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Decentralized impulsive control for a class of uncertain interconnected systems*

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Abstract: A great deal of stabilization criteria has been obtained from study of stabilizing interconnected systems. The results obtained are usually based on continuous systems by state feedback. In this paper, decentralized impulsive control is presented to stabilize a class of uncertain interconnected systems based on Lyapunov theory. The system under consideration involves parameter uncertainties and unknown nonlinear interactions among subsystems. Some new criteria of stabilization under impulsive control are established. Two numerical examples are offered to prove the effectiveness and practicality of the proposed method.

Key words: Uncertain interconnected systems, Decentralized impulsive control, Stabilization
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INTRODUCTION

With the development of science and technology, the structures of control systems are getting more and more complicated. This kind of system has the characteristic of large dimension, and interconnection among subsystems. Generally, this kind of system is called interconnected large system. The study of interconnected systems is motivated by quite a number of practical applications ranging from power networks, transportation, aerospace, economics, management, and so on. The strategies to control interconnected systems are divided into two kinds: centralized control and decentralized control. Although centralized control has good effect, the complexity among subsystems makes the information transfer restricted so that it is difficult to realize. Decentralized control is different from centralized control, which designs controller for each subsystem using only local information. Utilizing this control strategy facilitates design of controllers. There is a large body of literature on interconnected large system (Wang and Zhang, 1993; Liu and Guan, 1995; Yan *et al.*, 1998). Until now, most of the results obtained on interconnected systems are usually based on continuous system; only a few results on stability of large scale systems with impulse effect have been obtained. Liu and Guan (1995), Guan and Liu (1994), Guan (1999) studied the stability of large scale systems with impulse effect based on differential equations. In these papers, impulses were regarded as perturbations.

Significant progress has been made on the theory of impulsive differential equations (Laksh-

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mikantham *et al.*, 1989; Lakshmikantham and Liu, 1989). People try to use impulsive control to control practical systems because such controls may be simpler to implement and involve cheaper control mechanisms. Impulsive control was widely applied in chaos (Guan *et al.*, 2002; Liu and Kok, 2002), but no corresponding papers on interconnected large system have appeared yet. The objective of this paper is to present decentralized impulsive controllers to stabilize a class of interconnected large system based on Lyapunov theory and to develop some stabilization criteria. Theoretical analysis showed the method is feasible. The simulation results of two numerical examples proved the effectiveness of the method.

SUPPORTING RESULTS

This section introduces the basic principles of impulsive control and several lemmas. Consider the following impulse nonlinear system

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}), & t \neq t_k \\ \Delta \boldsymbol{x} = \boldsymbol{I}_k(\boldsymbol{x}), & t = t_k \\ \boldsymbol{x}(t_0^+) = \boldsymbol{x}_0 \end{cases},$$

where $t \in R_+, \boldsymbol{x} \in R^n$ is the state variable, $\boldsymbol{f}(t, \boldsymbol{x})$: $R_+ \times R^n \to R^n$ is a continuous function in its domains of definition; $\Delta \boldsymbol{x} = \boldsymbol{x}(t_k^+) - \boldsymbol{x}(t_k)$, $\boldsymbol{x}(t_k^+) = \lim_{t \to t_k^+} \boldsymbol{x}(t)$, $\boldsymbol{I}_k(\boldsymbol{x}): R^n \to R^n$. t_k is the moment when the impulse occurs. Assume that $\{t_k\}$, a set of discontinuity points, satisfies:

$$0 < t_0 < t_1 < t_2 \cdots < t_k < \cdots, \text{ and } \lim_{k \to \infty} t_k = \infty .$$

The state is changed under impulsive control, hence the solution trajectories of the system is changed as well.

Lemma 1 Suppose $\boldsymbol{P} \in R^{n \times n}$ is a positive definite matrix, $\boldsymbol{Q} \in R^{n \times n}$ is a symmetric matrix; then for any $\boldsymbol{x} \in R^n$, we have

$$\lambda_{\min}(\boldsymbol{P}^{-1}\boldsymbol{Q})\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x} \leq \boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x} \leq \lambda_{\max}(\boldsymbol{P}^{-1}\boldsymbol{Q})\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x} .$$

Proof $\boldsymbol{P}$ is a positive definite matrix, so there is a full rank matrix $\boldsymbol{P}_1$ to make $\boldsymbol{P} = \boldsymbol{P}_1^{\mathrm{T}}\boldsymbol{P}_1$.

Let $\boldsymbol{P}_1\boldsymbol{x} = \boldsymbol{y}$,

hence $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x} = \boldsymbol{y}^{\mathrm{T}}(\boldsymbol{P}_1^{-1})^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{P}_1^{-1}\boldsymbol{y}$.

Since

$$\begin{aligned}(\boldsymbol{P}_1^{-1})^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{P}_1^{-1} &= \boldsymbol{P}_1\boldsymbol{P}_1^{-1}(\boldsymbol{P}_1^{\mathrm{T}})^{-1}\boldsymbol{Q}\boldsymbol{P}_1^{-1} = \boldsymbol{P}_1(\boldsymbol{P}_1^{\mathrm{T}}\boldsymbol{P}_1)^{-1}\boldsymbol{Q}\boldsymbol{P}_1^{-1} \\ &= \boldsymbol{P}_1\boldsymbol{P}^{-1}\boldsymbol{Q}\boldsymbol{P}_1^{-1}\end{aligned}.$$

Hence matrix $(\boldsymbol{P}_1^{-1})^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{P}_1^{-1}$ is analogous with $\boldsymbol{P}^{-1}\boldsymbol{Q}$, as they have the same eigenvalues.

Hence

$$\begin{aligned}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x} &= \boldsymbol{y}^{\mathrm{T}}(\boldsymbol{P}_1^{-1})^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{P}_1^{-1}\boldsymbol{y} \\ &\leq \lambda_{\max}((\boldsymbol{P}_1^{-1})^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{P}_1^{-1})\boldsymbol{y}^{\mathrm{T}}\boldsymbol{y} \\ &= \lambda_{\max}(\boldsymbol{P}^{-1}\boldsymbol{Q})\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}\end{aligned}.$$

Similarly, we have $\lambda_{\min}(\boldsymbol{P}^{-1}\boldsymbol{Q})\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x} \leq \boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x}$.

Lemma 2 Suppose $\boldsymbol{x}, \boldsymbol{y} \in R^n$, $\varepsilon > 0$, then we have

$$\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} + \boldsymbol{y}^{\mathrm{T}}\boldsymbol{x} \leq \varepsilon \boldsymbol{y}^{\mathrm{T}}\boldsymbol{y} + \frac{\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}}{\varepsilon} .$$

Lemma 3

1) Let

$$\begin{aligned}&\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n, \\ &\beta_1 \leq \beta_2 \leq \cdots \leq \beta_n, \\ &\gamma_1 \leq \gamma_2 \leq \cdots \leq \gamma_n\end{aligned}$$

be the eigenvalues of the symmetric matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C} = \boldsymbol{A} + \boldsymbol{B}$ respectively. Then

$$\alpha_i + \beta_1 \leq \gamma_i \leq \alpha_i + \beta_n,\ i = 1,2,\cdots,n .$$

2) Let λ be the eigenvalue of matrix $\boldsymbol{A}$; then for any integer k, λ^k is the eigenvalue of matrix $\boldsymbol{A}^k$.

Lemma 4 Suppose $\boldsymbol{P}\in R^{n\times n}$ is a positive definite matrix, $\boldsymbol{f}(\boldsymbol{x})\in R^{n}$, is a continuous nonlinear function. Assume $\boldsymbol{f}(\boldsymbol{x})$ satisfies $\|\boldsymbol{f}(\boldsymbol{x})\|\le L\|\boldsymbol{x}\|$, then

$$\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{P}\boldsymbol{x}\le L\sqrt{\frac{\lambda_{\mathrm{M}}(\boldsymbol{P})}{\lambda_{\mathrm{m}}(\boldsymbol{P})}}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}.$$

Proof $\boldsymbol{P}$ is a positive definite matrix, so there is a full rank $\boldsymbol{P}_1$ to make $\boldsymbol{P}=\boldsymbol{P}_1^{\mathrm{T}}\boldsymbol{P}_1$. Since $\boldsymbol{P}_1^{\mathrm{T}}\boldsymbol{P}_1$ is analogous to $\boldsymbol{P}_1\boldsymbol{P}_1^{\mathrm{T}}$, $\lambda_{\mathrm{M}}(\boldsymbol{P}_1\boldsymbol{P}_1^{\mathrm{T}})=\lambda_{\mathrm{M}}(\boldsymbol{P})$.
Furthermore $\|\boldsymbol{P}_1^{\mathrm{T}}\|=\sqrt{\lambda_{\mathrm{M}}(\boldsymbol{P})}$,
we have

$$\begin{aligned}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{P}\boldsymbol{x}&\le\|\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x})\|\|\boldsymbol{P}\boldsymbol{x}\|\le L\|\boldsymbol{x}\|(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x})^{\frac{1}{2}}\\&=L(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x})^{\frac{1}{2}}(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}_1^{\mathrm{T}}(\boldsymbol{P}_1\boldsymbol{P}_1^{\mathrm{T}})\boldsymbol{P}_1\boldsymbol{x})^{\frac{1}{2}}\\&\le L\lambda_{\mathrm{m}}^{-\frac{1}{2}}(\boldsymbol{P})(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x})^{\frac{1}{2}}\lambda_{\mathrm{M}}^{\frac{1}{2}}(\boldsymbol{P})(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x})^{\frac{1}{2}}\\&=L\sqrt{\frac{\lambda_{\mathrm{M}}(\boldsymbol{P})}{\lambda_{\mathrm{m}}(\boldsymbol{P})}}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}\end{aligned}$$

MAIN RESULTS

Consider the following uncertain interconnected systems under impulsive control

$$\begin{cases}\dot{\boldsymbol{x}}_i=\boldsymbol{A}_i\boldsymbol{x}_i+\boldsymbol{f}_i(\boldsymbol{x}_i,t)+\sum\limits_{j=1,j\ne i}^{N}(\boldsymbol{A}_{ij}\boldsymbol{x}_j+\boldsymbol{h}_{ij}(\boldsymbol{x}_j,t)),\\ \qquad\qquad i=1,2,\cdots,N,\ t\ne t_k,\\ \boldsymbol{x}_i(t_k^+)=\boldsymbol{x}_i(t_k)+\boldsymbol{u}_i(t_k,\boldsymbol{x}_i(t_k)),\qquad t=t_k\end{cases}\tag{1}$$

where $t\in J=[t_0,+\infty)(t_0\ge 0)$, $\boldsymbol{x}_i\in R^{n_i}$ is the state variable of the ith subsystem, $\boldsymbol{A}_i$ is an $n_i\times n_i$ matrix, $\boldsymbol{A}_{ij}$ is the conjunction matrix of appropriate dimension which denotes the jth subsystem's relation with the ith subsystem. $\boldsymbol{f}_i(\boldsymbol{x}_i,t)$, $\boldsymbol{h}_{ij}(\boldsymbol{x}_j,t)$ are the unknown vector fields, the first denotes the uncertainty of the ith subsystem and the later denotes the uncertainty relationship of the jth subsystem to the ith subsystem.

Suppose $\boldsymbol{f}_i(0,t)=0$, $\boldsymbol{h}_{ij}(0,t)=0$, and $\|\boldsymbol{f}_i(\boldsymbol{x}_i,t)\|\le\alpha_i\|\boldsymbol{x}_i\|$, $\|\boldsymbol{h}_{ij}(\boldsymbol{x}_j,t)\|\le\alpha_{ij}\|\boldsymbol{x}_j\|$, $\boldsymbol{x}(t_k^+)=\lim\limits_{t\to t_k^+}\boldsymbol{x}(t)$. Impulsive control law $\boldsymbol{u}_i(t_k,\boldsymbol{x}_i(t_k))$ has the effect of suddenly changing the state of the system at the points t_k, where $t_1<t_2\cdots<t_k<\cdots$, $\lim\limits_{k\to\infty}t_k=\infty$.

Definition 1 System

$$\begin{cases}\dot{\boldsymbol{x}}_i=\boldsymbol{A}_i\boldsymbol{x}_i+\boldsymbol{f}_i(\boldsymbol{x}_i,t),\ i=1,2,\cdots,N,\ t\ne t_k,\\ \boldsymbol{x}_i(t_k^+)=\boldsymbol{x}_i(t_k)+\boldsymbol{u}_i(t_k,\boldsymbol{x}_i(t_k)),\qquad t=t_k\end{cases}\tag{2}$$

is called the isolated subsystem of system (1).

Definition 2 Suppose there exist impulsive control laws $\boldsymbol{u}_i(t_k,\boldsymbol{x}_i(t_k))$, which make system (1), (2) all asymptotically stable, then $\boldsymbol{u}_i(t_k,\boldsymbol{x}_i(t_k))$ is called the decentralized impulsive controller of system (1).

In this paper, $\boldsymbol{u}_i(t_k,\boldsymbol{x}_i(t_k))$ can be chosen as the form of state feedback $\boldsymbol{B}_{ik}\boldsymbol{x}_i(t_k)$, where $\boldsymbol{B}_{ik}$ are $n_i\times n_i$ constant matrices. The objective of this paper is to design decentralized impulsive control matrices $\boldsymbol{B}_{ik}$ and impulse occurring time interval (t_k–t_{k-1}) to make system (1), (2) all asymptotically stable. For convenience, define the following notation:

$$\begin{aligned}&\lambda_i=\lambda_{i\max}[\boldsymbol{P}_i^{-1}(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i\boldsymbol{A}_i)],\ \lambda=\max[\lambda_1,\cdots,\lambda_N],\\&\beta_{ik}=\lambda_{i\max}[\boldsymbol{P}_i^{-1}(\boldsymbol{I}+\boldsymbol{B}_{ik})^{\mathrm{T}}\boldsymbol{P}_i(\boldsymbol{I}+\boldsymbol{B}_{ik})],\\&\beta_i=\max[\beta_{i1},\cdots,\beta_{ik}],\ \beta=\max[\beta_1,\cdots,\beta_N],\\&\lambda_{i\mathrm{M}}=\lambda_{i\max}(\boldsymbol{P}_i),\ \lambda_{\mathrm{M}}=\max[\lambda_{1\mathrm{M}},\cdots,\lambda_{N\mathrm{M}}],\\&\lambda_{i\mathrm{m}}=\lambda_{i\min}(\boldsymbol{P}_i),\ \lambda_{\mathrm{m}}=\min[\lambda_{1\mathrm{m}},\cdots,\lambda_{N\mathrm{m}}],\\&\lambda_{ji}=\lambda_{\max}[\boldsymbol{P}_i^{-1}(\boldsymbol{A}_{ji}^{\mathrm{T}}\boldsymbol{A}_{ji})],\\&\lambda_{ji\mathrm{M}}=\max\sum_{j=1,j\ne i}^{N}\lambda_{ji},\ \alpha=\max[\alpha_1,\cdots,\alpha_N],\\&\alpha_{ij\mathrm{M}}=\max\sum_{j=1,j\ne i}^{N}\alpha_{ij},\ \Delta_{\min}=\inf[t_2-t_1,\cdots,t_{k+1}-t_k],\\&\Delta_{\max}=\sup[t_2-t_1,\cdots,t_{k+1}-t_k],\qquad i,j=1,2,\cdots,N,\end{aligned}$$

where $\boldsymbol{P}_i$ are $n_i\times n_i$ positive definite matrices, $\boldsymbol{I}$ is the $n_i\times n_i$ identity matrix, $\lambda_{\max}(\boldsymbol{M})$ is the maximal eigenvalue of matrix $\boldsymbol{M}$. $\Delta_{\min}$, $\Delta_{\max}$ denote the smallest and the largest time interval of the impulsive control respectively.

Theorem 1 Assume there exist positive definite matrices $\boldsymbol{P}_i$, control matrices $\boldsymbol{B}_{ik}$ and a positive constant ε to make the following conditions hold

1) If $\beta \geq \beta_i \geq 1$,

$$\frac{\ln\beta}{\Delta_{\min}}+\lambda+2\alpha\sqrt{\frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{m}}}}+\frac{\lambda_{ji\mathrm{M}}}{\varepsilon}+2\varepsilon(N-1)\lambda_{\mathrm{M}}+\frac{\alpha_{ij\mathrm{M}}^2}{\lambda_{\mathrm{m}}}<0,$$

then the systems (1), (2) are all asymptotically stable .

2) If $\beta_i \leq \beta \leq 1$,

$$\frac{\ln\beta}{\Delta_{\max}}+\lambda+2\alpha\sqrt{\frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{m}}}}+\frac{\lambda_{ji\mathrm{M}}}{\varepsilon}+2\varepsilon(N-1)\lambda_{i\mathrm{M}}+\frac{\alpha_{ij\mathrm{M}}^2}{\lambda_{\mathrm{m}}}<0,$$

then the systems (1), (2) are all asymptotically stable.

Proof Construct a Lyapunov function $\boldsymbol{V}_i(\boldsymbol{x}_i)=\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i$ of each isolated subsystem. When $t \neq t_k$, we have

$$\begin{aligned} D^{+}\boldsymbol{V}_i(\boldsymbol{x}_i)\big|_{(2)} &= \dot{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i+\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\dot{\boldsymbol{x}}_i \\ &= \boldsymbol{x}_i^{\mathrm{T}}(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i\boldsymbol{A}_i)\boldsymbol{x}_i \\ &\quad +\boldsymbol{f}_i^{\mathrm{T}}(x_i)\boldsymbol{P}_i\boldsymbol{x}_i+\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{f}_i(\boldsymbol{x}_i). \end{aligned} \tag{3}$$

By Lemma 1 and Lemma 4, we have

$$D^{+}\boldsymbol{V}_i(\boldsymbol{x}_i)\big|_{(2)} \leq (\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})\boldsymbol{V}_i(\boldsymbol{x}_i),\ t\neq t_k, \tag{4}$$

which implies that

$$\begin{aligned} \boldsymbol{V}_i(t) &\leq \boldsymbol{V}_i(t_{k-1}^{+})\exp[\int_{t_{k-1}}^{t_k}(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})\mathrm{d}s] \\ &= \boldsymbol{V}_i(t_{k-1}^{+})\mathrm{e}^{(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})(t_k-t_{k-1})}, \\ & t\in(t_{k-1},t_k],\quad k=1,2,\cdots. \end{aligned} \tag{5}$$

When $t=t_k$, we have

$$\begin{aligned} \boldsymbol{V}_i(\boldsymbol{x}_i+\boldsymbol{B}_{ik}\boldsymbol{x}_i)\big|_{t=t_k} &= (\boldsymbol{x}_i+\boldsymbol{B}_{ik}\boldsymbol{x}_i)^{\mathrm{T}}\boldsymbol{P}_i(\boldsymbol{x}_i+\boldsymbol{B}_{ik}\boldsymbol{x}_i) \\ &= \boldsymbol{x}_i^{\mathrm{T}}(\boldsymbol{I}+\boldsymbol{B}_{ik}^{\mathrm{T}})\boldsymbol{P}_i(\boldsymbol{I}+\boldsymbol{B}_{ik})\boldsymbol{x}_i \\ &\leq \beta_{ik}\boldsymbol{V}_i(\boldsymbol{x}_i). \end{aligned} \tag{6}$$

when $t\in(t_0, t_1]$,

$$\boldsymbol{V}_i(t)\leq \boldsymbol{V}_i(t_0^{+})\mathrm{e}^{(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})(t-t_0)}, \tag{7}$$

which leads to

$$\boldsymbol{V}_i(t_1)\leq \boldsymbol{V}_i(t_0^{+})\mathrm{e}^{(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})(t_1-t_0)},$$

and

$$\boldsymbol{V}_i(t_1^{+})\leq \beta_{i1}\boldsymbol{V}_i(t_1)\leq \beta_{i1}\boldsymbol{V}_i(t_0^{+})\mathrm{e}^{(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})(t_1-t_0)}. \tag{8}$$

Similarly, for $t\in(t_1, t_2]$,

$$\boldsymbol{V}_i(t)\leq \boldsymbol{V}_i(t_1^{+})\mathrm{e}^{(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})(t-t_1)} \leq \beta_{i1}\boldsymbol{V}_i(t_0^{+})\mathrm{e}^{(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})(t-t_0)}. \tag{9}$$

In general, for $t\in(t_k, t_{k+1}]$,

$$\begin{aligned} \boldsymbol{V}_i(t) &\leq \beta_{i1}\cdots\beta_{ik}\boldsymbol{V}_i(t_0^{+})\mathrm{e}^{(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})(t-t_0)} \\ &\leq \beta_i^{k}\boldsymbol{V}_i(t_0^{+})\mathrm{e}^{(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})(t-t_0)} \\ &= \mathrm{e}^{k\ln\beta_i}\boldsymbol{V}_i(t_0^{+})\mathrm{e}^{(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})(t-t_0)}. \end{aligned} \tag{10}$$

Hence, when $\beta_i \geq 1$, we have

$$\boldsymbol{V}_i(t)\leq \mathrm{e}^{\frac{t-t_0}{\Delta_{\min}}\ln\beta_i}\boldsymbol{V}_i(t_0^{+})\mathrm{e}^{(\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}})(t-t_0)}. \tag{11}$$

Furthermore, when

$$\frac{\ln\beta_i}{\Delta_{\min}}+\lambda_i+2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}}<0 \tag{12}$$

is satisfied, the isolated subsystem is asymptotically stable.

Similarly, if $\beta_i \le 1$, $\frac{\ln\beta_i}{\Delta_{\max}} + \lambda_i + 2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}} < 0$ holds, the isolated subsystem is asymptotically stable.

Construct a Lyapunov function $\boldsymbol{V}(\boldsymbol{x}) = \sum_{i=1}^{N}(\boldsymbol{x}_i^{\mathrm{T}} \cdot \boldsymbol{P}_i\boldsymbol{x}_i)$ of the large-scale system; then we have

$$\begin{aligned}\dot{\boldsymbol{V}}(\boldsymbol{x}) &= \sum_{i=1}^{N}(\dot{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i + \boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\dot{\boldsymbol{x}}_i) \\ &= \sum_{i=1}^{N}[(\boldsymbol{A}_i\boldsymbol{x}_i + \boldsymbol{f}_i(\boldsymbol{x}_i,t) + \sum_{j=1,j\ne i}^{N}(\boldsymbol{A}_{ij}\boldsymbol{x}_j + \boldsymbol{h}_{ij}(\boldsymbol{x}_j,t)))^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i \\ &\quad + \boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i(\boldsymbol{A}_i\boldsymbol{x}_i + \boldsymbol{f}_i(\boldsymbol{x}_i,t) + \sum_{j=1,j\ne i}^{N}(\boldsymbol{A}_{ij}\boldsymbol{x}_j + \boldsymbol{h}_{ij}(\boldsymbol{x}_j,t)))] \\ &= \sum_{i=1}^{N}[\boldsymbol{x}_i^{\mathrm{T}}(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i\boldsymbol{A}_i)\boldsymbol{x}_i + \boldsymbol{f}_i^{\mathrm{T}}(\boldsymbol{x}_i,t)\boldsymbol{P}_i\boldsymbol{x}_i \\ &\quad + \boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{f}_i(\boldsymbol{x}_i,t) + \sum_{j=1,j\ne i}^{N}(\boldsymbol{x}_j^{\mathrm{T}}\boldsymbol{A}_{ij}^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i + \boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{A}_{ij}\boldsymbol{x}_j) \\ &\quad + \sum_{j=1,j\ne i}^{N}(\boldsymbol{h}_{ij}^{\mathrm{T}}(\boldsymbol{x}_j,t)\boldsymbol{P}_i\boldsymbol{x}_i + \boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{h}_{ij}(\boldsymbol{x}_j,t))]. \end{aligned} \tag{13}$$

By the identical equation

$$\sum_{i=1}^{N}\sum_{j=1,j\ne i}^{N}\boldsymbol{x}_j^{\mathrm{T}}\boldsymbol{A}_{ij}^{\mathrm{T}}\boldsymbol{A}_{ij}\boldsymbol{x}_j = \sum_{i=1}^{N}\sum_{j=1,j\ne i}^{N}\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{A}_{ji}^{\mathrm{T}}\boldsymbol{A}_{ji}\boldsymbol{x}_i,$$

and Lemma 1 and Lemma 2, we have

$$\begin{aligned}\dot{\boldsymbol{V}}(\boldsymbol{x}) &\le \sum_{i=1}^{N}[\lambda_i\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i + 2\alpha_i\sqrt{\frac{\lambda_{i\mathrm{M}}}{\lambda_{i\mathrm{m}}}}\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i \\ &\quad + \sum_{j=1,j\ne i}^{N}(\frac{\lambda_{ji}}{\varepsilon}\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i + 2\varepsilon\lambda_{i\mathrm{M}}\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i \\ &\quad + \frac{\alpha_{ij}^2}{\lambda_{i\mathrm{m}}}\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}_i)] \le (\lambda + 2\alpha\sqrt{\frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{m}}}} + \frac{\lambda_{ji\mathrm{M}}}{\varepsilon} \\ &\quad + 2\varepsilon(N-1)\lambda_{i\mathrm{M}} + \frac{\alpha_{ij\mathrm{M}}^2}{\lambda_{\mathrm{m}}})V, \quad t\in(t_{k-1}, t_k] \end{aligned} \tag{14}$$

When $t = t_k$, we have

$$\begin{aligned}\boldsymbol{V}(\boldsymbol{x})\big|_{t=t_k} &= \sum_{i=1}^{N}(\boldsymbol{x}_i + \boldsymbol{B}_i\boldsymbol{x}_i)^{\mathrm{T}}\boldsymbol{P}_i(\boldsymbol{x}_i + \boldsymbol{B}_i\boldsymbol{x}_i) \\ &= \sum_{i=1}^{N}\boldsymbol{x}_i^{\mathrm{T}}(\boldsymbol{I} + \boldsymbol{B}_{ik}^{\mathrm{T}})\boldsymbol{P}_i(\boldsymbol{I} + \boldsymbol{B}_{ik})\boldsymbol{x}_i \le \beta V(x). \end{aligned} \tag{15}$$

Hence, when $\beta \ge 1$,

$$\frac{\ln\beta}{\Delta_{\min}} + \lambda + 2\alpha\sqrt{\frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{m}}}} + \frac{\lambda_{ji\mathrm{M}}}{\varepsilon} + 2\varepsilon(N-1)\lambda_{i\mathrm{M}} + \frac{\alpha_{ij\mathrm{M}}^2}{\lambda_{\mathrm{m}}} < 0 \tag{16}$$

holds, then the interconnected system (1) is asymptotically stable as well.

Comparing Eq.(12) with Eq.(16), we know, when Eq.(16) holds, Eq.(12) is certainly satisfied.

Hence, when Eq.(16) holds, the interconnected system (1) and the isolated subsystem (2) are all asymptotically stable.

Similarly, if $\beta \le 1$,

$$\frac{\ln\beta}{\Delta_{\max}} + \lambda + 2\alpha\sqrt{\frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{m}}}} + \frac{\lambda_{ji\mathrm{M}}}{\varepsilon} + 2\varepsilon(N-1)\lambda_{i\mathrm{M}} + \frac{\alpha_{ij\mathrm{M}}^2}{\lambda_{\mathrm{m}}} < 0$$

holds, then the systems (1) and (2) are all asymptotically stable.

This completes the proof.

Corollary 1 In Theorem 1, suppose $N = 1$, $\boldsymbol{A}_{ij} = 0$, $\boldsymbol{h}_{ij} = 0$; the following conditions hold

(1) If $\beta \ge 1$,

$$\frac{\ln\beta}{\Delta_{\min}} + \lambda + 2\alpha\sqrt{\frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{m}}}} < 0,$$

then the system (1)is asymptotically stable .

(2) If $\beta \le 1$,

$$\frac{\ln\beta}{\Delta_{\max}} + \lambda + 2\alpha\sqrt{\frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{m}}}} < 0,$$

then the system (1) is asymptotically stable.

This is the special case of a single system.

In practical application, in order to predigest design, control matrices $\boldsymbol{B}_{ik}$ are chosen as constant matrices $\boldsymbol{B}_i$, matrix $\boldsymbol{P}_i$ is chosen as identity matrix. Positive constant $\varepsilon = 1$. We have the following theorem.

Theorem 2 Suppose the impulsive control interval is a constant such that $t_k - t_{k-1} = \Delta$, control matrices $\boldsymbol{B}_{ik}$ are chosen as constant matrices $\boldsymbol{B}_i$ (i=1, 2,$\cdots$, N) whose largest eigenvalue is $\lambda_{\boldsymbol{B}\mathrm{M}}$. Matrices

$\boldsymbol{P}_i$ (i=1, 2, $\cdots$, N) are chosen as identity matrix. Positive constant ε = 1 to make the following hold

$$2\frac{\ln|1+\lambda_{\boldsymbol{B}\mathrm{M}}|}{\Delta}+\lambda+2\alpha+\lambda_{ji\mathrm{M}}+2(N-1)+\alpha_{ij\mathrm{M}}^2<0,$$

then the isolated and the interconnected systems (1), (2) are all asymptotically stable.

By Lemma 3, we can proof Theorem 2; the method is similar to Theorem 1.

NUMERICAL VERIFICATION

Example 1 Consider system (1), the parameters as follows:

$$\boldsymbol{A}_1=\boldsymbol{A}_2=\begin{bmatrix}2 & 1\\ 1 & 0\end{bmatrix}, \boldsymbol{A}_{12}=\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}, \boldsymbol{A}_{21}=\begin{bmatrix}-2 & 1\\ 0 & 0\end{bmatrix},$$

$$\boldsymbol{f}_1(\boldsymbol{x}_1, t)=\begin{bmatrix}0.5\boldsymbol{x}_{11}\cos\boldsymbol{x}_{11}\\ \boldsymbol{x}_{12}\sin\boldsymbol{x}_{12}\end{bmatrix}, \boldsymbol{f}_2(\boldsymbol{x}_2, t)=\begin{bmatrix}0.5\boldsymbol{x}_{21}\cos\boldsymbol{x}_{21}\\ \boldsymbol{x}_{22}\sin\boldsymbol{x}_{22}\end{bmatrix},$$

$$\boldsymbol{h}_{12}(\boldsymbol{x}_2, t)=\begin{bmatrix}0.25\boldsymbol{x}_{21}\cos\boldsymbol{x}_{21}\\ 0.25\boldsymbol{x}_{22}\sin\boldsymbol{x}_{22}\end{bmatrix}, \boldsymbol{h}_{21}(\boldsymbol{x}_1, t)=\begin{bmatrix}0.25\boldsymbol{x}_{11}\cos\boldsymbol{x}_{11}\\ 0.25\boldsymbol{x}_{12}\sin\boldsymbol{x}_{12}\end{bmatrix},$$

It is easily verified

$$\|\boldsymbol{f}_1(\boldsymbol{x}_1, t)\|\le\|\boldsymbol{x}_1\|, \quad \|\boldsymbol{f}_2(\boldsymbol{x}_1, t)\|\le\|\boldsymbol{x}_2\|,$$
$$\|\boldsymbol{h}_{12}(\boldsymbol{x}_2, t)\|\le 0.25\|\boldsymbol{x}_2\|, \quad \|\boldsymbol{h}_{21}(\boldsymbol{x}_1, t)\|\le 0.25\|\boldsymbol{x}_1\|,$$

In this example, we choose ε=1, control matrix $\boldsymbol{B}_i$ as

$$\boldsymbol{B}_1=\begin{bmatrix}-0.72 & 0\\ 0 & -0.65\end{bmatrix}, \boldsymbol{B}_2=\begin{bmatrix}-0.78 & 0\\ 0 & -0.90\end{bmatrix},$$

We have N=2, α=1, $\alpha_{ij\mathrm{M}}$=0.25, $\lambda=\lambda_i$=4.828 (i=1, 2), when i=1, $\sum_{j=1,j\ne i}^{2}\lambda_{j1}=5$; when i=2, $\sum_{j=1,j\ne i}^{2}\lambda_{j2}=5.30$. By Theorem 2, we have $\Delta\le$0.146, choose Δ =0.1, initial value $(\boldsymbol{x}_{11}, \boldsymbol{x}_{12}, \boldsymbol{x}_{21}, \boldsymbol{x}_{22})$ =(10, 10, 10, 10)$^{\mathrm{T}}$. The system states trajectories without and under impulsive control are shown in Fig.1 and Fig.2 respectively.

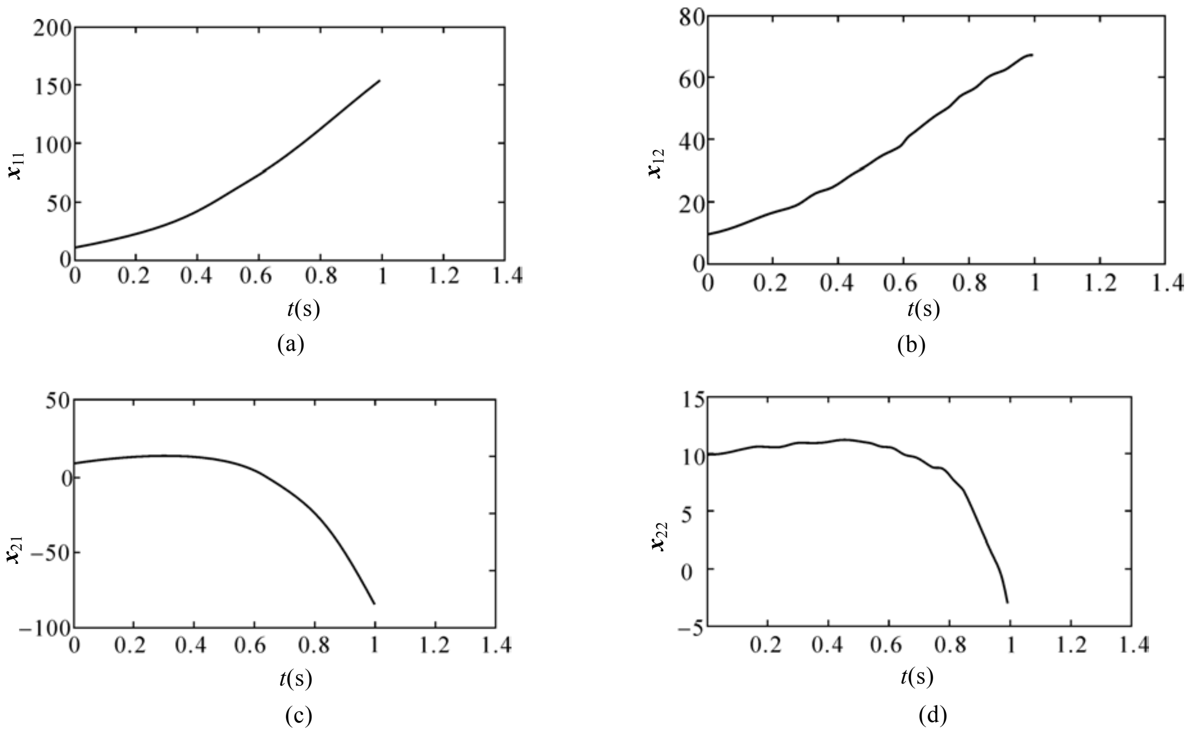

Fig.1 State trajectories of the system without impulsive control

(a) $\boldsymbol{x}_{11}$ trajectory; (b) $\boldsymbol{x}_{12}$ trajectory; (c) $\boldsymbol{x}_{21}$ trajectory; (d) $\boldsymbol{x}_{22}$ trajectory

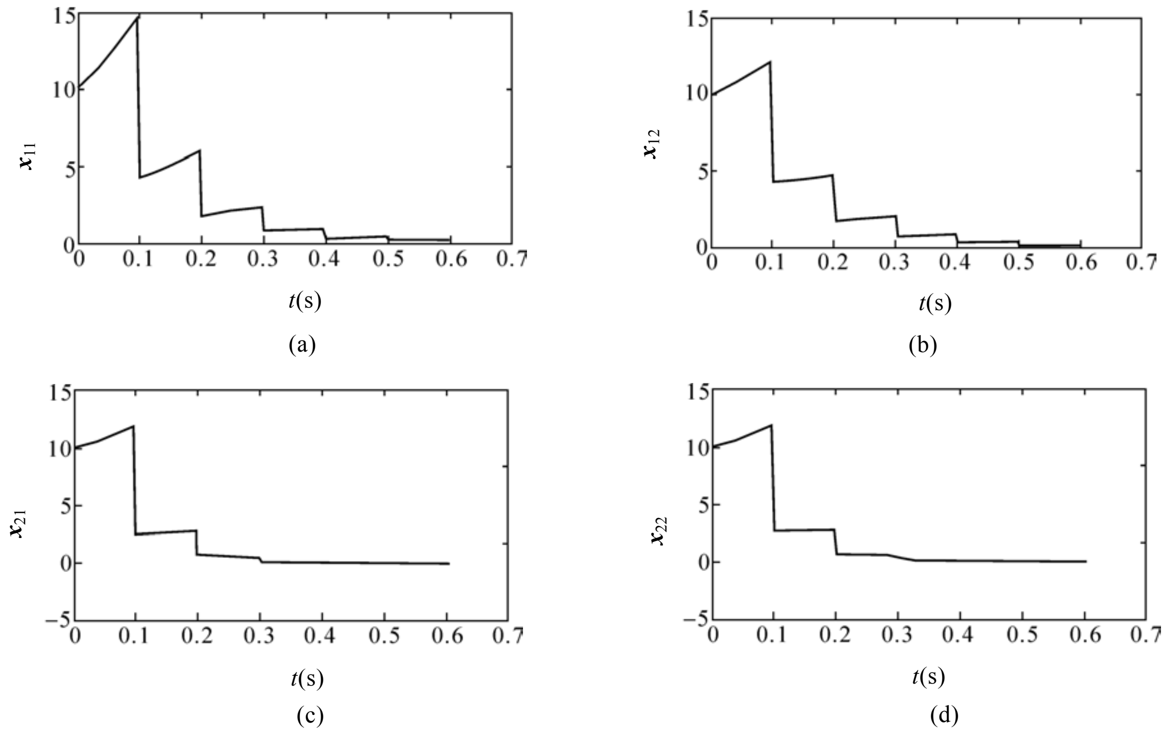

Fig.2 State trajectories of the system under impulsive control

(a) $\boldsymbol{x}_{11}$ trajectory; (b) $\boldsymbol{x}_{12}$ trajectory; (c) $\boldsymbol{x}_{21}$ trajectory; (d) $\boldsymbol{x}_{22}$ trajectory

Example 2 Consider system (1), the parameters as follows:

$$\boldsymbol{A}_1=\boldsymbol{A}_2=\begin{bmatrix}2 & 2\\ 1 & 3\end{bmatrix}, \boldsymbol{A}_{12}=\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}, \boldsymbol{A}_{21}=\begin{bmatrix}-2 & 1\\ 0 & 0\end{bmatrix},$$

$$\boldsymbol{f}_1(\boldsymbol{x}_1,t)=\begin{bmatrix}0.5\boldsymbol{x}_{11}\cos\boldsymbol{x}_{11}\\ 0.5\boldsymbol{x}_{12}\end{bmatrix}, \boldsymbol{f}_2(\boldsymbol{x}_2,t)=\begin{bmatrix}0.5\boldsymbol{x}_{21}\cos\boldsymbol{x}_{21}\\ 0.4\boldsymbol{x}_{22}\sin\boldsymbol{x}_{22}\end{bmatrix},$$

$$\boldsymbol{h}_{12}(\boldsymbol{x}_2,t)=\begin{bmatrix}0.25\boldsymbol{x}_{21}\\ 0.25\boldsymbol{x}_{22}\sin\boldsymbol{x}_{22}\end{bmatrix}, \boldsymbol{h}_{21}(\boldsymbol{x}_1,t)=\begin{bmatrix}0.5\boldsymbol{x}_{11}\cos\boldsymbol{x}_{11}\\ 0.25\boldsymbol{x}_{12}\end{bmatrix},$$

It is easily verified

$$\|\boldsymbol{f}_1(\boldsymbol{x}_1,t)\|\le 0.5\|\boldsymbol{x}_1\|, \quad \|\boldsymbol{f}_2(\boldsymbol{x}_1,t)\|\le 0.5\|\boldsymbol{x}_2\|,$$
$$\|\boldsymbol{h}_{12}(\boldsymbol{x}_2,t)\|\le 0.25\|\boldsymbol{x}_2\|, \quad \|\boldsymbol{h}_{21}(\boldsymbol{x}_1,t)\|\le 0.5\|\boldsymbol{x}_1\|,$$

In this example, we choose ε=1, control matrix $\boldsymbol{B}_i$ as

$$\boldsymbol{B}_1=\begin{bmatrix}-0.42 & 0\\ 0 & -0.25\end{bmatrix}, \boldsymbol{B}_2=\begin{bmatrix}-0.38 & 0\\ 0 & -0.40\end{bmatrix},$$

We have

N=2, α=0.5, $\alpha_{ij\mathrm{M}}$=0.5, λ=λ_i=8.16 (i=1, 2),

when $i=1$, $\sum_{j=1,j\ne i}^{2}\lambda_{j1}=5$;

when i=2, $\sum_{j=1,j\ne i}^{2}\lambda_{j2}=5.30$.

By Theorem 2, we have $\Delta\le 0.0387$; choose $\Delta=0.02$, initial value $(\boldsymbol{x}_{11},\boldsymbol{x}_{12},\boldsymbol{x}_{21},\boldsymbol{x}_{22})$=(10, 10, 10, 10)$^{\mathrm{T}}$. The system states trajectories without and under impulsive control are shown in Fig.3 and Fig.4 respectively.

CONCLUSION

This paper utilizes decentralized impulsive control to stabilize a class of uncertain interconnected systems and provides some new criteria of stabilization. The impulsive controller is easy to design. The theory analysis and simulation results show the method is effective.

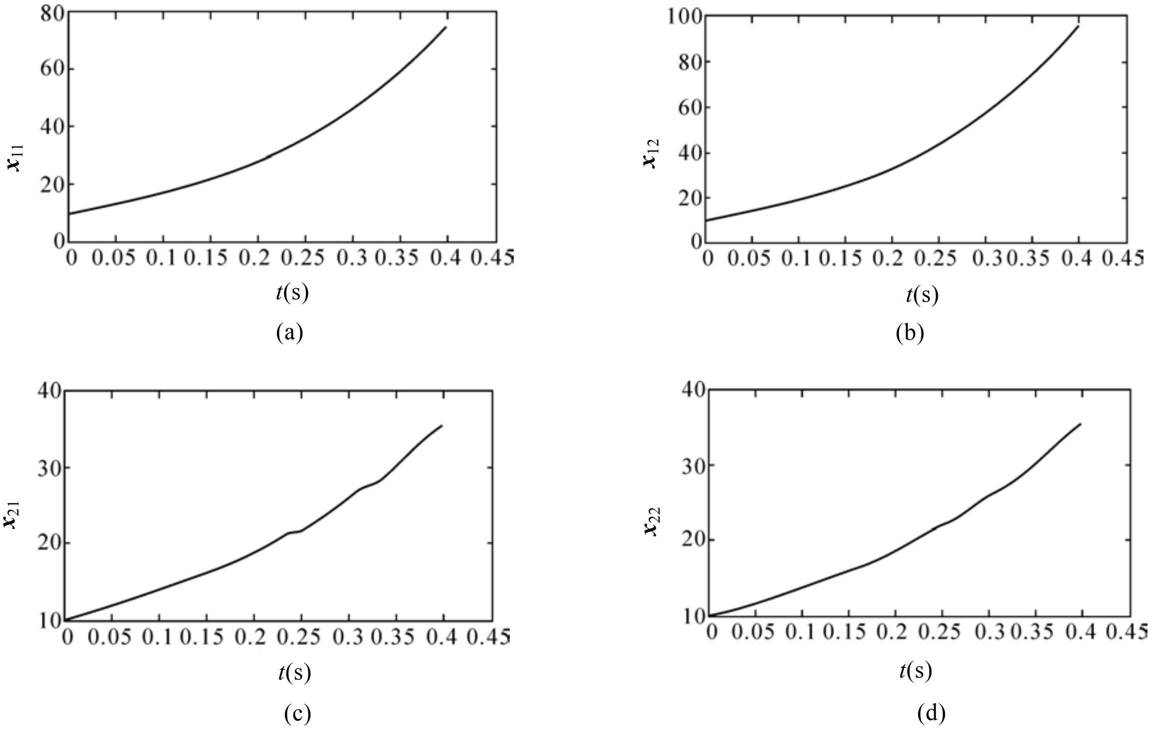

Fig.3 State trajectories of the system without impulsive control

(a) $\boldsymbol{x}_{11}$ trajectory; (b) $\boldsymbol{x}_{12}$ trajectory; (c) $\boldsymbol{x}_{21}$ trajectory; (d) $\boldsymbol{x}_{22}$ trajectory

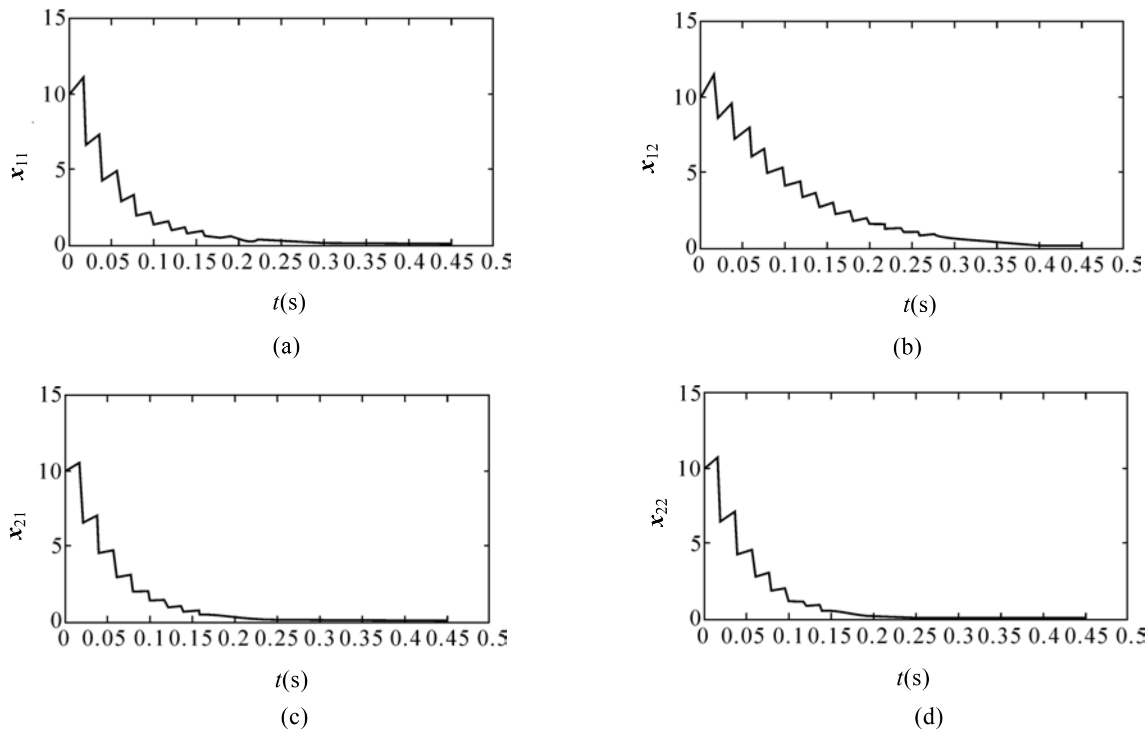

Fig.4 State trajectories of the system under impulsive control

(a) $\boldsymbol{x}_{11}$ trajectory; (b) $\boldsymbol{x}_{12}$ trajectory; (c) $\boldsymbol{x}_{21}$ trajectory; (d) $\boldsymbol{x}_{22}$ trajectory

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