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Sliding mode control for synchronization of chaotic systems with structure or parameters mismatching*

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Abstract: This paper deals with the synchronization of chaotic systems with structure or parameters difference. Nonlinear differential geometry theory was applied to transform the chaotic discrepancy system into canonical form. A feedback control for synchronizing two chaotic systems is proposed based on sliding mode control design. To make this controller physically realizable, an extended state observer is used to estimate the error between the transmitter and receiver. Two illustrative examples were carried out: (1) The Chua oscillator was used to show that synchronization was achieved and the message signal was recovered in spite of parametric variations; (2) Two second-order driven oscillators were presented to show that the synchronization can be achieved and that the message can be recovered in spite of the strictly different model.

Key words: Chaos synchronization, Sliding mode control, Extended state observer, Secure communication
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INTRODUCTION

Synchronization of chaotic systems and its application to secure communication has recently received much attention. Several chaos synchronization schemes have been successfully established (Liao and Huang, 1999; Jiang, 2002; Zhao *et al*., 2004). However, to our best knowledge, most of the chaos synchronization strategies have been studied in similar oscillators. In fact, from the perspective of hardware implementation, the resistance and capacitance of the resistors and the capacitors, respectively, in the transmitter circuit will always differ slightly from those in the receiver circuit. Hence, the two and chaotic circuits will not be identical due to the mismatch in the system's parameters. Therefore, the synchronization of two chaotic systems with parametric uncertainties or different structure is an important issue. Femat proposed an extended form and several schemes have been reported (Femat and Alvarez-Ramírez, 1997; Femat *et al*., 2000; Femat and Jauregui-Ortiz, 2001) based on extend state observer theory to resolve the problem. On the other hand, Liao and Tsai (2000) resolved the chaos synchronization of a class of nonlinear systems with disturbances and unknown parameters by deriving an adaptive observer-based driven system via a scalar transmitted signal.

Sliding mode control is a nonlinear control scheme widely used for controlling uncertain nonlinear systems (Liao and Huang, 1997; Yau *et al*., 2000). In view of the above developments, this work is aimed at studying the synchronization of two chaotic systems with parametric uncertainties or different structure based on the sliding mode control scheme and the extended state observer theory. In this paper, the structure of the extended state observer presented by Femat and Alvarez-Ramírez (1997), Femat *et al*.(2000), and Femat and Jauregui-Ortiz (2001) is

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generalized and the minimum phase assumption is considered. The two-channel transmission method (Jiang, 2002) was used in the numeric simulation to verify the application in secure communication of this chaos synchronization method.

CANONICAL FORM OF NONLINEAR SYSTEM

Consider the following nonlinear system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u, \quad y = h(\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{x}\in\mathrm{R}^n$; y, $u\in\mathrm{R}$; $\boldsymbol{f}$, $\boldsymbol{g}$ are n dimensions smooth vector field; $h(\boldsymbol{x})$ is smooth function.

Define Lie derivative of function $h(\boldsymbol{x})$ with respect to $\boldsymbol{f}$ as $L_f h(\boldsymbol{x})$, and $L_f^0 h(\boldsymbol{x}) = h(\boldsymbol{x})$, $L_f^{k+1} h(\boldsymbol{x}) = L_f(L_f^k h(\boldsymbol{x}))$.

Definition 1 (relative degree) Let us assume that $\boldsymbol{x}_0\in\boldsymbol{X}$, $\boldsymbol{V}$ is the neighborhood of $\boldsymbol{x}_0$ and constant $r>0$, system Eq.(1) has relative degree r at $\boldsymbol{x}_0$ if

(i) $L_g L_f^k h(\boldsymbol{x}) = 0,\ \forall \boldsymbol{x}\in\boldsymbol{V},\ 0\le k < r-1$;

(ii) $L_g L_f^{r-1} h(\boldsymbol{x}) \ne 0,\ \forall \boldsymbol{x}\in\boldsymbol{V}$.

Proposition (Feng and Fei, 1998) Suppose system Eq.(1) has a relative degree $r\le n$ at $\boldsymbol{x}_0$, it is always possible to find $n-r$ functions ϕ_{r+1}, ϕ_{r+2}, …, ϕ_n that the mapping $\boldsymbol{\Phi}(x) = [h(\boldsymbol{x}), L_f h(\boldsymbol{x}), \cdots, L_f^{r-1} h(\boldsymbol{x}), \phi_{r+1}, \cdots, \phi_n]$ has a Jacobin matrix which is nonsingular at $\boldsymbol{x}_0$, and it is always possible to choose ϕ_{r+1}, ϕ_{r+2}, …, ϕ_n in such that $L_{\boldsymbol{g}}\phi_i(\boldsymbol{x})=0$ for all $r+1\le i\le n$. This implies that there exists a globally invertible coordinate transformation $(\boldsymbol{z}, \boldsymbol{v})=\boldsymbol{\Phi}(\boldsymbol{x})$ such that the system Eq.(1) can be globally transformed into the canonical form

$$\begin{aligned} &\dot{z}_i = z_{i+1},\ i = 1,2,\cdots,r-1 \\ &\dot{z}_r = \alpha(\boldsymbol{z},\boldsymbol{v}) + \beta(\boldsymbol{z},\boldsymbol{v})u \\ &\dot{\boldsymbol{v}} = q(\boldsymbol{z},\boldsymbol{v}) \\ &y = z_1 \end{aligned} \tag{2}$$

where $\alpha(\boldsymbol{z},\boldsymbol{v}) = L_f^r h(\boldsymbol{x}), \beta(\boldsymbol{z},\boldsymbol{v}) = L_g L_f^{r-1} h(\boldsymbol{x})$, $\boldsymbol{v}=[\phi_{r+1}, \phi_{r+2}, \ldots, \phi_n]$. The system is fully linearizable when $r=n$. The system is partly linearizable when $r<n$ and $\dot{\boldsymbol{v}} = q(0,\boldsymbol{v})$ is called zero dynamics of system Eq.(2).

BASE THEORY

Normalization of the chaotic discrepancy

Consider the following chaotic transmitter:

$$\dot{\boldsymbol{X}}_1 = \boldsymbol{f}_1(\boldsymbol{X}_1;\pi_1), \quad y_1 = h(\boldsymbol{X}_1) \tag{3}$$

and the following receiver:

$$\dot{\boldsymbol{X}}_2 = \boldsymbol{f}_2(\boldsymbol{X}_2;\pi_2) + \boldsymbol{g}u, \quad y_2 = h(\boldsymbol{X}_2) \tag{4}$$

where $\boldsymbol{X}_1$, $\boldsymbol{X}_2\in\mathrm{R}^n$ are state vectors; $\boldsymbol{\pi}_1$, $\boldsymbol{\pi}_2\in\mathrm{R}^m$ are parameter sets; $\boldsymbol{g}\in\mathrm{R}^n$ is a smooth vector field; u is the control; y_1 is the output of system Eq.(3).

By defining $\boldsymbol{x}=\boldsymbol{X}_1-\boldsymbol{X}_2$, the following uncertain dynamical error system can be obtained:

$$\dot{\boldsymbol{x}} = \Delta\boldsymbol{f} - \boldsymbol{g}u, \quad y = h(\boldsymbol{x}) \tag{5}$$

where $\Delta\boldsymbol{f}=\boldsymbol{f}_1(\boldsymbol{X}_1;\boldsymbol{\pi}_1)-\boldsymbol{f}_2(\boldsymbol{X}_2;\boldsymbol{\pi}_2)$.

We make the following assumptions.

Assumption A1 The order of systems Eq.(3) and Eq.(4) is the same.

Assumption A2 Systems Eq.(3) and Eq.(4) have different structure or identical structure but with parametric variations.

Assumption A3 System Eq.(5) is minimum phase, i.e., the subsystem of the zero dynamics, $\dot{\boldsymbol{v}} = \boldsymbol{q}(0,\boldsymbol{v})$ where $\boldsymbol{v}\in\mathrm{R}^{n-r}$, is asymptotically stable.

Now the synchronization problem becomes: Is there any smooth function u such that the uncertain nonlinear system Eq.(5) is asymptotically stable at the origin?

There are uncertain model mismatches between the transmitter and the receiver, which implies that the discrepancy model is unknown.

Definition 2 (nominal function) Function $\beta_0(\boldsymbol{x})$ is called the nominal function of function $\beta(\boldsymbol{x})$ if $\beta(\boldsymbol{x})$ is an uncertain function, and $\beta(\boldsymbol{x})=\beta_0(\boldsymbol{x})\beta_1(\boldsymbol{x})$, while $\beta_0(\boldsymbol{x})$ is a certain function, and sign[$\beta(\boldsymbol{x})$]=sign[$\beta_0(\boldsymbol{x})$].

The following lemma (Femat and Alvarez-Ramírez, 1997; Femat and Jauregui-Ortiz, 2001) is essential to our paper.

Lemma Let us assume that there exists a coordination transformation $(\boldsymbol{z}, \boldsymbol{v})=\boldsymbol{\Phi}(\boldsymbol{x})$ such that the uncertain nonlinear system Eq.(5) can be transformed into the canonical form Eq.(2). Now suppose that γ is the nominal function of function $\beta(z, v)$. Let us define $\delta(z, v)=\beta(z, v)-\gamma$, $\eta=\Theta(z, v, u)$, $\Theta(z, v, u)=\alpha(z, v)+\delta(z, v)u$. Then there exists an invariant manifold such that the nonlinear system Eq.(2) can be rewritten in the following form:

$$\begin{cases} \dot{z}_i = z_{i+1},\ i=1,2,\cdots,r-1 \\ \dot{z}_r = \eta+\gamma u \\ \dot{\eta} = \Gamma(\boldsymbol{z},\boldsymbol{v},\eta,u,\dot{u}) \end{cases} \tag{6.1}$$

$$\dot{\boldsymbol{v}} = \boldsymbol{q}(\boldsymbol{z},\boldsymbol{v}) \tag{6.2}$$

where $\Gamma(\boldsymbol{z},\boldsymbol{v},\eta,u,\dot{u}) = \sum_{i=1}^{r-1}[z_{i+1}\partial_i\Theta(\boldsymbol{z},\boldsymbol{v},u)]+[\eta+\gamma]\cdot\partial_r\Theta(\boldsymbol{z},\boldsymbol{v},u)+\delta(\boldsymbol{z},\boldsymbol{v})\dot{u}+\partial_{\boldsymbol{v}}\Theta(\boldsymbol{z},\boldsymbol{v},u)\boldsymbol{q}(\boldsymbol{z},\boldsymbol{v})$; $y=z_1$ is the output.

System Eq.(6) is dynamically equivalent to system Eq.(5). Now the synchronization problem becomes: Is there any smooth function u such that system Eq.(6) is asymptotically stable at the origin?

Sliding surface design and associated control law

Subsystem Eq.(6.1) and Eq.(6.2) can be decoupled fully when subsystem Eq.(6.1) becomes a linear stability system under the control, hence the whole system is asymptotically stable at the origin according to Assumption A3, and the synchronization of systems Eq.(3) and Eq.(4) is achieved.

The proposed design method for sliding mode control is as follows.

Let $z_{r+1}=\eta+\gamma u$, subsystem Eq.(6.1) becomes:

$$\begin{aligned} \dot{z}_i &= z_{i+1},\ i=1,2,\cdots,r-1 \\ \dot{z}_r &= z_{r+1} \\ \dot{z}_{r+1} &= \dot{\eta}+\gamma\dot{u} \end{aligned} \tag{7}$$

Based on the control law proposed by Yau *et al.* (2000), the sliding surface can be defined as

$$S = z_{r+1} - z_{0(r+1)} + \int\sum_{i=1}^{r+1}c_i z_i \mathrm{d}t = 0 \tag{8}$$

where $z_{0(r+1)}$ denotes the initial state of z_{r+1}. Eq.(8) can also be formulated as

$$\dot{z}_{r+1} = -\sum_{i=1}^{r+1}c_i z_i \tag{9}$$

with initial condition $z_{r+1}(0)=z_{0(r+1)}$ and the sliding mode dynamics can be described by the following system of equations

$$\begin{aligned} \dot{z}_j &= z_{j+1},\ j=1,2,\cdots,r \\ \dot{z}_{r+1} &= -\sum_{i=1}^{r+1}c_i z_i \end{aligned} \tag{10}$$

By defining $\boldsymbol{Z}=[z_1\ z_2\ \ldots\ z_{r+1}]^{\mathrm{T}}$, Eq.(10) can be described in matrix equation form as

$$\dot{\boldsymbol{Z}} = \boldsymbol{AZ} \tag{11}$$

where $\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ -c_1 & -c_2 & \cdots & \cdots & -c_{r+1} \end{bmatrix}$.

The design parameters c_i (i=1, 2, …, r+1) can be determined by choosing the eigenvalues $\boldsymbol{A}$ such that the corresponding characteristic polynomial $P_{r+1}(s)=s^{r+1}+c_{r+1}s^r+\ldots+c_2s+c_1$ is Hurwitz. Thus, system Eq.(11) is asymptotically stable. The sliding surface is a stable surface.

The reaching law can be chosen such that (Liao and Huang, 1997)

$$\dot{S} = \alpha S - \beta\,\mathrm{sgn}(S) \tag{12}$$

where $0\leq\alpha<1$ and $\beta>0$.

From Eqs.(7), (8) and (12), we have

$$\dot{u} = \frac{1}{\gamma}\left(\alpha S - \beta\,\mathrm{sgn}(S) - \sum_{i=1}^{r+1}c_i z_i - \dot{\eta}\right) \tag{13}$$

This results in

$$u = \int_0^t \frac{1}{\gamma}\left(\alpha S - \beta\,\mathrm{sgn}(S) - \sum_{i=1}^{r+1}c_i z_i - \dot{\eta}\right)\mathrm{d}t \tag{14}$$

In general, the initial state of the control u is zero. After substituting Eq.(13) into Eq.(7), the close-loop system dynamics can be described as

$$\begin{aligned} \dot{z}_i &= z_{i+1},\ i=1,2,\cdots,r-1 \\ \dot{z}_r &= z_{r+1} \end{aligned}$$

$$\dot{z}_{r+1} = \alpha S - \beta \operatorname{sgn}(S) - \sum_{i=1}^{r+1} c_i z_i \tag{15}$$

Let the Lyapunov function of the system be $V=S^2/2$, then its first derivative with respect to time is

$$\dot{V} = S(\alpha S - \beta \operatorname{sgn}(S)) \le |S|(|S|-\beta) \tag{16}$$

From Eq.(8), we know that S is bounded. Therefore, if $\beta \ge |S|$, then $\dot{V} \le 0$, that is, there exist sliding mode dynamics. In many situations, $\beta \ge |S|$ can be satisfied by choosing a large enough switching gain β. Thus, the error system Eq.(7) reaches the neighborhood of the sliding surface S=0 and remains within a band (Liao and Huang, 1997) under the control, the synchronization of systems Eq.(3) and Eq.(4) is ensured. The controller Eq.(14) requires a priori knowledge about the augmented state η. According to Assumption A2, that is an unrealistic situation. Consequently, estimated values of the states ($\boldsymbol{z}$, η) are required for practical implementation. Choose a continuously differential nonlinear function $m(x)$ such that $m'=\mathrm{d}m(x)/\mathrm{d}x\neq 0$ and $m(0)=0$. Apply extended state observer theory to design the following Luenberger-like state estimator to reconstruct the states ($\boldsymbol{z}$, η) from the measurement of z_1 (Zhao, 2004):

$$\begin{aligned}
\dot{\hat{z}}_i &= \hat{z}_{i+1} - L^i \frac{k_i}{m'(\hat{z}_1 - z_1)} m(\hat{z}_1 - z_1), \quad i = 1, 2, \ldots, r-1 \\
\dot{\hat{z}}_i &= \hat{\eta} - L^r \frac{k_r}{m'(\hat{z}_1 - z_1)} m(\hat{z}_1 - z_1) + \gamma u \\
\dot{\hat{\eta}} &= -L^{r+1} \frac{k_{r+1}}{m'(\hat{z}_1 - z_1)} m(\hat{z}_1 - z_1)
\end{aligned} \tag{17}$$

where ($\hat{\boldsymbol{z}}$, $\hat{\eta}$) are estimated values of ($\boldsymbol{z}$, η), respectively. The parameters k_i, i=1, 2, …, r+1 are chosen such that they are coefficients of Hurwitz polynomial $P_{r+1}(s)=s^{r+1}+k_1 s^r+\ldots+k_r s+k_{r+1}$, and L>0 be the high-gain estimation parameter. Therefore, the controller is:

$$u = \int_0^t \frac{1}{\gamma}\left(\alpha S - \beta \operatorname{sgn}(S) - \sum_{i=1}^{r} c_i \ \hat{z}_i - c_{i+1}(\hat{\eta} + \gamma u) - \dot{\hat{\eta}} \right) \mathrm{d}t \tag{18}$$

ILLUSTRATIVE EXAMPLES

After the synchronization of two chaotic systems, we can use various chaotic secure communication schemes to transmit information signal. In the simulation, we use the two-channel transmission method (Jiang, 2002). We present two examples in this section.

Synchronization of two Chua's circuits with parametric variations

Consider two Chua circuits (Kennedy, 1992) in the form: $\boldsymbol{X}_{1,2}=(x_{1,2}, y_{1,2}, w_{1,2})$;

$$\begin{aligned}
\dot{x}_{1,2} &= \rho_{1,2}[y_{1,2} - x_{1,2} - f(x_{1,2})] \\
\dot{y}_{1,2} &= x_{1,2} - y_{1,2} - w_{1,2} \\
\dot{w}_{1,2} &= -\sigma_{1,2} y_{1,2}
\end{aligned} \tag{19}$$

where $f(x)=bx+(a-b)(|x+1|-|x-1|)/2$.

Suppose the transmitter's output is x_1, $\boldsymbol{g}=[1\ 0\ 0]^{\mathrm{T}}$. Defining the error variables as follows: $e_1=x_1-x_2$, $e_2=y_1-y_2$, $e_3=w_1-w_2$, then the discrepancy system can be obtained as follows:

$$\begin{aligned}
&\dot{e}_1 = \Delta f_1 - u, \dot{e}_2 = \Delta f_2, \dot{e}_3 = \Delta f_3 \\
&y = e_1
\end{aligned} \tag{20}$$

By simple calculation we know Eq.(20) has relative degree 1. Define coordination transform as: $z_1=e_1$, $v_1=e_2$, $v_2=e_3$, let $\boldsymbol{v}=(v_1, v_2)$, we have

$$\begin{aligned}
\dot{z}_1 &= \Delta f_1 - u \\
\dot{\boldsymbol{v}} &= \boldsymbol{C}\boldsymbol{v} + \Delta_2 \\
y &= z_1
\end{aligned} \tag{21}$$

where $\boldsymbol{C} = \begin{bmatrix} -1 & 1 \\ -2\theta & 0 \end{bmatrix}$, $\Delta_2 = \begin{bmatrix} z_1 \\ \delta \end{bmatrix}$, $\theta=(\sigma_1+\sigma_2)/2$, $\delta=\sigma_1 y_2+\sigma_2 y_1$. It is clear that θ>0, δ is bounded. As $z_1\to 0$ (zero dynamics), $\boldsymbol{\Delta}_2$ is bounded, hence the zero dynamics subsystem $\dot{\boldsymbol{v}} = \boldsymbol{C}\boldsymbol{v} + \Delta_2$ is asymptotically stable. That is, the discrepancy is a minimum phase system. z_1 is the measurable error. By calculating we know $\beta(\boldsymbol{z}, \boldsymbol{v})=-1$, so $\gamma=-1$. Let $\eta=\Delta f_1$, the control becomes:

$$\dot{\hat{z}}_1 = \hat{\eta} - L\frac{k_1}{m'(\hat{z}_1 - z_1)}m(\hat{z}_1 - z_1) - u$$
$$\dot{\hat{\eta}} = -L^2\frac{k_2}{m'(\hat{z}_1 - z_1)}m(\hat{z}_1 - z_1)$$
$$\dot{S} = \alpha S - \beta\,\mathrm{sgn}(S)$$
$$u = \int_0^t -(\alpha S - \beta\,\mathrm{sgn}(S) - c_1\hat{z}_1 - c_2(\hat{\eta} - u) - \dot{\hat{\eta}})\mathrm{d}t \quad (22)$$

For numerical simulation, we choose $m(x)=x$, and choose the following initial conditions: $S(0)=0$, $\boldsymbol{X}_1(0)=(0.1,\ -0.5,\ 0.5)$, $\boldsymbol{X}_2(0)=(0.6,\ 0.4,\ 0.8)$, $(\hat{z}_1(0), \hat{\eta}(0)) = (0,0)$, $u(0)=0$, and select the following parameter values: $\alpha=0.01$, $\beta=0.05$, $L=30$, $k_1=2$, $k_2=1$, $c_1=900$, $c_2=60$, $\rho_1=10$, $\sigma_1=14.28$, $a=-1.27$, $b=-0.68$, $\rho_2=0.9\rho_1$, $\sigma_2=0.9\sigma_1$. The information signal is chosen as $s=\sin(t)+\sin(2t)+\sin(3t)$. We take the following encryption function and decryption function (Jiang, 2002):

$$s_e = x_1^2 + (1+\cos^2(t))s,\ s_d = (s_e - x_2^2)/(1+\cos^2(t)).$$

Fig.1 shows the synchronization of two Chua's circuits in spite of the response system parameters being modified by 10%. Note that the sliding mode control yields complete synchronization, and the information signal s is recovered quite accuracy.

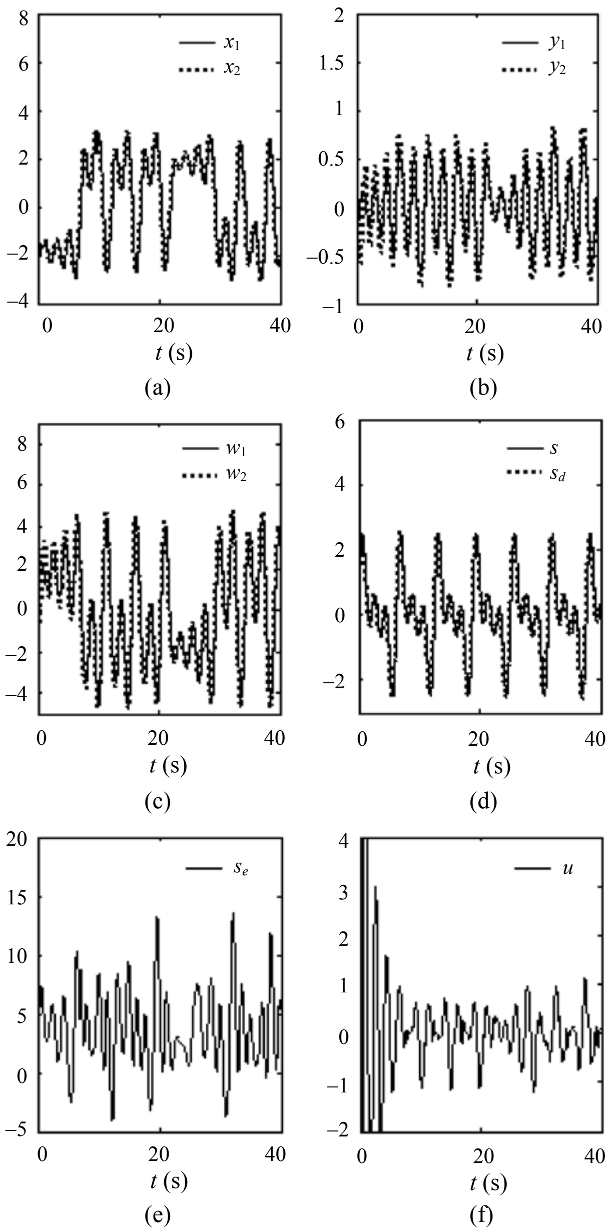

Fig.1 Synchronization of two Chua's circuit with parametric variations and the signal transmitted
(a) States x_1 and x_2; (b) States y_1 and y_2; (c) States w_1 and w_2; (d) Information signal s and recovered signal s_d; (e) Encrypted signal s_e; (f) Control u

Synchronization of two different chaotic systems

We choose the Duffing equation as the transmitter and the Vander Pol oscillator as receiver. The drive system is:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_1 - \delta_\mathrm{D}x_2 - x_1^3 + \gamma_\mathrm{D}\cos w_\mathrm{D}t\end{aligned} \quad (23)$$

The response system is given by:

$$\begin{aligned}\dot{y}_1 &= y_2\\ \dot{y}_2 &= \delta_\mathrm{V}(1-y_1^2)y_2 - y_1^3 + \gamma_\mathrm{V}\cos w_\mathrm{V}t + u\end{aligned} \quad (24)$$

Suppose the output of Eq.(23) is x_1. Defining $e_1=x_1-y_1$, $e_2=x_2-y_2$, one yields the following discrepancy system

$$\begin{aligned}\dot{e}_1 &= e_2\\ \dot{e}_2 &= \Delta f - u\end{aligned} \quad (25)$$

where $\Delta f = x_1 - \delta_\mathrm{D}x_2 - x_1^3 + \gamma_\mathrm{D}\cos w_\mathrm{D}t - (\delta_\mathrm{V}(1-y_1^2)y_2 - y_1^3 + \delta_\mathrm{V}\cos w_\mathrm{V}t)$.

System Eq.(25) has relative degree 2. Note that system Eq.(25) is fully linearizable. Thus the coordinate transformation is given by $z_1=e_1$, $z_2=e_2$. Defining $\eta=\Delta f$, then system Eq.(25) can be constructed and the control becomes:

$$\dot{\hat{z}}_1 = \hat{z}_2 - L\frac{k_1}{m'(\hat{z}_1 - z_1)} m(\hat{z}_1 - z_1)$$
$$\dot{\hat{z}}_2 = \hat{\eta} - u - L^2\frac{k_2}{m'(\hat{z}_1 - z_1)} m(\hat{z}_1 - z_1)$$
$$\dot{\hat{\eta}} = -L^3\frac{k_3}{m'(\hat{z}_1 - z_1)} m(\hat{z}_1 - z_1)$$
$$\dot{S} = \alpha S - \beta\,\mathrm{sgn}(S)$$
$$u = \int_0^t -(\alpha S - \beta\,\mathrm{sgn}(S) - c_1\hat{z}_1 - c_2(\hat{\eta} - u) - \dot{\hat{\eta}})\mathrm{d}t \quad (26)$$

For the purpose of simulation, we choose $m(x)=x$, and make the following choice of initial conditions: $(x_1, x_2)=(0.3, 2.1)$, $u(0)=0$, $S(0)=0$, $(\hat{z}_1(0), \hat{z}_2(0), \hat{\eta}(0)) = (-0.01, 0.01, 0.01)$, and select the following parameter values: $\delta_D=0.15$, $\gamma_D=1.75$, $w_D=2/3$, $\delta_V=0.1$, $w_V=1$, $\gamma_V=0.3$, $\alpha=0.01$, $\beta=3.2$, $L=500$, $k_1=3$, $k_2=3$, $k_3=1$, $c_1=500^3$, $c_2=7500$, $c_3=1500$. Fig.2 shows the performance of the controller Eq.(26). Systems Eq.(23) and Eq.(24) is synchronized in spite of different structure; the information signal s and the recovered signal s_d are almost identical.

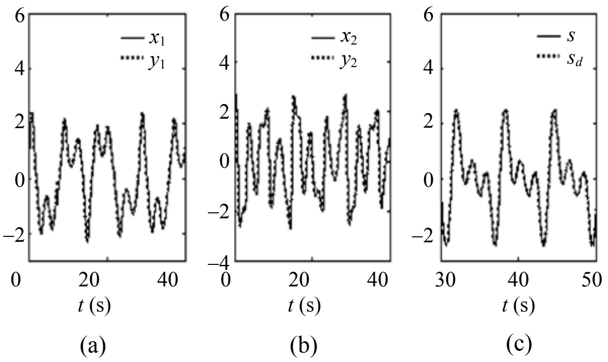

Fig.2 Synchronization of two different chaotic systems and the signal transmitted
(a) States x_1 and y_1; (b) States x_2 and y_2; (c) Information signal s and recovered signal s_d

CONCLUSION

In this work, we designed a strategy to synchronize two chaotic systems with structure or parameters difference. To this end, uncertainties in the discrepancy system are lumped into a nonlinear function and interpreted as an augmented state. The synchronization problem was addressed as one of chaos suppression. Then the chaos suppression problem was solved by means of a feedback control based on sliding mode control scheme. To make this controller physically realizable, an extended state observer was used to estimate the error between the transmitter and receiver. In addition, the proposed scheme allows for message signal recovery in spite of parametric variations and strictly different model of transmitter and receiver.

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