Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Nonlinear control for a class of hydraulic servo system

YU Hong (余 宏)†, FENG Zheng-jin (冯正进), WANG Xu-yong (王旭永)

(*School of Mechanical Engineering, Shanghai Jiaotong University, Shanghai 200030, China*)

†E-mail: mailyu@sina.com

Received Aug. 20, 2003; revision accepted Dec. 5, 2003

Abstract: The dynamics of hydraulic systems are highly nonlinear and the system may be subjected to non-smooth and discontinuous nonlinearities due to directional change of valve opening, friction, etc. Aside from the nonlinear nature of hydraulic dynamics, hydraulic servo systems also have large extent of model uncertainties. To address these challenging issues, a robust state-feedback controller is designed by employing backstepping design technique such that the system output tracks a given signal arbitrarily well, and all signals in the closed-loop system remain bounded. Moreover, a relevant disturbance attenuation inequality is satisfied by the closed-loop signals. Compared with previously proposed robust controllers, this paper's robust controller based on backstepping recursive design method is easier to design, and is more suitable for implementation.

Key words: Nonlinear system, Electro hydraulic servo system, Robust control, Backstepping, Lyapunov function
doi:10.1631/jzus.2004.1413 **Document code:** A **CLC number:** TP273.31

INTRODUCTION

Electro hydraulic servo systems (EHSS) have been used in industry in a wide number of applications due to their small size-to-power ratio and the ability to apply very large force and torque. However, the dynamics of hydraulic systems are highly nonlinear (Merritt, 1967), the system may be subjected to non-smooth and discontinuous nonlinearities due to control input saturation, directional change of valve opening, friction, and valve overlap. Aside from the nonlinear nature of hydraulic dynamics, EHSS also have large extent of model uncertainties, such as the external disturbances and leakage that cannot be modeled exactly; and the nonlinear functions that describe them may not be known. Thus, nonlinear robust control techniques, which can deliver high performance in spite of the uncertainties of the system, are essential for successful operation of high-performance hydraulic systems (Plummer and Vaughan, 1996; Tsao and Tomizuka, 1994; Vossoughi and Donath, 1995).

During the last decade, backstepping based design have emerged as powerful tools for stabilizing nonlinear systems for tracking and regulation purposes (Krstic *et al*., 1995). The key idea of backstepping is very simple. At every step of backstepping, a new Control Lyapunov Function (CLF) is constructed by augmentation of the CLF from the previous step by a term, which penalizes the error between a state variable and its desired value (Isidori, 1995; Kokotavic and Murat, 2001). A major advantage of backstepping is the construction of a Lyapunov function whose derivative can be made negative definite by a variety of control laws rather that by a specific control law. The systematic construction of a Lyapunov function for the closed loop allows analysis of its stability properties (Lin and Shen, 1999).

In this paper, we propose a simple backstepping-based robust scheme for an EHSS such that the system output tracks a given signal arbitrarily well, and all signals in the closed-loop system remain bounded. Moreover, a relevant disturbance attenuation inequality is satisfied by the closed-loop signals.

The paper is organized as follows. In Section 2 the dynamic equations of the system under study are presented. The nonlinear controllers are developed in Section 3. In Section 4 simulation results are discussed and finally conclusions are drawn in Section 5.

SYSTEM DYNAMICS

The differential equations governing the dynamics of the hydraulic actuator in Fig.1 are given in (Merritt, 1967).

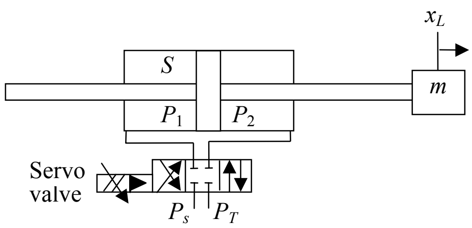

Fig.1 A hydraulic actuator with four-way valve configuration

$$\frac{V_t}{4\beta_e}\dot{P}_L = -S\dot{x}_L - C_{tm}P_L + Q_L \tag{1}$$

where S is the ram area of cylinder, V_t is the total volume of the cylinder and the hoses between the cylinder and servovalve, P_L=P_1−P_2 is the load pressure, β_e is the effective bulk modulus, C_{tm} is the coefficient of the total internal leakage of the cylinder due to pressure, and Q_L is the load flow. Q_L is related to the spool valve displacement of the servo valve, x_v, repressed by

$$Q_L = C_d\omega x_v\sqrt{\frac{P_s - \mathrm{sgn}(x_v)P_L}{\rho}} = Cx_v\sqrt{P_s - \mathrm{sgn}(x_v)P_L} \tag{2}$$

where C_d is the discharge coefficient, ω is the spool valve area gradient, and P_s is the supply pressure of the fluid, $C = C_d\omega\big/\sqrt{\rho}$. x_v is the response of the valve to a command signal u. In particular, the system that we have experimented with, the valve dynamics can be considered to be a second order system (FitzSimons and Palazzolo, 1996):

$$\ddot{x}_v = -\omega_e^2 x_v - 2\xi_e\omega_e\dot{x}_v + \omega_e^2 u \tag{3}$$

The dynamics of the inertia load can be described by

$$P_L S = m\ddot{x}_L + f_f \tag{4}$$

where m is the total mass of the actuator and the load, x_L represents the displacement, and f_f is an unknown nonlinear function, which is the combination of damping friction, viscous friction, the external disturbance, etc. Since the extents of uncertainties can be predicted, we make the following practical assumption $\left|f_f\right| < \Delta(x_L, \dot{x}_L)$. $\Delta(x_L, \dot{x}_L)$ is a known positive nonlinear function, which are allow grow faster than linear, like $\Delta(x_L, \dot{x}_L) = \xi_1 x_L^2 + \xi_2\dot{x}_L^2$, where ξ_1, ξ_2 are positive constants. For ease of presentation, it will be assumed that $\Delta(0,0) = 0$, and that the derivative of $\Delta(x_L, \dot{x}_L)$ exists and is zero at (0,0) (Alleyne and Hedrick, 1995).

Eqs.(1)–(4) completely described the fifth order nonlinear dynamics of the system under study. The corresponding state space representation of these dynamics follows. By defining

$$x_1 = x_L,\ x_2 = \dot{x}_L,\ x_3 = P_L,\ x_4 = x_v,\ x_5 = \dot{x}_v$$

One can write

$$\begin{aligned} &\dot{x}_1 = x_2,\ \ \dot{x}_2 = \frac{S}{m}x_3 - \frac{1}{m}f_f(x) \\ &\dot{x}_3 = \frac{4\beta_e}{V}(-Sx_2 - C_{tm}x_3 + Cx_4\sqrt{P_s - \mathrm{sgn}(x_4)x_3}) \\ &\dot{x}_4 = x_5,\ \ \dot{x}_5 = -\omega_e^2 x_4 - 2\xi_e\omega_e x_5 + \omega_e^2 u \end{aligned} \tag{5}$$

CONTROLLER DESIGN

The nonlinear system described by Eq.(5) is in so-called strict feedback form. This special form allows the use of recursive backstepping procedure for the controller design (Krstic *et al*., 1995). The method basically provides a recursive framework to construct a CLF and corresponding control action for the system stabilization (Jiang and Hill, 1999). In the rest of this section, this idea is adopted to design a nonlinear controller for position tracking in a hydraulic servo system.

Let $e_i=x_i-x_{id}$, i=1,2,…,5 and $e=x-x_d$. The design procedure is beginning with defining the following Lyapunov-like function

$$V_1 = e_1^2/2 \tag{6}$$

In the first equation of Eq.(5), if $x_2=x_{2d}(x_1,x_{1d})$ were our control, it would be able to seek $x_{2d}(x_1,x_{1d})$ to render the derivative of $\dot{V}_1$ negative

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(x_{2d} - \dot{x}_{1d})$$

Let

$$x_{2d} = \dot{x}_{1d} - k_1 e_1 \tag{7}$$

then

$$\dot{V}_1 = -k_1 e_1^2 < 0$$

where k_1 stand for weighting parameters. Now, in order to go one step ahead, a new Lyapunov-like function V_2 is defined as

$$V_2 = V_1 + e_2^2/2 \tag{8}$$

By taking the derivative of Eq.(8)

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + e_2\dot{e}_2\\ &= -k_1e_1^2 + e_2(\frac{S}{m}x_{3d} - \frac{1}{m}f_f - \dot{x}_{2d})\\ &\le -k_1e_1^2 + e_2\frac{S}{m}x_{3d} - e_2\dot{x}_{2d} + |e_2|\frac{1}{m}\Delta(x_1,x_2)\end{aligned}$$

If x_{3d} is chosen as

$$x_{3d} = \frac{m}{S}(-k_2e_2 + \dot{x}_{2d} - \mathrm{sgn}(e_2)\frac{1}{m}\Delta(x_1,x_2)) \tag{9}$$

$\dot{V}_2$ is simplified to

$$\dot{V}_2 \le -k_1e_1^2 - k_2e_2^2 \le 0$$

The new weighting parameter k_2 is also introduced. Let V_3 be defined as follows

$$V_3=V_2+ e_3^2/2 \tag{10}$$

By taking the derivative of V_3, one may write

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + e_3\dot{e}_3 = \dot{V}_2 + e_3(\dot{x}_3 - \dot{x}_{3d})\\ &\le -k_1e_1^2 - k_2e_2^2 + e_3(\dot{x}_3 - \dot{x}_{3d})\end{aligned}$$

where $\dot{x}_{3d}$ exists because of the assumptions on Δ. In this step, because function sgn(x_4) in discontinuous, the desired x_4 is non-differentiable, the backstepping cannot be applied here anymore. If we made a smooth modification to the function sgn(x_4) by replacing it with tanh(x_4/δ) (δ is a small enough positive constant, this modification is reasonable because of the leakage of the valve spool) (Yao *et al*., 1998), the backstepping procedure can be continued.

If x_{4d} is chosen as

$$\begin{aligned}x_{4d} &= \frac{1}{C\sqrt{P_s - \tanh(x_4/\delta)x_3}}\\ &\quad\times(\frac{V}{4\beta_e}(-k_3e_3 + \dot{x}_{3d}) + Sx_2 + C_{tm}x_3)\end{aligned} \tag{11}$$

$\dot{V}_3$ is simplified to

$$\dot{V}_3 \le -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 \le 0$$

Because of the smoothing modification, x_{4d} is differentiable. k_3 represent new weight parameters.

Let V_4 be defined as follows

$$V_4 = V_3 + e_4^2/2 \tag{12}$$

By taking the derivative of V_4, one may write

$$\dot{V}_4 = \dot{V}_3 + e_4\dot{e}_4 = \dot{V}_3 + e_4(x_{5d} - \dot{x}_{4d})$$
$$\le -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_4(x_{5d} - \dot{x}_{4d})$$

If x_{5d} is chosen as

$$x_{5d} = \dot{x}_{4d} - k_4e_4 \tag{13}$$

then

$$\dot{V}_4 \le -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 \le 0$$

where k_4 also represent weight parameters. Let us go one step ahead, and let V_5 be defined as follows

$$V_5 = V_4 + e_5^2/2 \tag{14}$$

By taking the derivative of V_5, one may write

$$\dot{V}_5 = \dot{V}_4 + e_5\dot{e}_5 = \dot{V}_4 + e_5(\dot{x}_5 - \dot{x}_{5d})$$
$$= \dot{V}_4 + e_5(-\omega_e^2x_4 - 2\xi_e\omega_ex_5 + \omega_e^2u - \dot{x}_{5d})$$
$$\le -k_1e_1^2 - k_2e_2^2 - k_3e_3^2$$
$$+ e_5(-\omega_e^2x_4 - 2\xi_e\omega_ex_5 + \omega_e^2u - \dot{x}_{5d})$$

If u is chosen as

$$u = \frac{1}{\omega_e}(\dot{x}_{5d} + \omega_e^2x_4 + 2\xi_e\omega_ex_5 - k_5e_5) \tag{15}$$

then

$$\dot{V}_5 \le -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - k_5e_5^2 \le 0$$

Note that Eq.(12) is a Lyapunov function for the system defined by Eq.(5); and that the control law given by Eqs.(7), (9), (11), (15) renders its derivative negative semidefinite. It is easy to show that e=0 is the largest invariant set in $E = \{e \in \Omega \mid \dot{V}_5(e) = 0\}$. So, using LaSalle's principle, the tracking errors which include position and velocity tracking errors, converge to zero asymptotically (Re and Isidori, 1995).

SIMULATION RESULTS

The results of simulations are presented in this section. The following were the system parameters used for simulation: m=3.3 kg, s=0.002 m^2, C_{tm}=0.1, P_s=21 Mpa, β_e=700 Mpa, δ=0.002, C=1.5×10^{-4}, f_f =0.1x_2−5$x_1$sgn(x_1), V=0.001 m^3, $\Delta = 10x_1^2 + 0.3x_2^2$, k_1=20, k_2=100, k_3=1000, k_4=1000, k_5=800, ξ_e=0.6, ω_e=255 rad/sec.

Figs.2, 3, 4 and 5 present the tracking behavior of the actuator controlled by the nonlinear controller that we developed in the previous section.

CONCLUSON

This paper dealt with the nonlinear control of an EHSS consisting of an electrohydraulic servo valve and a hydraulic cylinder. We proposed a robust nonlinear controller for this nonlinear system via backstepping approach. The simulation result proved that the desired control objective was accomplished with very fast convergence of the control signal to its nominal value.

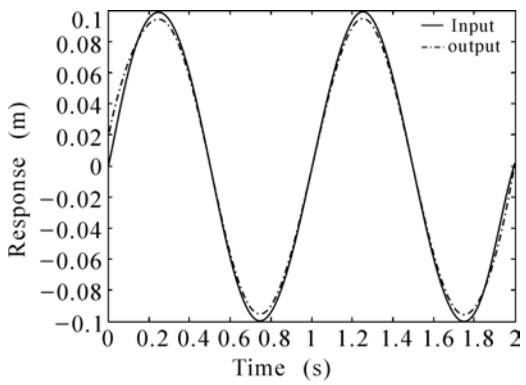

Fig.2 Simulation result of system obtained by using 0.1sin(2πt) as reference input

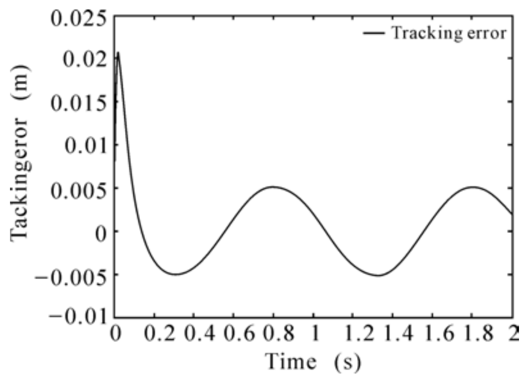

Fig.3 Tracking error of system obtained by using 0.1sin(2πt) as reference input

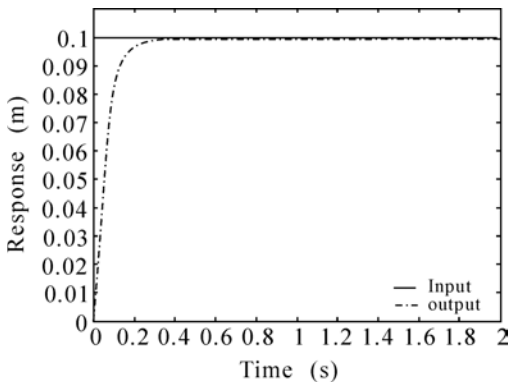

Fig.4 Simulation result of system obtained by using a step as reference input

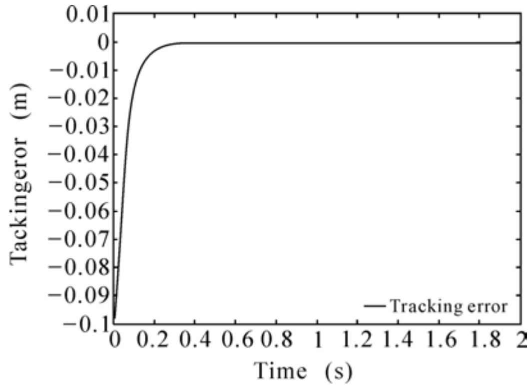

Fig.5 Simulation result of system obtained by using a step as reference input

References

Alleyne, A., Hedrick, J.K., 1995. Nonlinear adaptive control of active suspension. *IEEE Trans. on Control Systems Technology*, **3**(1):94-101.

FitzSimons, P.M., Palazzolo, J.J., 1996. Part I: Modeling of a one-degree-of-freedom active hydraulic mount; Part II: Control. *ASME J. Dynamic Systems, Measurement, and Control*, **118**(3):439-448.

Isidori, A., 1995. Nonlinear Control Systems. Third Edition. Springer, London.

Jiang, Z.P., Hill, D.J., 1999. A robust adaptive backstepping scheme for nonlinear systems with unmodeled dynamics. *IEEE Trans. On Automatic Controller*, **44**(9):1705-1711.

Kokotovic, P.V., Murat, A., 2001. Constructive nonlinear control: a historical perspective. *Automatica*, **37**(5):637-662.

Krstic, M., Kanellakopaulos, I., Kokotovic, P.V., 1995. Nonlinear and Adaptive Control Design. Willey, New York.

Lin, W., Shen, T., 1999. Robust passivity and feedback design for minimum-phase nonlinear systems with structural uncertainty. *Automatic*, **35**(1):35-48.

Merritt, H.E., 1967. Hydraulic Control Systems. Willey, New York.

Plummer, A.R., Vaughan, N.D., 1996. Robust adaptive control for hydraulic servosystems. *ASME J. Dynamic Systems, Measurement, and Control*, **118**(2):237-244.

Re, L.D., Isidori, A., 1995. Performance enhancement of nonlinear drives by feedback linearization of linear-bilinear cascade models. *IEEE Trans. On Control Systems Technology*, **3**(3):299-308.

Tsao, T.C., Tomizuka, M., 1994. Robust adaptive and repetitive digital control and application to hydraulic servo for noncircular machining. *ASME J. Dynamic Systems, Measurement, and Control*, **116**(1):24-32.

Vossoughi, R., Donath, M., 1995. Dynamic feedback linearization for electro-hydraulically actuated control system. *ASME J. Dynamic Systems, Measurement, and Control*, **117**(3):468-477.

Yao, B., Bu, F.P., Chiu, G.T.C., 1998. Nonlinear Adaptive Robust Control of Electro-Hydraulic Servo System with Discontinuous Projection. Proceeding of the 37th IEEE Conference on Decision & Control Tampa, Florida, USA, p.2265-2270.