reasonable and important to imcorporate the effects of load and gas (fuel) price into the electricity price model.

MODELS

Prices for most common commodities, such as copper and oil, are mostly modeled using a stochastic process, in which the price at a future time is a random function of the current price. A common price process model is Geometric Brownian Motion (sometimes called Wiener Process) which is a continuous-time process as described below:

$$dP_t = \mu_t P_t dt + \sigma P_t dw$$

where, P_t, spot price at time t; μ_t, drift of spot price; σ, volatility of spot price; dw, random brownian motion.

Another price process model is the mean-reverting process. Whereas a random walk can deviate far from the starting point and not return for a very long time, a mean-reverting process will have a tendency to return to a mean value over time. The mean-reverting process may be described in discrete time as follows:

Geometric Mean reversion:

$$dP_t = k(\mu_t - \Psi(t))P_t dt + \sigma P_t dw,$$

where, $u_t \equiv \alpha_t + \sigma^2/2$, k is the mean reversion factor. $\Psi(t)$ is the equilibrium process for $P(t)$.

Here, we represent the electricity load and the electricity price with time mean reversion model. And for the fuel or gas price we adopt the geometric brownian motion model, because this model is widely used in the energy (except electricity) market by many researchers.

1. The electricity load model

The first state variable is a demand variable. The load stochastic process can be decribed as:

$$dq_t = k(\theta_q(t) - \ln q_t)q_t dt + \sigma q_t dw_q \quad (1)$$

Here we define $\mu(q_t, t) \equiv k(\theta_q(t) - \ln q_t)$.

Assume that $y_t = \ln q_t$, then rewrite Eq.(1) as

$$dy_t = k(\theta_q(t) - y_t)dt + \sigma dw_y \quad (2)$$

where,

y_t: the log of the electricity load;

k: the mean reversion factor;

$\theta_q(t)$: the equilibrium process for y_t;

σ: the volatility of the log electricity load;

dw_y: the stochastic variable for log electricity load.

y_t reverts to a time-varying mean $\theta_q(t)$. In Fig.2 we can see that the historical log load curve looks like a function of the sum of sine terms. Here we express $\theta_q(t)$ as a sum of sine terms to reflect seasonal, predictable variations in electricity load (this assumption was first proposed by Craig Pirrong in 1998).

In order to simplify the problem, we can take all parameters as constants.

Let $\theta(t_i) = \ln q_i$ and $\theta(t) = x_1\exp(-x_2 t) + x_3\sin(x_4 t + x_5) + x_6\sin(x_7 t + x_8)$

Using the maximum-likelihood method to estimate the parameters in the following expressions, and assuming that

$$\theta(t_i) = x_1\exp(-x_2 t_i) + x_3\sin(x_4 t_i + x_5) + x_6\sin(x_7 t_i + x_8) + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2)$$

the equation becomes:

$$\theta(t_i) \sim N(x_1\exp(-x_2 t_i) + x_3\sin(x_4 t_i + x_5) + x_6\sin(x_7 t_i + x_8) + \sigma^2)$$

where $\theta(t_i)$, $\theta(t_j)$ are independent each other, here $i \neq j$.

Then the combination function is:

$$L = \prod_{i=1}^{365} \frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{1}{2\sigma^2}[x(t_i) - (x_1\exp(-x_2 t_i) + x_3\sin(x_4 t_i + x_5) + x_6\sin(x_7 t_i + x_8))]^2\right\}$$

To obtain the maximum of the above is equivalent to solving the following:

$$\text{Min: } \sum_{i=1}^{365}[x(t_i) - (x_1\exp(-x_2 t_i) + x_3\sin(x_4 t_i + x_5) + x_6\sin(x_7 t_i + x_8))]^2 \equiv \theta(x_1, x_2, \cdots, x_8)$$

The parameters $x_1, x_2, \cdots, x_8$ can be obtained by solving the above equation.

Fig.3 shows the recorded log load curve in 1999 in CalPX and the simulated curve obtained by using the estimated parameters.

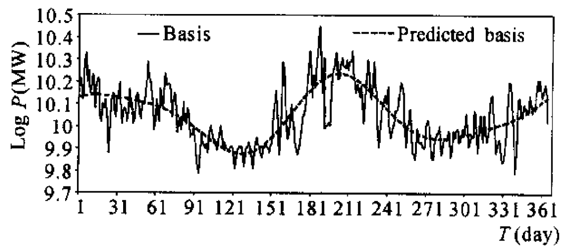

Fig.3 Comparation of original and simulated curves

2. The gas(fuel) price Model

The second state variable is gas(fuel) price. The geometric brownian motion model is frequently used to model the price behaviour in normal commodities markets (including fuel and gas market). The Gas prices also have the characteristic of time mean reversion; the half-life is always about a few months, much longer than the half-life of electricity load (about a few hours). The risk-neutralised process for the price of the marginal fuel or the gas is:

$$\frac{dg_t}{g_t} = \chi(g_t, t)dt + \sigma_g(g_t, t)dw \quad (3)$$

g_t: the gas or fuel price at time t

χ: drift of the gas price

σ: the volatility of the gas price

dw: standard Brownian motion.

Assume that $z_t = \ln g_t$, then

$$dz_t = \chi(z_t, t)dt + \sigma_z(z_t, t)dw_z$$

To simplify the problem, we can assume that all paparameters are constants.

3. The price model

In any forward contract, it is assumed that the forward price is $P(q_t, g_t, t)$, which is a function of t, q_t and g_t; then the forward price satisfies the following PDE (Partial Differential Equation):

$$dP = \frac{\partial P}{\partial q_t}dq_t + \frac{\partial P}{\partial g_t}dg_t + \frac{\partial P \partial P}{\partial q_t \partial g_t}dq_t dg_t + 0.5\frac{\partial^2 P}{\partial q_t^2}(dq_t)^2 + 0.5\frac{\partial^2 P}{\partial g_t^2}(dg_t)^2 \quad (4)$$

Combining Eqs.(1) and (4) into one yields

$$dP = \frac{\partial P}{\partial q_t}q_t[\mu(q_t, t)dt + \sigma_q du] + \frac{\partial P}{\partial g_t}g_t \cdot [\chi(g_t, t)dt + \sigma_g(g_t, t)dw] + \frac{\partial P \partial P}{\partial q_t \partial g_t}q_t \cdot [\mu(q_t, t)dt + \sigma_q du]g_t[\chi(g_t, t)dt + \sigma_g(g_t, t)dw] + 0.5\frac{\partial^2 P}{\partial q_t^2}(q_t[\mu(q_t, t)dt + \sigma_q du])^2 + 0.5\frac{\partial^2 P}{\partial g_t^2}(g_t[\chi(g_t, t)dt + \sigma_g(g_t, t)dw])^2 \quad (5)$$

written in another form, Eq.(5) becomes

$$dP = \frac{\partial P}{\partial q_t}q_t\mu(q_t, t)dt + \frac{\partial P}{\partial g_t}g_t\chi(g_t, t)dt + \frac{\partial P}{\partial q_t}q_t\sigma_q du + \frac{\partial P}{\partial g_t}g_t\sigma_g(g_t, t)dw + \frac{\partial P \partial P}{\partial q_t \partial g_t} \cdot q_t g_t \mu(q_t, t)\chi(g_t, t)dt^2 + 0.5\frac{\partial^2 P}{\partial q_t^2}q_t^2\mu^2(q_t, t)dt^2 + 0.5\frac{\partial^2 P}{\partial g_t^2}g_t^2\chi^2(g_t, t)dt^2 + \frac{\partial P \partial P}{\partial q_t \partial g_t} \cdot \mu(q_t, t)\sigma_g(g_t, t)dwdt + \sigma_q\chi(g_t, t)dudt + \frac{\partial P \partial P}{\partial q_t \partial g_t}\sigma_q\sigma_g(g_t, t)dudw + 0.5\frac{\partial^2 P}{\partial q_t^2}q_t^2\sigma_q^2 du^2 + 0.5\frac{\partial^2 P}{\partial g_t^2}g_t^2\sigma_g^2(g_t, t)dw^2 + \frac{\partial^2 P}{\partial q_t^2}q_t^2\mu(q_t, t) \cdot \sigma_q dtdu + \frac{\partial^2 P}{\partial g_t^2}g_t^2\chi(g_t, t)\sigma_g(g_t, t)dtdw \quad (6)$$

as $dt \to 0$, dt^2 and $dt^{1.5}$ can be ignored compared with dt. And we know that: $dw^2 = dt$, $du^2 = dt$ and $dwdt = dt^{1.5}$, $dudt = dt^{1.5}$ and assume $dwdu = \rho_{pg}dt$, then the above equation becomes:

$$dP = \frac{\partial P}{\partial q_t}q_t\mu(q_t, t)dt + \frac{\partial P}{\partial g_t}g_t\chi(g_t, t)dt + 0.5\frac{\partial^2 P}{\partial q_t^2}q_t^2\sigma_q^2 dt + 0.5\frac{\partial^2 P}{\partial g_t^2}g_t^2\sigma_g^2(g_t, t)dt + \frac{\partial P \partial P}{\partial q_t \partial g_t}\sigma_q\sigma_g(g_t, t)\rho_{qg}dt + \frac{\partial P}{\partial q_t}q_t\sigma_q du + \frac{\partial P}{\partial g_t}g_t\sigma_g(g_t, t)dw \quad (7)$$

then we can assume that

$$dP = \phi(q_t, g_t, t)dt + \frac{\partial P}{\partial q_t}q_t\sigma_q du + \frac{\partial P}{\partial g_t}g_t\sigma_g(g_t, t)dw \quad (8)$$

with

$$\phi(q_t, g_t, t) = \frac{\partial P}{\partial q_t}q_t\mu(q_t, t) + \frac{\partial P}{\partial g_t}g_t\chi(g_t, t) + \frac{\partial P \partial P}{\partial q_t \partial g_t}\sigma_q\sigma_g(g_t, t)\rho_{qg} + 0.5\frac{\partial^2 P}{\partial q_t^2}q_t^2\sigma_q^2 + 0.5\frac{\partial^2 P}{\partial g_t^2}g_t^2\sigma_g^2(g_t, t) \quad (9)$$

and the variance is

$$\sigma^2(q_t, g_t, t) = \left(\frac{\partial P}{\partial q_t}\right)^2 q_t^2\sigma_q^2 + \left(\frac{\partial P}{\partial g_t}\right)^2 g_t^2\sigma_g^2 + 2\frac{\partial P \partial P}{\partial q_t \partial g_t}\sigma_q\sigma_g q_t g_t\rho_{qg} \quad (10)$$

Forward Price $P(q_t, g_t, t)$ can be written as follows

$$P(q_t, g_t, t) = (m_0 + m_1 g_t)\exp(n_0(t) + n_1 q_t + n_2 q_t^2)$$

Where

g_t: gas or fuel price at time t

q_t: the electricity load at time t.

Again, using the maximum-likelihood method, we can estimate the parameters m_0, m_1, n_1, n_2, $n_0(t)$. Here $n_0(t)$ can exhibit seasonal effects.

Rewrite the Eq.(8) as follows,

$$dP = \phi(q_t, g_t, t)dt + (n_1 + 2n_2 q_t)q_t\sigma_q P du + m_1 g_t\sigma_g P dw \quad (11)$$

where

$$\phi(q_t, g_t, t) = (n_1 + 2n_2 q_t)q_t\mu(q_t, t)P + m_1 g_t\chi(g_t, t)P + m_1(n_1 + 2n_2 q_t)\sigma_q\sigma_g(g_t, t)\cdot \rho_{qg}P + 0.5(2n_2 + (n_1 + 2n_2 q_t)^2)Pq_t^2\sigma_q^2 + 0.5m_1^2 g_t^2\sigma_g^2 P \quad (12)$$

and the variance is

$$\sigma^2(q_t, g_t, t) = ((n_1 + 2n_2 q_t)q_t\sigma_q P)^2 + (m_1 g_t\sigma_g P)^2 + 2(n_1 + 2n_2 q_t)q_t\sigma_q P m_1 g_t\sigma_g P\rho_{qg} \quad (13)$$

Define $X \equiv \mathrm{Ln}(P)$, the electricity price process can be described as,

$$dX = \phi_{new}(q_t, g_t, t)dt + (n_1 + 2n_2 q_t)q_t\sigma_q du + m_1 g_t\sigma_g dw \quad (14)$$

with

$$\phi_{new}(q_t, g_t, t) = (n_1 + 2n_2 q_t)q_t\mu(q_t, t) + m_1 g_t\chi(g_t, t) + m_1(n_1 + 2n_2 q_t)\sigma_q\sigma_g(g_t, t)\cdot \rho_{qg} + 0.5(2n_2 + (n_1 + 2n_2 q_t)^2)q_t^2\sigma_q^2 + 0.5m_1^2 g_t^2\sigma_g^2 \quad (15)$$

and the new variance is

$$\sigma_{new}^2(q_t, g_t, t) = ((n_1 + 2n_2 q_t)q_t\sigma_q)^2 + (m_1 g_t\sigma_g)^2 + 2(n_1 + 2n_2 q_t)q_t\sigma_q m_1 g_t\sigma_g\rho_{qg} \quad (16)$$

There are stochastic motions in the electricity price model; one is for load process and the other is for gas (fuel) price process.

Among many methods that can be used to solve the model (PDE) are the Monte Carlo method and finite diffence mesh method. The explicit methods include bionomial trees provided by Cox *et al*. (1979), and trinomial trees (Hull and White, 1994). Although the explicit methods are not so accurate compared with finite difference methods, they are very simple to implement and useful for futher research on options or future's electricity prices.

SUMMARY

This paper first analyses some characteristics of the electricity load and prices, then it describes the relation between electricity price and load and gas (fuel) prices. It is obvious that electricity prices are strongly dependent on load and gas price, so we construct the model for electricity price based on the effects of these two factors. In this paper we also present methods for parameters estimation, propose some methods for solving the model.

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