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Time-dependent response of laminated isotropic strips with viscoelastic interfaces*

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Abstract: The two dimensional problem of simply supported laminated isotropic strips with viscoelastic interfaces and under static loading was studied. Exact solution was derived based on the exact elasticity equation and the Kelvin-Voigt viscoelastic interfacial model. Numerical computations were performed for a strip consisting of three layers of equal thickness. Results indicated that the response of the laminate was very sensitive to the presence of viscoelastic interfaces.

Keywords: Viscoelastic interfaces, Isotropic laminated strips, Exact solution
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INTRODUCTION

A traditional premise in the theory of composites is the continuity of tractions and displacements at constituent interfaces. However, the interfaces are generally weaker than the plies in laminates, which then frequently suffer from failure (such as delamination, interlaminar slip, etc.) due to high stress concentration at the interfaces. Before the final failure, these interfaces are usually weakened and become imperfect due to the emergence of microcracks. There are numerous papers in the literature dealing with various aspects of composites with imperfect interfaces (Benveniste, 1985; Hashin, 1991a; Zhong and Meguid, 1996; Yu and Zhong, 1999; Shu and Soldatos, 2001). In most works, such as those mentioned above, the response of laminates under static loading does not vary with time. More recently, He and Jiang (2003) derived an exact two-dimensional solution for isotropic laminates with viscous interfaces and showed that the response of laminates varies remarkably with time, especially at the initial stage. Chen and Lee (2004) proposed an efficient and accurate semi-analytical method for the analysis of angle-ply laminates in cylindrical bending with viscous interfaces. Both studies revealed that, as time approaches infinity, the viscous interfaces will lose the ability of transferring shear stress totally. However, this seems unsuitable for certain types of practical composites, especially within the framework of small deformation. According to Hashin (1991b), a viscoelastic interface will be more appropriate for characterizing the creep and relaxation behavior of interlaminar bonding material under high temperature circumstance.

In this paper, we discuss the two-dimensional responses of a simply-supported laminated isotropic strip (or rectangular plate in cylindrical bending) with viscoelastic interfaces of Kelvin-Voigt type model, subjected to sinusoidal loading. As a primary exploration, we assume that each layer in the strip is elastically isotropic. An

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exact solution is derived and numerical results are given and discussed.

SOLUTION PROCEDURE

Consider an n-layered simply-supported laminated isotropic strip (plate in cylindrical bending) as shown in Fig.1. The strip has width of l, and is simply supported at x=0 and x=l. The Young's modulus and Poisson's ratio of the kth layer are E_k and μ_k respectively. The constitutive law of the kth layer is then written as (He and Jiang, 2003)

$$\sigma_x^{(k)}=\frac{E_k}{(1-2\mu_k)(1+\mu_k)}\left[(1-\mu_k)\frac{\partial u^{(k)}}{\partial x}+\mu_k\frac{\partial w^{(k)}}{\partial z}\right]$$
$$\sigma_z^{(k)}=\frac{E_k}{(1-2\mu_k)(1+\mu_k)}\left[(1-\mu_k)\frac{\partial w^{(k)}}{\partial z}+\mu_k\frac{\partial u^{(k)}}{\partial x}\right]$$
$$\tau_{xz}^{(k)}=\frac{E_k}{2(1+\mu_k)}\left(\frac{\partial u^{(k)}}{\partial z}+\frac{\partial w^{(k)}}{\partial x}\right) \quad (1)$$

where u and w are the displacements in x- and z-directions, respectively. σ_x and σ_z are the normal stresses, and τ_{xz} is the shear stress. The equilibrium equations can be written in terms of displacements as

$$2(1-\mu_k)\frac{\partial^2 u^{(k)}}{\partial x^2}+(1-2\mu_k)\frac{\partial^2 u^{(k)}}{\partial z^2}+\frac{\partial^2 w^{(k)}}{\partial x\partial z}=0$$
$$2(1-\mu_k)\frac{\partial^2 w^{(k)}}{\partial z^2}+(1-2\mu_k)\frac{\partial^2 w^{(k)}}{\partial x^2}+\frac{\partial^2 u^{(k)}}{\partial x\partial z}=0 \quad (2)$$

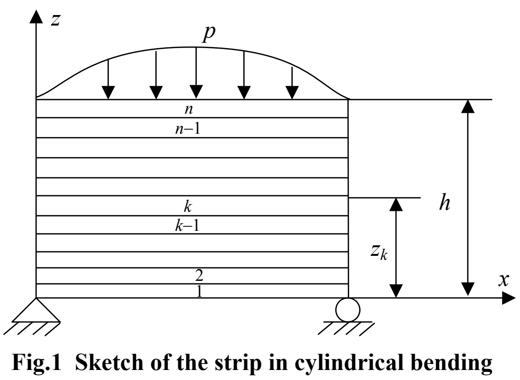


Fig.1 Sketch of the strip in cylindrical bending

Considering a sinusoidal loading p=$p_0\sin(\alpha x)$ (α=π/l) applied on the top surface, we will solve Eq.(2) under the following boundary and interfacial conditions

$$\sigma_z^{(n)}=-p_0\sin\alpha x,\ \tau_{xz}^{(n)}=0,\ \text{at}\ z=h$$
$$\sigma_z^{(1)}=0,\ \tau_{xz}^{(1)}=0,\ \text{at}\ z=0 \quad (3)$$
$$\sigma_x^{(k)}=w^{(k)}=0,\ \text{at}\ x=0\ \text{and}\ x=l \quad (4)$$
$$\sigma_z^{(k+1)}=\sigma_z^{(k)},\ \tau_{xz}^{(k+1)}=\tau_{xz}^{(k)},$$
$$u^{(k+1)}=u^{(k)}+\delta^{(k)},\ w^{(k+1)}=w^{(k)},\ \text{at}\ z=z_k \quad (5)$$

where $\delta^{(k)}$ is the relative sliding displacement at the kth interface. In this paper, we assume that the shear stress and sliding obey the Kelvin-Voigt viscoelastic law

$$\tau_{xz}^{(k)}=\eta_0^{(k)}\delta^{(k)}+\eta_1^{(k)}\dot{\delta}^{(k)}, \quad (6)$$

where $\dot{\delta}^{(k)}$ is the sliding velocity (the dot over a quantity denotes differentiation with respect to time), and $\eta_0^{(k)}$ and $\eta_1^{(k)}$ are the elastic constant and viscous coefficient, respectively. Setting $\eta_0^{(k)}=0$, we get the viscous model studied by He and Jiang (2003). The solution to Eq.(2) had already been derived by He and Jiang (2003) as

$$u^{(k)}=f^{(k)}\cos\alpha x,\ w^{(k)}=g^{(k)}\sin\alpha x, \quad (7)$$

with

$$f^{(k)}(z)=(C_1^{(k)}+zC_2^{(k)})e^{\alpha z}+(C_3^{(k)}+zC_4^{(k)})e^{-\alpha z},$$
$$g^{(k)}(z)=\frac{1}{\alpha}[\alpha C_1^{(k)}+(\alpha z-3+4\mu_k)C_2^{(k)}]e^{\alpha z}$$
$$-\frac{1}{\alpha}[\alpha C_3^{(k)}+(\alpha z+3-4\mu_k)C_4^{(k)}]e^{-\alpha z} \quad (8)$$

where $C_i^{(k)}$ are integral constants to be determined. The corresponding expressions for stresses are

$$\sigma_x^{(k)}=-\frac{E_k}{1+\mu_k}\left\{\left[\alpha C_1^{(k)}+(\alpha z+2\mu_k)C_2^{(k)}\right]e^{\alpha z}\right.$$
$$\left.+\left[\alpha C_3^{(k)}+(\alpha z-2\mu_k)C_4^{(k)}\right]e^{-\alpha z}\right\}\sin\alpha x,$$

$$\sigma_z^{(k)} = \frac{E_k}{1+\mu_k}\Big\{\Big[\alpha C_1^{(k)} + (\alpha z - 2 + 2\mu_k)C_2^{(k)}\Big]\mathrm{e}^{\alpha z} + \Big[\alpha C_3^{(k)} + (\alpha z + 2 - 2\mu_k)C_4^{(k)}\Big]\mathrm{e}^{-\alpha z}\Big\}\sin\alpha x,$$

$$\tau_{xz}^{(k)} = \frac{E_k}{1+\mu_k}\Big\{\Big[\alpha C_1^{(k)} + (\alpha z - 1 + 2\mu_k)C_2^{(k)}\Big]\mathrm{e}^{\alpha z} - \Big[\alpha C_3^{(k)} + (\alpha z + 1 - 2\mu_k)C_4^{(k)}\Big]\mathrm{e}^{-\alpha z}\Big\}\cos\alpha x \quad (9)$$

Further, as in the viscous model (He and Jiang, 2003), we assume the following form for the sliding displacement

$$\delta^{(k)} = \bar{\delta}^{(k)}\cos\alpha x\,, \quad (10)$$

where $\bar{\delta}^{(k)}$ is function of t.

The above solution has satisfied the simply supported conditions in Eq.(4), and there are still $4n$ conditions in Eqs.(3) and (5) to be considered with totally $4n$ unknown constants $C_i^{(k)}$ (i=1, 2, 3, 4; k=1, 2, …, n). These constants are directly related to $\bar{\delta}^{(k)}$, which can be exactly solved from Eq.(6), a first-order ordinary differential equation (or equation set for multiple viscous interfaces). In this paper, we assume that the relative sliding displacement is zero at the initial time. This corresponds to the case that the load has already been applied on the laminate before the interfaces begin to exhibit a viscoelastic character.

NUMERICAL COMPUTATION

We consider a symmetric three-layered strip, for which the Poisson's ratio of each layer (of same thickness) is the same and is denoted by μ. We take μ=0.3, l=10h, E_2=3E_1=3E_3, $\eta_0^{(1)} = \eta_0^{(2)}$, $\eta_1^{(1)} = \eta_1^{(2)}$ in the numerical calculation.

The twelve unknown constants $C_i^{(k)}$ (i=1, 2, 3, 4; k=1, 2, 3) are obtained in terms of $\bar{\delta}^{(1)}$ and $\bar{\delta}^{(2)}$, which can be determined from Eq.(6) as

$$\bar{\delta}^{(1)} = \frac{p_0 h}{2E_1\lambda_1\lambda_2}(a_1\mathrm{e}^{\lambda_1 t/t_0} + b_1\mathrm{e}^{\lambda_2 t/t_0} + c_1)\,,$$

$$\bar{\delta}^{(2)} = \frac{p_0 h}{2E_1\lambda_1\lambda_2}(a_1\mathrm{e}^{\lambda_1 t/t_0} - b_1\mathrm{e}^{\lambda_2 t/t_0} + c_2)\,, \quad (11)$$

where

λ_1=−0.00622826238719−B
λ_2=−0.02154074160281−B
a_1=0.17022582+7.902617B
b_1=−0.00011002−0.018017B
c_1=−0.0850578−3.9423B
c_2=−0.08516802−3.960317B (12)

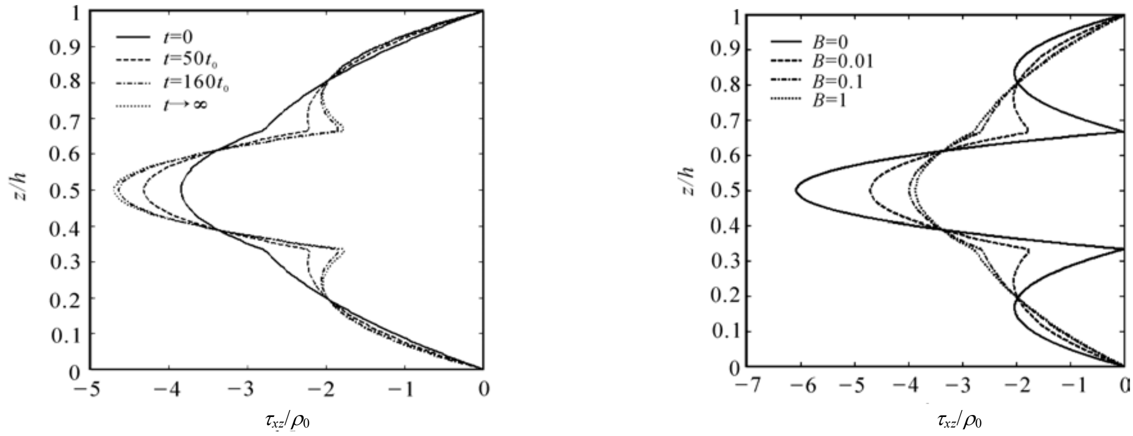
in which $t_0 = h\eta_1^{(1)}/E_1$, $B = \eta_0^{(1)}h/E_1$.

Most results obtained are similar to that obtained by He and Jiang (2003) for viscous interfaces, except for the magnitude, and hence are not given here for brevity. The most significant difference is that the viscoelastic interface will not lose the ability of transferring shear stress when $t\to\infty$, which should be more realistic for certain practical situations. The distribution of shear stress along the thickness direction is highlighted in Fig.2 for different parameters. The curve of B=0 shown in Fig.2b corresponds to the degenerated viscous interfaces, for which the shear stress τ_{xz} becomes zero when $t\to\infty$.

CONCLUSION

Using Kelvin-Voigt model, the response of simply-supported laminated strip with viscoelastic interfaces was investigated under sinusoidal transverse loading. An exact solution is obtained by extending the existent analysis for viscous interfaces. The prominent feature of viscoelastic interfaces is that they always hold the function as interlaminar bonds, although weakened. This should be more realistic for composites under an environment of relatively higher temperature.

In this paper, we only solve a two-dimensional problem, and each layer in the strip is elastically isotropic. The present work may provide a useful means of comparison for future research on more complicated problems.



(a) (b)

Fig.2 Through-thickness distribution of τ_{xz}(*l*/4,*z*) (a) B=0.01; (b) $t\to\infty$

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