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# A two-step rectification algorithm for airborne linear images with POS data*

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**Abstract:** Rectification for airborne linear images is an indispensable preprocessing step. This paper presents in detail a two-step rectification algorithm. The first step is to establish the model of direct georeference position using the data provided by the Positioning and Orientation System (POS) and obtain the mathematical relationships between the image points and ground reference points. The second step is to apply polynomial distortion model and Bilinear Interpolation to get the final precise rectified images. In this step, a reference image is required and some ground control points (GCPs) are selected. Experiments showed that the final rectified images are satisfactory, and that our two-step rectification algorithm is very effective.

**Key words:** Airborne linear image, POS data, Two-step rectification algorithm, Direct georeference position, Polynomial distortion model

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## INTRODUCTION

Image rectification is an important preprocessing step in many fields, such as computer vision, remote sensing, data fusion and classification; and had been studied by many researchers (Hartley, 1999; Fusiello and Trucco, 2000; Dong and Wang, 2000; Chen *et al.*, 2003). There are severe distortions in airborne linear images because of the ununiformity and turbulence of the velocity, the gradient, and the altitude of the airplane during flight. These data can only be useful if they are rectified.

The rectification process requires knowledge of the camera interior and exterior orientation parameters (three coordinates of the perspective center, and three rotation angles known as roll, pitch, and yaw angle). Airborne linear images are characterized by each line having its own projection center and a different set of values of the six exterior orientation elements, which lead variation in the displacement of each line so that the rectification process becomes much more difficult. The POS is carried in an airplane, and provides precise position and attitude parameters (Skaloud, 1999). Many researches have discussed sensor modeling to estimate high precise sensor external orientation parameters from POS observations (Lee *et al.*, 2000; Chen, 2001; Daniela, 2002; Gruen and Zhang, 2002; Hinsken *et al.*, 2002).

This paper presents a two-step rectification algorithm based on POS data, which we use for establishing the model of direct georeference position and for adjusting the displacement of each line, then applying polynomial distortion model to determine the geometric relationships of all the pixels in image space corresponding to those on the ground.

## TWO-STEP RECTIFICATION METHOD

### First step: model of direct georeferencing position for each line

According to the theory of photography, it is

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known that each line of airborne linear data has a perspective center and its own exterior orientation elements. At one certain interval, the six parameters for one line are recorded by the POS on board of the aircraft.

First, we try to get the ground coordinate of the perspective center. Suppose that ($X_G$, $Y_G$, $Z_G$) is the coordinate of the perspective center $G$, $\varphi$ is the pitch angle of the aircraft, $\omega$ is the roll angle and $\kappa$ is the yaw angle obtained by POS. Line $AB$ is one line in image space, and $C$ is the center of line $AB$ with the coordinate ($x_C$, $y_C$) in image coordinate system. $Q$ is the ground corresponding point of $C$. The geometry among the perspective center $G$, projective points in image space, and corresponding points on the ground for one line is shown in Fig.1.

In Fig.1, line $QR$ is perpendicular to $Y$-axis; point $R$ is the intersection. The coordinate ($X_Q$, $Y_Q$) of point $Q$ can be obtained as:

$$X_Q = X_G + |QR| = X_G + Z_G tg\omega / \cos\varphi \quad (1)$$

$$Y_Q = Y_G + |OR| = Y_G + Z_G tg\varphi \quad (2)$$

Assume one point $D$ selected at random from line $AB$, and that $D$'s coordinate is ($x_C$, $y$, $f$) in image coordinate system. Call its corresponding point in ground space as $P$. The next aim is to get the ground coordinate ($X_P$, $Y_P$) of $P$.

Line $PN$ is perpendicular to $Y$-axis, and point $N$ is the intersection. Line $QT$ is perpendicular to line $PN$, and point $T$ is the intersection. Let $\theta$ be the included angle of line $GQ$ and $GP$, starting from the same point $G$. Assume that the across-track IFOV (Instantaneous

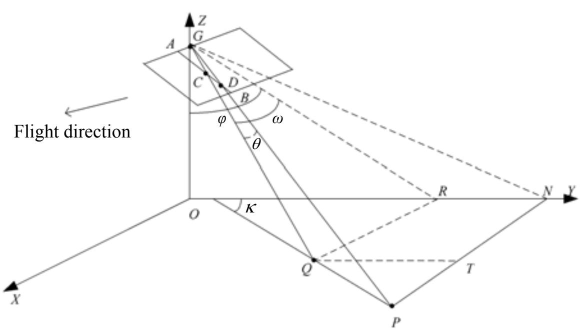



**Fig.1 Geometry among the perspective center $G$, projective points in image space, and corresponding points in ground space**

Field Of View) is $\rho$ mrad, so

$$\theta = (y - y_C)\rho(180/1000\pi) \text{ (Unit is degree)} \quad (3)$$

Letting $|GQ|=l$, $|GP|=s$ and $|QP|=d$, we have,

$$X_P = X_Q + d\sin\kappa \quad (4)$$

$$Y_P = Y_Q + d\cos\kappa \quad (5)$$

From the above, we obtain,

$$|GQ| = l = Z_G / (\cos\varphi\cos\omega) \quad (6)$$

According to Fig.1,

$$|PN| = l\sin\omega + d\sin\kappa \quad (7)$$

$$|ON| = l\cos\omega\sin\varphi + d\cos\kappa \quad (8)$$

In $RT\Delta GNP$, we have

$$GN^2=GP^2-PN^2 \quad (9)$$

and in $RT\Delta GON$, we get

$$GN^2=OG^2+ON^2 \quad (10)$$

Substitute Eqs.(7) and (8) into Eqs.(9) and (10), we obtain the following equation:

$$s^2 - (l\sin\omega + d\sin\kappa)^2 = (l\cos\omega\cos\varphi)^2 + (l\cos\omega\sin\varphi + d\cos\kappa)^2 \quad (11)$$

Simplifying Eq.(11), yields

$$l^2 + 2ld(\sin\omega\sin\kappa + \cos\omega\sin\varphi\cos\kappa) + d^2 - s^2 = 0 \quad (12)$$

In $\Delta GQP$, we have

$$d^2 = l^2 + s^2 - 2ls\cos\theta \quad (13)$$

Let

$$\beta = \sin\omega\sin\kappa + \cos\omega\sin\varphi\cos\kappa \quad (14)$$

Combining Eqs.(12) and (13) to solve for $d$, yields

$$d = l\frac{\mathrm{tg}\theta}{\left(\sqrt{1-\beta^2} - \beta\mathrm{tg}\theta\right)} = \frac{Z_G}{\cos\varphi\cos\omega}\frac{\mathrm{tg}\theta}{(\sqrt{1-\beta^2} - \beta\mathrm{tg}\theta)} \quad (15)$$

Then, the coordinate ($X_P$, $Y_P$) of point $P$ can be obtained as:

$$X_P = X_G + Z_G \mathrm{tg}\omega / \cos\varphi + d\sin\kappa \quad (16)$$

$$Y_P = Y_G + Z_G \mathrm{tg}\varphi + d\cos\kappa \quad (17)$$

If point $P$ is at the left of $Q$, the coordinate ($X_P$, $Y_P$) of $P$ is

$$X_P = X_G + Z_G \mathrm{tg}\omega / \cos\varphi - d\sin\kappa \quad (18)$$

$$Y_P = Y_G + Z_G \mathrm{tg}\varphi - d\cos\kappa \quad (19)$$

When each pixel ($i$, $j$) in image space is transformed to get its corresponding coordinate ($X$, $Y$) in ground space, resampling and interpolation techniques can be used to get the first-step rectified image.

**Second step: rectification using polynomial distortion model**

Though the first step can rectify some errors caused by certain factors and the expected rectification image can be obtained, the next rectification step is an essential process using polynomial distortion model for further applications, such as fusion and classification.

In this step, a reference image is required and some GCPs are selected. Let ($X$, $Y$) be a point in the above rectified image $\hat{F}$, $(\varepsilon, \eta)$ be the corresponding point in the reference image $\hat{G}$. Also let $(\hat{X}, \hat{Y})$ be the estimate of ($X$, $Y$) by polynomial transformation. The general form of the model is expressed as follows:

$$\hat{X} = \sum_{i=0}^{n}\sum_{j=0}^{n-i} a_{i,j}\varepsilon^i\eta^j \quad (20)$$

$$\hat{Y} = \sum_{i=0}^{n}\sum_{j=0}^{n-i} b_{i,j}\varepsilon^i\eta^j \quad (21)$$

where $a_{i,j}$ and $b_{i,j}$ are unknown coefficients, and $n$ is the degree of the model.

If the degree of polynomial model is $n$, we must have a set of $M=(n+1)(n+2)/2$ GCPs at least to solve Eqs.(20) and (21). Suppose that $\{(\varepsilon_i, \eta_i)$: $i=1, \ldots, L\}$ and $\{(X_i, Y_i)$: $i=1, \ldots, L\}$ are GCPs selected from $\hat{G}$ and $\hat{F}$ respectively. The least square method can be used to estimate the coefficients $a_{i,j}$ and $b_{i,j}$. Then the transformation between $\hat{F}$ and $\hat{G}$ is determined. Bilinear Interpolation is applied to get the final rectified image.

For the analysis of the rectification accuracy, the error of mean square (*EMS*) is given as:

$$EMS = \frac{\sqrt{\sum_{i=1}^{L}((\hat{X}_i - X_i)^2 + (\hat{Y}_i - Y_i)^2)}}{L} \quad (22)$$

**Algorithm**

In this part, we give in detail the two-step rectification algorithm for airborne linear images.

(1) Input airborne linear image with size of $I\times J$ and defined as $\boldsymbol{F}$.

(2) Let $\boldsymbol{M}$ be the matrix of referencing POS data with size of $6\times J$, in which the elements of $j$th line are the six exterior orientation parameters of $j$th line in $\boldsymbol{F}$, $j=1, \ldots, J$.

(3) For the $j$th line, the center in image space is $(I/2, j)$. First get the perspective center coordinate ($X_{Gj}$, $Y_{Gj}$) in ground space by Eqs.(1) and (2). Then for all point $(i, j)$, $i=1, \ldots, I$, compute its corresponding point coordinate ($X_{Pj}$, $Y_{Pj}$) in ground space according to Eqs.(16) and (17) or Eqs.(18) and (19).

(4) For each line of image $\boldsymbol{F}$, repeat Step 3. Then get the first rectified image defined as $\hat{F}$ by using resampling and interpolation techniques.

(5) Select some GCPs $\{(\varepsilon_i, \eta_i)$: $i=1, \ldots, L\}$ and $\{(X_i, Y_i)$: $i=1, \ldots, L\}$ from the reference image and $\hat{F}$ respectively. Use polynomial distortion model and Bilinear Interpolation to obtain the final rectified image.

## EXPERIMENTS

In this section, we give some results of the two-step rectification model.

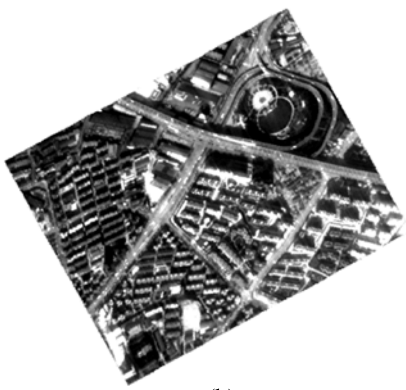
In our experiments, the images were all from airborne pushbroom hyperspectral imager, developed by the Shanghai Institute of Technical Physics, Chinese Academy of Sciences. Each line of this kind of image is collected at a different instant of time and has its own perspective center. We use the algorithm proposed above to adjust the displacement of each line. The results are shown in the following images.

Fig.2a is the raw image. Fig.2b is the first-step rectified image of Fig.2a.

For further applications such as fusion and classification, the second step of rectification is applied. Here, the reference image (Fig.3) is selected from one band of a color infrared image, and the test image is Fig.2b. We select a set of 12 GCPs from Fig.3 and Fig.2b and label them in Fig.3 and Fig.4. Table 1 lists the selected GCPs. Fig.5a shows the result of final rectification of Fig.4 using one-degree polynomial distortion model, and Fig.5b is the result of Fig.4 obtained by two-degree model. Fig.5 shows that the geometric relationships of all the pixels in the rectified image are almost as same as those in the reference image.

The estimates and *EMS* obtained by one-degree polynomial model and two-degree model are also listed in Table 1 showing that the *EMS* obtained by using two-degree model is far less than that obtained by using one-degree model.

(a)



(b)

**Fig.2 (a) The raw image; (b) The first-step rectified image of Fig.2a**

**Fig.3 Reference image**

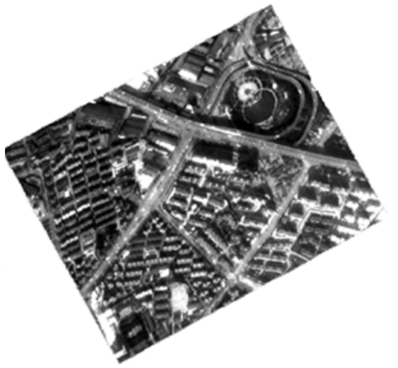
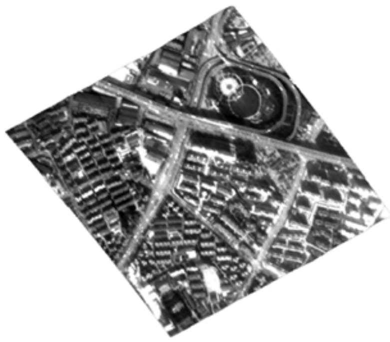


**Fig.4 First-step rectification image with labeled GCPs**



(a)

(b)

**Fig.5 Final rectification image obtained by one-degree polynomial model (a) and by two-degree polynomial model (b)**

**Table 1 GCPs, Estimates and *EMS***

| No. | Reference image ($\varepsilon$, $\eta$) | Test image ($X$, $Y$) | Estimate by one-degree polynomial model ($\hat{X}$, $\hat{Y}$) | Estimate by two-degree polynomial model ($\hat{X}$, $\hat{Y}$) |
|---|---|---|---|---|
| 1 | (1172.00,590.00) | (555.50,127.00) | (556.83, 127.76) | (555.28, 127.09) |
| 2 | (1235.00, 833.25) | (560.50, 200.00) | (557.93, 199.75) | (560.13, 198.72) |
| 3 | (1713.25,1055.75) | (681.50, 260.00) | (682.43, 260.31) | (681.83, 261.07) |
| 4 | (379.00, 916.50) | (301.00,236.00) | (302.27,233.73) | (300.41,235.29) |
| 5 | (782.50,1682.75) | (368.50, 460.50) | (363.09, 460.85) | (367.36, 459.93) |
| 6 | (566.50,1420.50 ) | (317.50, 382.00) | (318.58, 384.94) | (320.04, 384.32) |
| 7 | (1434.50,2297.25) | (514.50, 633.50) | (515.62, 631.30) | (513.41, 632.46) |
| 8 | (343.25,1840.00) | (221.50, 515.50) | (219.65, 516.74) | (220.55, 514.60) |
| 9 | (1890.50,2196.50 ) | (663.00, 587.50) | (660.74, 591.28) | (663.67, 588.18) |
| 10 | (2157.75,1875.50 ) | (761.00, 494.00) | (761.71, 492.36) | (760.80, 493.76) |
| 11 | (864.75,489.75) | (474.00, 100.50) | (475.43, 99.94) | (474.22, 100.99) |
| 12 | (877.50,2495.75) | (327.50, 706.00) | (331.72, 704.04) | (328.30, 706.61) |
| *EMS* | | | 3.076995 | 1.380354 |

## CONLUSION

Image acquisition based on airborne linear hyperspectral imagers has become a mature technology and is becoming the mainstream. As data in general have much distortions caused by many factors, the images can only be useful if they are effectively rectified.

In this paper, we present a two-step rectification algorithm. The first step is to use the POS data to establish the model of direct georeference position. The rectification result of this step is shown in Fig.2b. The ranges of raw image are adjusted greatly and effects are satisfactory. The second step is to apply polynomial distortion model to further correct the above rectified image. The geometric relationships of all the pixels in the final rectified image are almost the same as those in the reference image. It is shown that our two-step rectification algorithm is very effective.

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