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Optimal choice of parameters for particle swarm optimization*

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Abstract: The constriction factor method (CFM) is a new variation of the basic particle swarm optimization (PSO), which has relatively better convergent nature. The effects of the major parameters on CFM were systematically investigated based on some benchmark functions. The constriction factor, velocity constraint, and population size all have significant impact on the performance of CFM for PSO. The constriction factor and velocity constraint have optimal values in practical application, and improper choice of these factors will lead to bad results. Increasing population size can improve the solution quality, although the computing time will be longer. The characteristics of CFM parameters are described and guidelines for determining parameter values are given in this paper.

Key words: Particle swarm optimization (PSO), Constriction factor method (CFM), Parameter selection
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INTRODUCTION

Particle swarm optimization (PSO) is one of the evolutionary computational techniques. Since its introduction (Kennedy and Eberhart, 1995; Eberhart and Kennedy, 1995; Eberhart *et al.*, 1996), PSO has attracted much attention from researchers around the world. It is a population-based search algorithm and is initialized with a population of random solutions, called particles. Each particle in PSO moves over the search space at velocity dynamically adjusted according to the historical behaviors of the particle and its companions.

Suppose that the search space is D-dimensional, and the position of ith particle of the swarm can be represented by a D-dimensional vector, $\boldsymbol{x}_i=(x_{i1},\ldots,x_{id},\ldots,x_{iD})$. The velocity (position change per generation) of the particle $\boldsymbol{x}_i$ can be represented by another D-dimensional vector $\boldsymbol{v}_i=(v_{i1},\ldots,v_{id},\ldots,v_{iD})$. The best position previously visited by the ith particle is denoted as $\boldsymbol{p}_i=(p_{i1},\ldots,p_{id},\ldots,p_{iD})$. If the topology is defined such that all particles are assumed to be neighbors and g as the index of the particle visited the best position in the swarm, then $\boldsymbol{p}_g$ becomes the best solution found so far, and the velocity of the particle and its new position will be determined according to the following two equations:

$$v_{id}=v_{id}+c_1r_1(p_{id}-x_{id})+c_2r_2(p_{gd}-x_{id}) \quad (1)$$

$$x_{id}=x_{id}+v_{id}, \quad d=1,\ldots,d,\ldots,D \quad (2)$$

where c_1 and c_2 are acceleration coefficients regulating the relative velocity toward global and local best, r_1 and r_2 are two random numbers in [0, 1]. Besides, a maximum allowable velocity vector $\boldsymbol{V}_{\max}$ clamps velocities of particles on each dimension. If the acceleration causes the velocity on that dimension to exceed $\boldsymbol{V}_{\max}$ specified by the user, then the velocity on that dimension will be limited to $\boldsymbol{V}_{\max}$. In later studies, for insuring convergence, an analysis of the algorithm from mathematical aspects was given by Clerc (1999), who proposed the use of a constriction factor χ; the algorithm was named the constriction factor method (CFM). Let $\varphi=c_1+c_2$, and define

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$$\chi = \begin{cases} \dfrac{2k}{\varphi - 2 + \sqrt{\varphi^2 - 4\varphi}}, & \text{for } \varphi > 4 \\ k, & \text{otherwise} \end{cases} \tag{3}$$

where k is a positive constant, then Eq.(1) becomes

$$v_{id} = \chi(v_{id} + c_1 r_1 (p_{id} - x_{id}) + c_2 r_2 (p_{gd} - x_{id})) \tag{4}$$

Eberhart and Shi (2000) compared the performance of PSO with its different versions, and concluded that the best approach is to use the constriction factor while limiting the maximum velocity $\boldsymbol{V}_{max}$ to the dynamic range of the variable $\boldsymbol{X}_{max}$ on each dimension. For instance, it is convenient to take $\boldsymbol{V}_{max}=\boldsymbol{X}_{max}$. The above mentioned research on the CFM recommended parameters values based on reasonable inference, not precise theoretical analysis. Considering the necessity of finding proper parameters values which may improve the CFM performance, the effects of parameters on CFM implementation were investigated in our study to explore for better assignment of values to them.

SETUP OF EXPERIMENTS

The experiments were designed to study the behavior of CFM by varying parameter settings which might significantly affect the results. As suggested by Carlisle and Dozier (2001), a fully connected topology and asynchronous updates must be used in all cases.

Parameter settings

φ was considered as the first important parameter. Clerc and Kennedy (2002) gave some clues that CFM exhibits almost linear convergence for $\varphi>4$, so the test suite was run by varying φ from 4.0 to 4.4 with increment of 0.01. In the general case $k=1$ (Eberhart and Shi, 2000), χ can be determined by Eq.(3).

The second parameter is $\boldsymbol{V}_{max}$, which limits the maximum distance a particle can travel in one generation. Our experiment results suggested the following relation between $\boldsymbol{V}_{max}$ and $\boldsymbol{X}_{max}$

$$\boldsymbol{V}_{max}=\gamma\boldsymbol{X}_{max} \tag{5}$$

where γ is a proportional factor to be determined. The increment of γ is varied for different interval. That is, take interval of 0.001 from 0.001 to 0.01, and take interval of 0.01 from 0.01 to 0.1, etc.

The last parameter is the population size N. The population sizes were varied from 10 to 150 in steps of 10. The ranges and increments of parameters used are listed in Table 1.

Table 1 Parameter ranges

Parameter	Range	Increment
φ	[4.00, 4.40]	0.01
γ	[0.001, 1000]	–
N	[10, 150]	10

Functions used for experiments

A set of unconstrained real-valued benchmark functions was used to investigate the effect of the three parameters. Sphere, De Jong's f2, De Jong's f4 and Rosenbrock are unimodal functions. De Jong's f5, Schaffer's f6, Rastrigin, Griewank, and Ackley are multimodal functions (Clerc and Kennedy, 2002). The expressions of functions are given in Table 2, and the problem-specific parameters are given in Table 3. Functions were implemented in high dimension of 30 and low dimension of 10 except for f2, f5, and f6, which are given for two dimensions. In all cases except f5, the globally optimal function result is 0.0. For f5, the best known result is 0.998004. In all cases, the fitness value was taken equal to the function value.

Tests procedures

Parameter analysis of PSO in our study was carried out as follows.

1. Initialize a population of N particles. For the ith particle, its location $\boldsymbol{x}_i$ in the search space is randomly placed. Its velocity vector is $\boldsymbol{v}_i=(v_{i1}, \ldots, v_{id}, \ldots, v_{iD})$, in which the velocity in the dth dimension is v_{id}=rand()×$\boldsymbol{V}_{max}$, where rand() is the random number in the range [−1, 1]. $\boldsymbol{V}_{max}$ is calculated by Eq.(5), and $\boldsymbol{X}_{max}$ is given in Table 3.

2. Start the outer loop execution, assign φ and γ according to the range given in Table 1. Calculate χ according to Eq.(3), and assign $c_1=c_2=\varphi/2$. Implement 3 series of experiments by varying φ, γ and N with the given interval once a time in turn, and keep the other two parameters constant respectively.

Table 2 Functions used to test the effects of different parameters

Name	Expression
Sphere (De Jong's f1)	$f_1(x)=\sum_{d=1}^{D}x_d^2$
De Jong's f2	$f_2(x)=100(x_1^2-x_2)^2+(1-x_1)^2$
De Jong's f4–no noise	$f_4(x)=\sum_{d=1}^{D}d\cdot x_d^4$
Rosenbrock	$f_7(x)=\sum_{d=1}^{D}\left(100(x_{d+1}-x_d^2)^2+(x_d-1)^2\right)$
De Jong's f5	$f_5(x)=\frac{1}{0.002}+\sum_{j=1}^{25}\frac{1}{j+\sum_{d=1}^{2}(x_d-a_{dj})^6}$
Shaffer's f6	$f_6(x)=0.5+\frac{\sin\sqrt{x^2+y^2}-0.5}{(1.0+0.001(x^2+y^2))^2}$
Rastrigin	$f_8(x)=\sum_{d=1}^{D}\left(x_d^2-10\cos(2\pi x_d)+10\right)$
Griewank	$f_9(x)=\frac{1}{4000}\sum_{d=1}^{D}x_d^2-\prod_{d=1}^{D}\cos\left(\frac{x_d}{d}\right)+1$
Ackley	$f_{10}(x)=-20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{d=1}^{D}x_d^2}\right)-\exp\left(\frac{1}{D}\sum_{d=1}^{D}\cos(2\pi x_d)\right)+20+e$

Table 3 Function parameters for the test problems

Function	Dimension		Initial range of $\boldsymbol{x}_i$
	Low	High	
Sphere	10	30	±20
De Jong's f2	2		±50
De Jong's f4	10	30	±20
Rosenbrock	10		±10
De Jong's f5	2		±50
Shaffer's f6	2		±100
Rastrigin	10	30	±5.12
Griewank	10	30	±300
Ackley	10	30	±32

3. Start the inner loop, set the number of iteration t=1, and evaluate the fitness function for each particle. Let *pbest* equal the fitness value of each particle. Let *gbest* equal the index of the particle which gives best fitness value.

4. Compare the evaluated fitness value of each particle with its *pbest*. If the current value is better than *pbest*, then set the current location as the *pbest* location. Furthermore, if the current value is better than *gbest*, then reset *gbest* to the current index in the particle array.

5. Change the velocity and location of the particle according to Eqs.(4) and (2), respectively. Check if the velocity and location exceed the constraints.

6. Set t=t+1, repeat Steps 4–5 until the number of iteration is greater than the allowable maximum iteration number $T_{\max}$. In this study, $T_{\max}$ equals 2000.

7. Go to the outer loop. Recurrently implement Steps 2–6 until a stop criterion is met.

EXPERIMENTS RESULTS AND DISCUSSIONS

The results are shown in Figs.1 to 3. Each point is made from average values of over 30 repetitions.

Effect of φ

Fig.1 illustrates that the average fitness varied with φ from 4.0 to 4.4 for different functions when γ=0.1 and N=30, and that for unimodal functions, when dimension is high, the best range may be [4.1, 4.2]; and that the range may be extended to [4.05, 4.3] for low dimension. It is more complicated for multimodal functions. For De Jong's f5, the curve is irregular, and no rules can be extracted. For Shaffer's f6 and Ackley function of high dimension, the best values of φ approach to a constant value around 4.05, and the Ackley function of low dimension also indicates the easy trend. It was concluded that from both high and low dimensions of Rastrgin functions, too small value of φ was inappropriate, and that the upper limit was unclear. For Griewank function, the best range was [4.05, 4.1].

In summary, it appears that the setting of 4.05 for φ is appropriate for high multimodal functions and 4.1 for unimodal functions. This is probably little different from the previous setting of 4.1 commonly used (Carlisle and Dozier, 2001).

It is clear from Eq.(4) that increasing φ will decrease the value of constriction factor χ. As shown in Fig.1, too small or too large φ will generally lead to bad results. This is due to the fact that, a smaller χ means that the search distance of every step for each particle becomes smaller, which will direct more attention to local exploitation. On the contrary, a larger χ will increase the search distance that facilitates global exploration. Thus, taking larger value of φ for unimodal function can make the solution refined. However, due to the requirement that for more exploration ability for multimodal functions, smaller value of φ is the best choice. In this aspect, a PSO system attempts to balance exploration and exploitation by combining local and global search ability. In particular, with too large χ, the algorithm cannot achieve the refinement of the optimal solution, while with too small χ, the algorithm can hardly cover the search space.

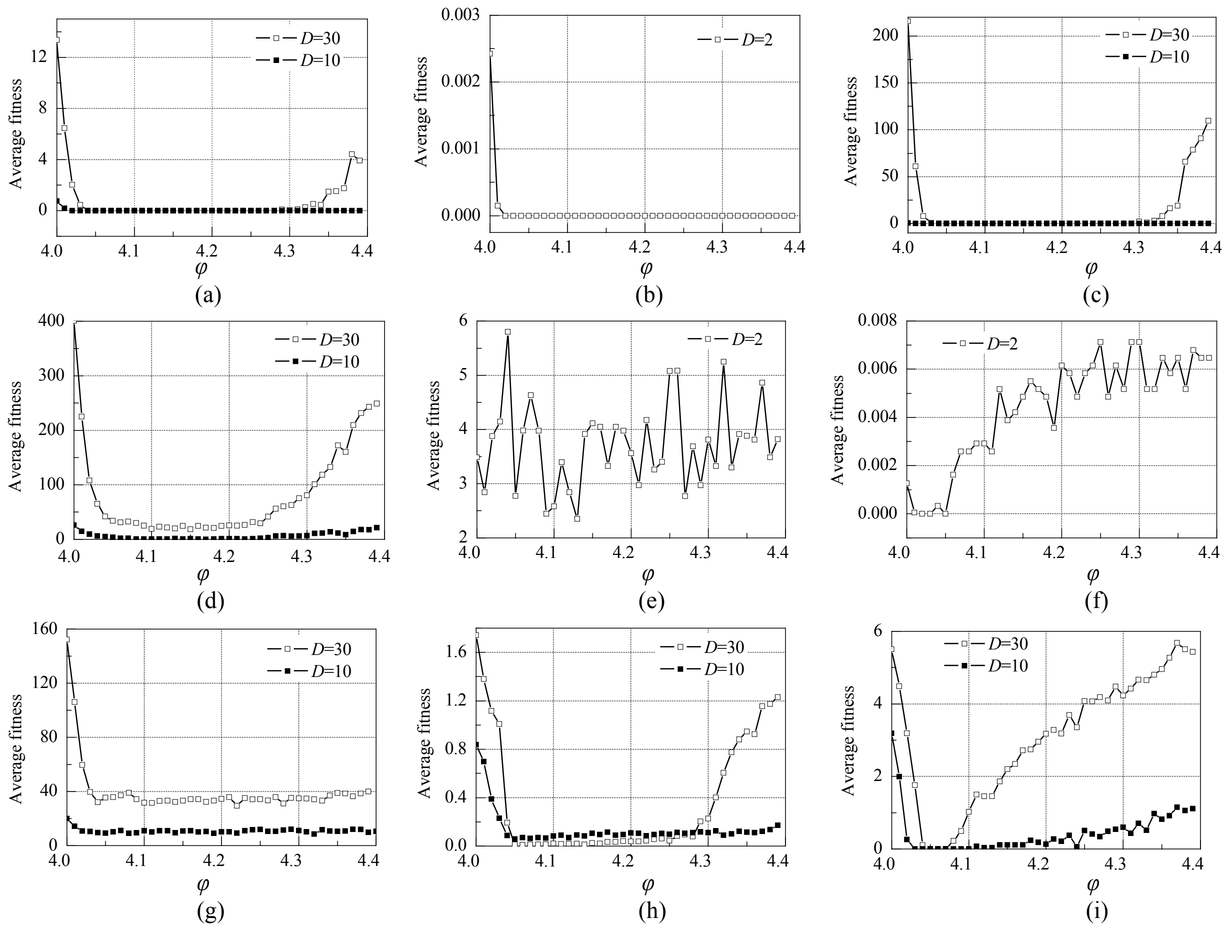

Fig.1 The average fitness variation with φ when γ=0.1 and N=30 for different functions
(a) Sphere; (b) De Jong's f2; (c) De Jong's f4; (d) Rosenbrock; (e) De Jong's f5; (f) Schaffer's f6; (g) Rastrigin; (h) Griewank; (i) Ackley

Influence of velocity coefficient γ

As shown in Fig.2, the average fitness varied with γ from 0.001 to 1000 when φ=4.1 and N=30. It appears that after γ was set larger than 5×10^{-3}, it has little influence on the performance of CFM for Sphere, De Jong's f2, and De Jong's f4. No rules can be drawn for Shaffer functions. In high dimension, for Rosenbrock and Rastrgin functions, the range of better γ located in [0.01, 0.1], while Griewank and Ackley functions located in [0.1, 1]. Also in low dimension, the trend of marginally moving leftward is clear. This phenomenon can be explained by the fact that, if $\boldsymbol{V}_{\max}$ is too large, particles might easily pass over but without catching good solutions. On the other hand, if $\boldsymbol{V}_{\max}$ is too small, particles may not have sufficient opportunity to explore more space beyond local regions. Therefore, the particles could be easily trapped in local optima, unable to move far enough to reach better positions globally. However, for De Jong's f5, larger γ probably produces better results. The extraordinary improvement appears to be related to the shape of the function. Since De Jong's f5 function is a multi-modal function, and has 25 local optima, which need more global search ability.

A series of observations showed that, $\boldsymbol{V}_{\max}$ is useful, and to set it at about 5% of the dynamic range of the variable on each dimension for unimodal function and 50% for multimodal functions is reasonable.

Size of swarm population

Fig.3 indicates that the average fitness varies with population size N when φ=4.1 and γ=0.1 for different functions. Except functions of Sphere, De

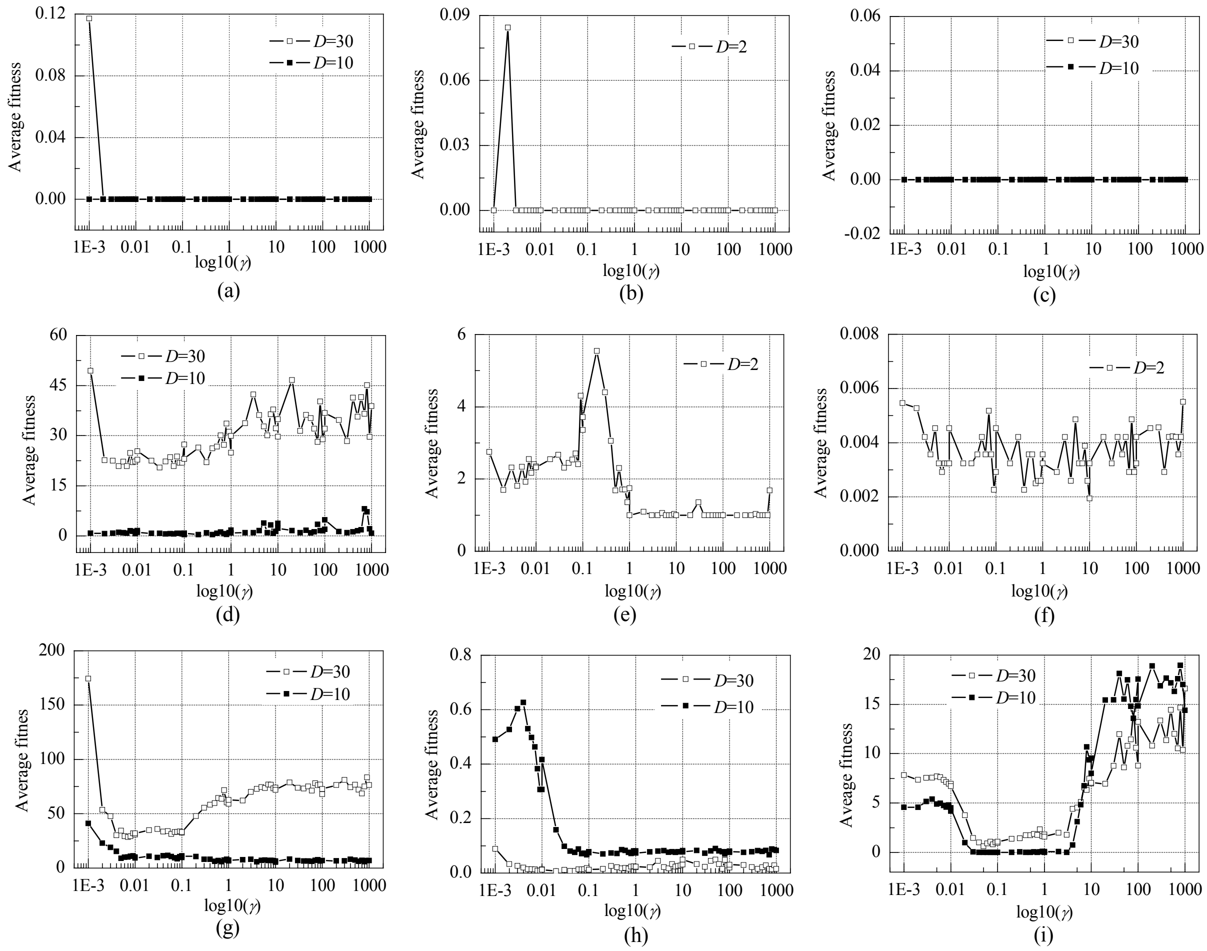

Fig.2 The average fitness variation with γ when φ=4.1 and N=30 for different functions
(a) Sphere; (b) De Jong's f2; (c) De Jong's f4; (d) Rosenbrock; (e) De Jong's f5; (f) Schaffer's f6; (g) Rastrigin; (h) Griewank; (i) Ackley

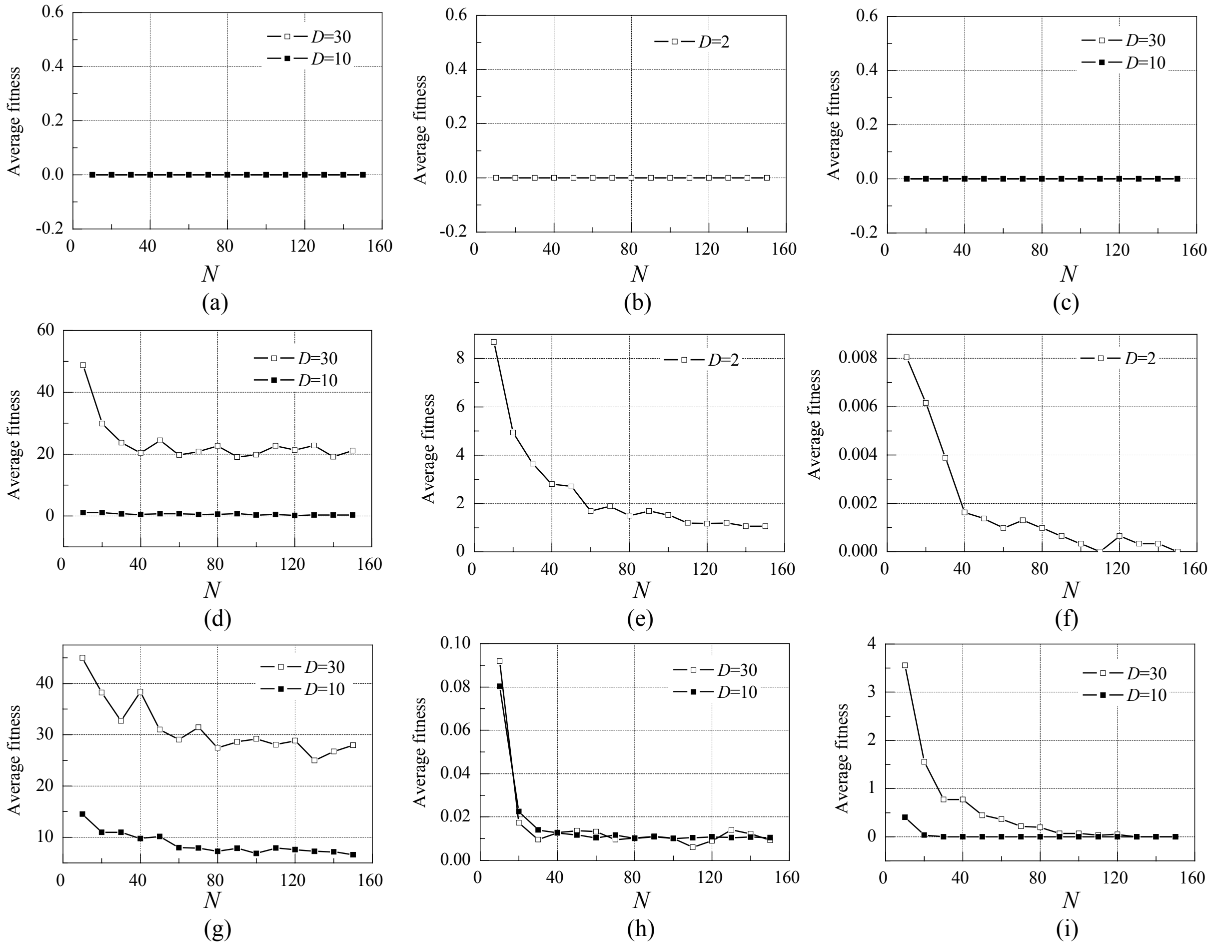

Fig.3 The average fitness variation with population size *N* when φ=4.1 and γ=0.1 for different functions
(a) Sphere; (b) De Jong's f2; (c) De Jong's f4; (d) Rosenbrock; (e) De Jong's f5;
(f) Schaffer's f6; (g) Rastrigin; (h) Griewank; (i) Ackley

Jong's f2, and De Jong's f4, it can be seen from other high dimension functions that the population size significantly affects the CFM in searching when the population size is smaller than 50. While when it is larger than 50, PSO is not sensitive to the population size. Except for the Griewank function, when dimension is low, the effect of population size is small.

Larger population sizes require more function evaluations and increase the computing efforts for convergence, but increase the reliability of the algorithm. The problem is to find a compromise between cost and reliability. It is recommended that the population size should not larger than 50 when the problem is complicated, and that a smaller size of 20 to 50 particles may be appropriate when the problem is simpler. The results obtained in our work were consistent with the results of other researchers (Eberhart and Shi, 2001).

CONCLUSION

PSO, including CFM, which is stochastic in nature and makes use of the memory of each particles as well as the knowledge gained by the swarm as a whole, has been proved to be powerful in solving many optimization problems. However, the algorithm includes some turning parameters that greatly influence the algorithm performance. In this work, how the parameters affect the performance of the PSO was examined, and some conclusions were derived.

1. Different problem may have its best choice of φ in CFM for PSO. Nevertheless, the choice of φ in the range [4.05, 4.3] is generally reasonable, and to take 4.05 for multimodal function and 4.1 for unimodal functions is probably the best choice.

2. The range of γ located in [0.01, 1] is a relative better choice in the general case, and taking the value

of 0.5 for multimodal functions and 0.05 for unimodal functions are appropriate. This implies that the particle should move with larger steps in more complicated search space.

3. For higher dimensional problems, it is better to select swarm size of 50, and for lower dimensional problems, to take swarm size of [20, 50] is appropriate and 30 may be a good choice.

In the present work, c_1 equal to c_2 was presumed, which implies each particle puts equal trust in the swarm and itself, while Carlisle and Dozier (2001) concluded that c_1/c_2 appeared to be 2.8/1.3. Therefore, further research should be conducted on this aspect. An adaptive algorithm for adjusting these factors based on present studies is under investigation now.

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