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Study on dynamic model of tractor system for automated navigation applications*

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Abstract: This research aims at using a dynamic model of tractor system to support navigation system design for an automatically guided agricultural tractor. This model, consisting of a bicycle model of the tractor system, has been implemented in the MATLAB environment and was developed based on a John Deere tractor. The simulation results from this MATLAB model was validated through field navigation tests. The accuracy of the trajectory estimation is strongly affected by the determination of the cornering stiffness of the tractor. In this simulation, the tractor cornering stiffness analysis was identified during simulation analysis using the MATLAB model based on the recorded trajectory data. The obtained data was used in simulation analyses for various navigation operations in the field of interest. The analysis on field validation test results indicated that the developed tractor system could accurately estimate wheel trajectories of a tractor system while operating in agricultural fields at various speeds. The results also indicated that the developed system could accurately determine tractor velocity and steering angle while the tractor operates in curved fields.

Key words: Tractor, Cornering stiffness, Automated navigation, Simulation, GPS
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INTRODUCTION

Automatic guidance of agricultural tractors has been studied over the past several decades. The potential benefits of automated agricultural tractors include increased productivity, increased application accuracy, and enhanced operation safety (Reid *et al.*, 2000). Various guidance technologies, including mechanical guidance, machine-vision guidance, radio navigation, and ultrasonic guidance, had been investigated (Reid *et al.*, 2000; Guo *et al.*, 2003). Most previous work on tractor dynamics considered the tractor alone in attempts to investigate its ride characteristics. However, there are few agricultural operations in which implements are not used. In recent years, several studies were conducted to include the effect of rear-mounted implements on tractor ride vibration (Bukta *et al.*, 1998; Bukta, 1998; Collins, 1991; Crolla, 1976; Sakai, 2000; Sakai and Aihra, 1994), but the results are still unsatisfactory in predicting the motion of the tractor-implement system. Bukta *et al.*(2002) investigated the role of the free play of the three-point hitch linkage system as a source of nonlinearity in tractor-implement systems. University of Illinois researchers have successfully developed and demonstrated a prototyped autonomous tractor that can perform autonomous planting and field cultivation (Zhang, 1999). A real-time kinematic (RTK) GPS and a Fiber Optic Gyroscope were used to provide tractor position, speed, and heading. In addition to the steering control, engine throttle, transmission speed, and 3-point hitch position were automatically controlled via a Controlled

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Area Network (CAN) bus based on field locations. Both the desired tractor path and the desired tractor functions (e.g., travel speed, hitch position) were developed off-line and then loaded into the navigation computer before the field operation started. Han and Zhang (2001) gave a map-based methodology used for the implementation of autonomous field operations of agricultural tractors.

In mechanical inter-row farming operations, especially during curve rows, accurate lateral control of the tractor-implement unit is required, since row space is only 76 cm (30 in). Lateral control of tractor-implement often requires estimates of tire side-slip angle and lateral velocity, which are difficult to measure directly. Several studies were conducted to estimate cornering stiffness for vehicles steering on the road. Sienel (1997) demonstrated how the cornering stiffness could be estimated by measuring dynamic vehicle parameters like lateral accelerations and yaw rate. Kitahama and Sakai (2000) presented a method for measurement method of the normalized equivalent cornering stiffness that dominated a vehicle's steering responses. Bevly *et al.*(2001) used integrated Inertial Navigation System (INS) sensors with GPS velocity measurements for estimating tire cornering stiffness.

The objective of this study was to use a dynamic model of tractor system to support navigation system design for an automatically guided agricultural tractor. This dynamic model consists of a bicycle model of the tractor. This paper showed how to estimate the cornering stiffness of the system, and how to use the estimated results for tractor navigation application.

MATERIAL AND METHODS

Nomenclature

U is velocity of tractor; V, lateral velocity of tractor; γ, yaw rate; m, mass of tractor; M, rolling moment; I_z, moment of inertia; a, distance from front axle to CG; b, distance from rear axle to CG; L, distance from front wheel to rear wheel; d, wheel base; C_{α_f}, front cornering stiffness; C_{α_r}, rear cornering stiffness; α_f, front slip angle; α_r, rear slip angle; $F_{r,x}$, longitudinal force on rear wheel; $F_{f,y}$, lateral force on front wheel; $F_{r,y}$, lateral force on rear wheel; $F_{f,x}$, longitudinal force on front wheel; δ, steering angle.

Mathematical modeling

This developed tractor system model is a general model for wheel-type tractor. It is independent of the maker, the type, and the size of the tractor. To simplify the modeling process, an approach of bicycle tractor model (as depicted in Fig.1) was used. A linear 2 degree of freedom transient bicycle model, which accounts for left and right tires, would be sufficient for simple maneuvering on flat ground. The equation (Wong, 1993) can be given by Eq.(1).

$$\begin{bmatrix}\dot{\boldsymbol{V}}\\ \dot{\gamma}\end{bmatrix}=\begin{bmatrix}\dfrac{C_{\alpha_f}+C_{\alpha_r}}{m\boldsymbol{U}} & \dfrac{aC_{\alpha_f}-bC_{\alpha_r}}{m\boldsymbol{U}}-\boldsymbol{U}\\ \dfrac{aC_{\alpha_f}-bC_{\alpha_r}}{I_z\boldsymbol{U}} & \dfrac{a^2C_{\alpha_f}+b^2C_{\alpha_r}}{I_z\boldsymbol{U}}\end{bmatrix}\begin{bmatrix}\boldsymbol{V}\\ \gamma\end{bmatrix}+\begin{bmatrix}-\dfrac{C_{\alpha_f}}{m} & -\dfrac{C_{\alpha_r}}{m}\\ -\dfrac{aC_{\alpha_f}}{I_z} & \dfrac{bC_{\alpha_r}}{I_z}\end{bmatrix}\begin{bmatrix}\delta\\ 0\end{bmatrix} \tag{1}$$

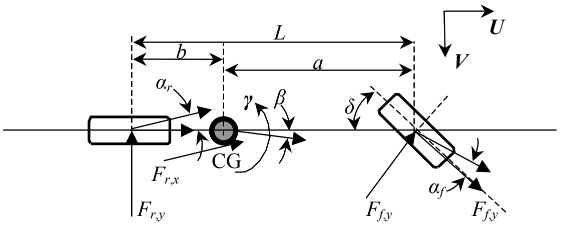

Fig.1 Bicycle model of tractor

Model implementation in MATLAB environment

This dynamic model of tractor system navigation has been implemented in the MATLAB environment. Fig.2 is the model's system block diagram showing that this model consists of six major elements: a parameter input block, a lateral force calculation block, a yaw and lateral acceleration model block, a yaw and lateral velocity model block, a heading angle block and a center gravity of tractor (CG) trajectories model block.

The parameter input block inputs two types of parameters: the implemental parameters and equipment parameters. The implemental parameters include the commending steering angle and the tractor speed. This set of parameters was the model's controlling variables for estimating the tractor system responses in following the desired the trajectory. The other set of required parameters are machinery parameters, including the mass of tractor, the cornering stiffness of tractor front/rear wheels, the moment of inertia of the tractor, the distances of wheel axles to the center of gravity of the tractor, and the wheel sizes.

The second block is the lateral force computation block. It calculates the lateral forces on two wheels based on machinery system parameters, the lateral velocity, and the yaw rates.

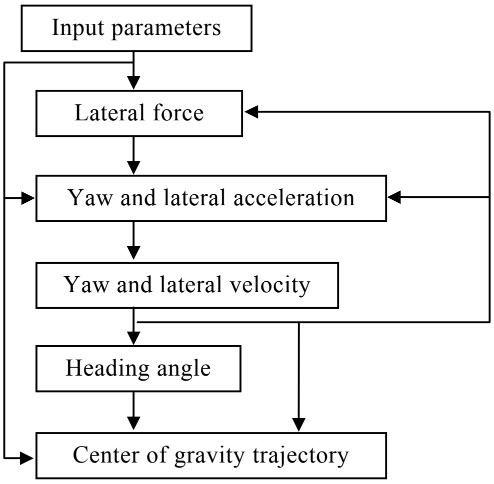

Fig.2 System block diagram of the tractor

The yaw and lateral acceleration block is used to compute the lateral acceleration, yaw accelerations of the tractor based on machinery system parameters and the lateral forces based on Eq.(1).

After the lateral and yaw velocities are calculated based on integral, the next two blocks compute the tractor heading angle; the CG and wheel trajectories of the tractor were calculated based on the tractor geometric dimensions in the last two blocks of the MATLAB implementation model.

Hardware

This MATLAB model of tractor system navigation has been validated against tractor system field test results. Fig.3 is the signal-flow diagram of the test system. The tested platform is based on John Deere 7700 tractor. To provide the test data, a Trimble MS750 RTK-DGPS with GPS antennas was installed on the top of the tractor cap (center gravity of tractor). This RTK-DGPS system with a local reference station can provide precise static and dynamic positioning and velocity information at the same time (horizontal accuracy of 2 cm and vertical accuracy of 3 cm at 20 Hz with a latency of less than 20 ms). An LP801 Potentiometer (OMEGA Engineering, USA), which can be used to measure linear position or displacement up to 1.2 m (48"), output 0–5 V, was installed on the right side front wheel. A JG-108FD Fiber Optic Gyroscope (FOG) (Japan Aviation Electronics, Tokyo, Japan), an angular rate sensor that outputs a relative angle with 0.01 degree error and 0.5 deg/h drift, was installed on both sides of front wheels. These two sensors were used to get accurate steering angle. From steering angle calibration analysis, it was found that the RMS errors of steering angle between the calibration data and the actual data on the right side was 0.0165, mean error was 0.5922 degree. The RMS errors on the left side was 0.0155, mean error was 0.4560 degree. The corresponding maximum errors were 1.3402 degree for left side and 1.5378 degree for right side. Both the maximum errors were found when the tractor underwent the largest turn (about 30 degrees). The Vehicle Control Unit (VCU) was developed as a data acquisition board, consisting of PHYTEC kitCON-167 single board computer (PHYTEC Meβtechnik GmbH, Germany) that sent data to a PC via RS232 serial interface. The PC was equipped with 1 GHz Intel Pentium III CPU and 256 MB memory.

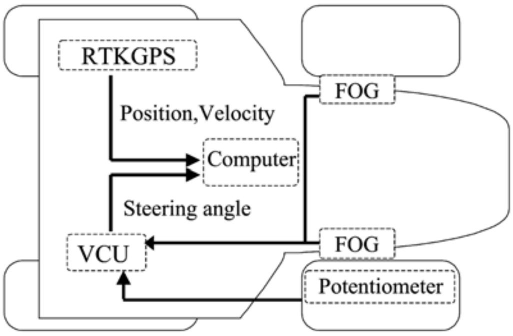

Fig.3 Diagram of hardware system

To provide representative trajectory pattern to validate the model, a series of U turns (either left or right turns) with different turning radius at various speeds were also performed. For controlling variables to the model to estimate the tractor system responses, the commending steering angle was tested by Fiber Optic Gyroscope and Potentiometer sensors, the tractor speed was tested by Trimble MS750 RTK-DGPS on the tractor's center of gravity. The RTK-DGPS receiver was also used to make precise tractor system positioning as the reference to trajectory calculated by the tractor system navigation model. The validation study indicated that this MATLAB model could predict wheel trajectories with reasonable accuracy. Since a set of preliminary test data obtained from the testing field was available for determining the cornering stiffness during turning, the accuracy of wheel trajectory prediction could be improved. A few randomly selected validation results are discussed in this paper.

RESULTS AND DISCUSSIONS

Fig.4 shows the theoretical and actual trajectories of tractor CG while making a 10° right turn (a complete circle turn) at 2.5 mph in a typical central Illinois field. In this evaluation, the actual field test data, including the commending steering angle (as time series data), tractor CG in the format of GPS longitude and latitudes readings, were recorded during field operation. This kind of tests was conducted to determine a set of appropriate wheel cornering stiffness values for specific operation conditions. A series of right and left circle turning tests at various traveling speeds was performed for this purpose. For the specific condition of 10° steering angle and 2.5 mph traveling speed on agricultural fields, it was found that the appropriate cornering stiffness values were 20700 N/rad for tractor front wheel and 40700 N/rad for tractor rear wheel. Using this set of stiffness values in the simulation, the estimated trajectories were matched the actual trajectories reasonably well. From error analysis, it was found that the RMS errors between the estimated and the actual trajectories were 0.02 m at tractor CG. The corresponding maximum errors were about 0.51 m for tractor CG. Both the maximum errors were found when the tractor was started to make the turn. Such high maximum errors may be caused by slower response of tractor steering system than the simulation model because the model did not take the steering actuation system dynamics into consideration.

To obtain tires cornering stiffness of tractor, a series of one complete circle turns (either left or right turns) at different speed (2.5 mile/hour, 3.5 mile/hour, 4.5 mile/hour, 5.5 mile/hour) and different steering angle (5 degrees, 10 degrees, 15 degrees, 20 degrees, 30 degrees) were performed. Then, with the dynamic model, based on specifications of John Deere 7700 tractor, cornering stiffness can be tuned to make the tractor model trajectory fit the real circles. Thus the cornering stiffness of front and rear wheel can be estimated based on different velocities and steering angles, shown in Table 1.

Fig.5 shows the theoretical and actual trajectories of tractor CG while the tractor was performing typical in-field operation (straight line in-field travel connected by end-field left or right U turns) at variable speeds in the typical central-Illinois field. In this particular case, the traveling speed was varying from

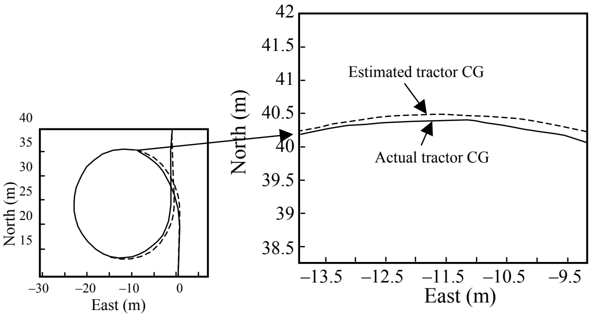

Fig.4 Actual and estimated tractor CG trajectories while the tractor makes a turn in agricultural field

Table 1 Cornering stiffness with different velocity and 30 degree steering angle

Velocity (mile/h)	Steering angle (degree)	Cornering stiffness of front wheel (N/rad)	Cornering stiffness of rear wheel (N/rad)
2.5	30	19800	39300
3.5	30	34800	42300
4.5	30	49400	42600
5.5	30	56000	45000
6.5	30	75000	46600

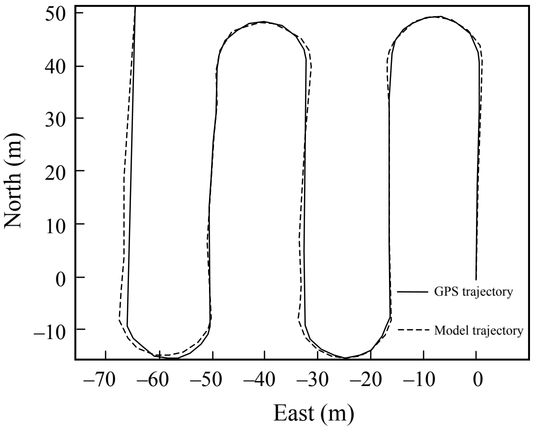

Fig.5 Theoretical and actual trajectories of tractor CG while the tractor was performing field operation

2.5 to 6.5 mph, and the steering angle for end-field turns was either +20° (right turn) or −20° (left turn). Based on the baseline test data obtained from the complete circle turn as discussed above, the appropriate cornering stiffness values were found to be between 20700 and 58200 N/rad for tractor front wheel, between 40700 and 42700 N/rad for tractor rear wheel. By using the above cornering stiffness values in the MATLAB model, and inputting the actual steering angle and tractor speeds to the model, the estimated trajectories of the tractor CG matched the test data fairly well. From the error analysis, it was found that the RMS errors between the estimated and the actual trajectories were 0.01 m at tractor CG. The corresponding maximum errors were 1.56 m for tractor CG. The maximum errors were found when the tractor was making the turn. This large error may be caused by the simplification of the model because the mathematical model of the tractor-implement system was based on the assumption of small steering angle, and the 20° turn obviously deviated from this assumption. While the tractor-implement system was traveling on the straight passes, the maximum error was less than 0.20 m even with the accumulated steering errors.

CONCLUSION

A dynamic model of tractor system was developed for providing a means to predict the wheel trajectories in support of navigation control for an automatically guided agricultural tractor. This tractor system model was developed based on a John Deere 7700 tractor with sufficient details without loss generality. This model, consisting of a bicycle model of the tractor sub-system, has been implemented in the MATLAB environment. The simulation results from this MATLAB model were validated by various field navigation tests. The results from the validation study indicated that the accuracy of trajectory estimation was strongly affected by the determination of the cornering stiffness of the wheels. Since its value was greatly affected by the characteristics of soil and wheel interactions, the wheel cornering stiffness in this simulation analysis was decided during the field test and the simulation. The obtained cornering stiffness was used in simulation analyses for various navigation operations in the field of interest. The analysis of field validation test results indicated that the developed tractor system could accurately estimate wheel trajectories (maximum error of 0.20 m) of a tractor system while operating in agricultural fields at various speeds and steering angles.

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