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A new digital approach to design multivariable robust optimal control systems*

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Abstract: This paper presents a new design of robust optimal controller for multivariable system. The row characteristic functions of a linear multivariable system and dynamic decoupling of its equivalent system, were applied to change the transfer function matrix of a closed-loop system into a normal function matrix, so that robust H^∞ optimal stability is guaranteed. Furthermore, for the decoupled equivalent control system the l^∞ optimization approach is used to have the closed-loop system embody optimal time domain indexes. A successful application on a heater control system verified the excellence of the new control scheme.

Key words: l^∞/H^∞, Dynamic decoupling, Normal matrix, Row characteristic function
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INTRODUCTION

It is well known that for an $m\times m$ rational reduced multivariable control system $\boldsymbol{G}$, there exist uni-modular matrices $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ such that $\boldsymbol{G}$ has Smith-McMillan decomposition form (Kaileth, 1980), $\boldsymbol{G}=\boldsymbol{U}_1\Lambda\boldsymbol{U}_2$. Since this decomposition method is a standard rule for dealing with linear rational fractional matrix functions, it is widely used in studies on optimization problems for linear multivariable systems, such as l^∞/l^1 optimization problems (Dahleh and Pearson, 1987; 1988; Dias and Dahleh, 1993), and so on. It had been pointed out (Hung and MacFarlane, 1982; Gao and Wu, 1998) that this method is not suitable for building a robust control system because uni-modular matrices are not unitary matrices. As the spectrum radius of a normal matrix is equal to its maximum singular value, if a stable closed-loop system is a normal matrix function then it embodies H^∞ robust stability (Gao and Wu, 1998).

In this paper, firstly, a new concept, row characteristic function, is presented for the realization of a normal multivariable closed-loop system. In a multivariable system, this scheme achieves normalization of its closed-loop system by robustly decoupling its equivalent system. Secondly, the normalized closed-loop system and its equivalent decoupled system have the same time-domain performance by modifying its right frame decomposition matrix into a constant real unitary matrix. Then the approach by Liu and Sun (2000) is adapted to make the robust closed-loop system obtain optimal time domain indexes. Thirdly, the new approach is applied to control the water pipe temperature of a heater. Robustness and optimization are achieved by the heater control system.

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NOTATION, DEFINITIONS AND LEMMA

z-transform The z-transform of a causal real sequence $x(n)$ is denoted as $\boldsymbol{x}$. Its expression is

$$\boldsymbol{x}=\sum_{n=0}^{\infty}x(n)z^{-n} \tag{1}$$

where z denotes a complex variable. With this notation the transfer function for an $m\times m$ causal linear discrete-time system is expressed as $\boldsymbol{P}=\sum_{i=0}^{\infty}P(k)z^{-k}$, generally $\boldsymbol{P}$ is always non-singular.

H^{∞} norm For an $m\times m$ causal linear discrete-time system $\boldsymbol{P}$, its H^{∞} norm is defined as

$$\|\boldsymbol{P}\|_{\infty}=\sup_{\omega}\bar{\sigma}(\boldsymbol{P}(\mathrm{e}^{\mathrm{j}\omega T})) \tag{2}$$

where $\bar{\sigma}(\boldsymbol{P})$ denotes the maximum singular value of $\boldsymbol{P}$. Obviously it needs no $\boldsymbol{P}$ poles outside the unit disc in the complex plane for guaranteeing the existence of its H^{∞} norm.

Row characteristic function For a row of an $m\times m$ causal linear discrete-time system $\boldsymbol{P}$, its row characteristic function is defined as a transfer function which embodies the common characters that belong to all of the transfer functions of this row.

Generally all of the non-minimum-phase zeros and unstable poles in the transfer functions of a row should be included in its row characteristic function. For a multivariable control system this concept is important for setting up its robust closed-loop system. Reasonable row characteristic functions can simplify the system's robust controller and achieve a relatively lower order.

Robust optimal system A robust optimal system has l^{∞} optimal time domain indexes in its standard model and H^{∞} robust stability in its perturbation model.

Lemma 1 For the functional matrices which have the same characteristic functions and no poles outside the unit disc of the complex plane, normal functional matrices have the minimum H^{∞} norm. The proof of the lemma is given by Gao and Wu (1998).

DESIGN OF ROBUST OPTIMAL SYSTEM

Given an $m\times m$ linear discrete-time system $\boldsymbol{P}$ and an $m\times m$ digital controller $\boldsymbol{K}$, a standard feedback control system can be constructed. If $\boldsymbol{P}$ is a causal system, then it is a real rational fractional complex matrix.

Theorem 1 For an $m\times m$ non-singular real rational fractional complex matrix $\boldsymbol{P}$, there always exists the following decomposition form:

$$\boldsymbol{P}=\boldsymbol{U}\boldsymbol{T}_1\boldsymbol{L} \tag{3}$$

where $\boldsymbol{U}$ is a constant real unitary matrix, $\boldsymbol{T}_1$ is a diagonal real rational fractional complex matrix, and $\boldsymbol{L}$ is a non-singular rational fractional complex matrix.

Proof For an $m\times m$ non-singular real rational fractional complex matrix $\boldsymbol{P}$, a right MFD (Matrix Fraction Description) decomposition form is written as

$$\boldsymbol{P}=\boldsymbol{N}\boldsymbol{D}^{-1} \tag{4}$$

where $\boldsymbol{N}$ and $\boldsymbol{D}$ are complex polynomial matrices.

There always exists a constant real unitary matrix $\boldsymbol{U}$, such as $\boldsymbol{U}=\boldsymbol{I}$ for one, that satisfies

$$\boldsymbol{N}=\boldsymbol{U}\boldsymbol{T}_0. \tag{5}$$

Clearly $\boldsymbol{T}_0$ is a non-singular complex matrix.

For each row of the matrix $\boldsymbol{T}_0\boldsymbol{D}^{-1}$, it needs to decide its row characteristic function such that a diagonal matrix $\boldsymbol{T}_1$ can be built up and the following equation is obtained.

$$\boldsymbol{T}_0\boldsymbol{D}^{-1}=\boldsymbol{T}_1\boldsymbol{L}. \tag{6}$$

In Eq.(6) $\boldsymbol{L}$ is a non-singular rational fractional complex matrix and $\boldsymbol{T}_1$ is a diagonal rational real fractional complex matrix consisting of the row characteristic functions of the rows of the matrix $\boldsymbol{T}_0\boldsymbol{D}^{-1}$ one-to-one correspondence.

Eqs.(4)~(6) can be used to derive that $\boldsymbol{P}=\boldsymbol{U}\boldsymbol{T}_1\boldsymbol{L}$.

With this we can see that for $\boldsymbol{P}$, if $\boldsymbol{U}$ and $\boldsymbol{T}_1$ are certain, then $\boldsymbol{L}$ can be exclusively determined.

With the above theorem for a closed-loop system $\boldsymbol{Q}$ with open-loop system $\boldsymbol{P}$ and the controller $\boldsymbol{K}$ to be solved, we can build its equivalent closed-loop system $\boldsymbol{Q}_1$ with the diagonal open-loop system $\boldsymbol{T}_1$ and diagonal controller

$$\boldsymbol{K}_1=\boldsymbol{L}\boldsymbol{K}\boldsymbol{U}. \tag{7}$$

It was reported in Liu *et al.*(2004)'s scheme that a relation exists between the transfer function $\boldsymbol{Q}$ of the closed-loop system, and its equivalent closed-loop system $\boldsymbol{Q}_1$:

$$\boldsymbol{Q}_1=\boldsymbol{U}^{-1}\boldsymbol{Q}\boldsymbol{U}=\boldsymbol{U}^{*}\boldsymbol{Q}\boldsymbol{U} \tag{8}$$

where $\boldsymbol{U}^{*}$ is the conjugate transpose of the unitary matrix $\boldsymbol{U}$. Note that $\boldsymbol{Q}_1$ is a diagonal matrix. By Lemma 1 it possesses H^{∞} robust stability if it is stable.

In this situation, by Eq.(8) $\boldsymbol{Q}$ is a normal matrix and also has H^{∞} robust stability.

The diagonal system $\boldsymbol{T}_1$ only needs to have optimal controllers for its diagonal elements correspondingly. The time-domain performance indexes such as overshoot, settling time, decreasing rate, etc., can be taken care of in the design of optimal controllers by using the approach for l^{∞} optimization introduced in Liu and Sun (2000)'s scheme. Of course the optimal controller $\boldsymbol{K}_1$ built up by these one-variable controllers is a diagonal square matrix. The approach in Eq.(7) can be used to achieve the robust optimal controller $\boldsymbol{K}$. To guarantee the same optimal time-domain indexes for $\boldsymbol{Q}$ and $\boldsymbol{Q}_1$, the unitary matrix $\boldsymbol{U}$ must be a constant matrix. To simplify the engineering design, approaches such as steady state decoupling and pseudo-diagonalization can be used to get a simplified $\boldsymbol{L}$, $\boldsymbol{L}=\boldsymbol{L}(0)$, $\boldsymbol{L}(1)$ for examples.

The steps in developing a robust optimal controller $\boldsymbol{K}$ are described in the following algorithm.

Algorithm 1 (1) For an $m\times m$ non-singular linear discrete-time system $\boldsymbol{P}$, by the approach introduced in Theorem 1, it is decomposed to $\boldsymbol{P}=\boldsymbol{U}\boldsymbol{T}_1\boldsymbol{L}$, where $\boldsymbol{U}$ is a real unitary matrix, $\boldsymbol{T}_1$ is a diagonal rational real fractional complex matrix consisting of all of the row characteristic functions of $\boldsymbol{U}^*\boldsymbol{P}$ one-to-one correspondence, and $\boldsymbol{L}$ is a non-singular rational fractional complex matrix. In industrial control, it is always possible that system $\boldsymbol{P}$ is nonsingular.

(2) For each element of the $m\times m$ diagonal system $\boldsymbol{T}_1$, using the approach introduced by Liu and Sun (2000) to achieve the corresponding one-variable controller for optimal time domain indexes, then the diagonal optimal controller $\boldsymbol{K}_1$ is obtained by the achieved optimal controllers one-to-one correspondence.

(3) Computing for $\boldsymbol{U}^{-1}$ and $\boldsymbol{L}^{-1}$ or $\boldsymbol{L}(0)^{-1}$, $\boldsymbol{L}(1)^{-1}$ for simplification.

(4) Modifying suitably to guarantee the robust optimal controller

$$\boldsymbol{K}=\boldsymbol{L}^{-1}\boldsymbol{K}_1\boldsymbol{U}^{-1}=\boldsymbol{L}^{-1}\boldsymbol{K}_1\boldsymbol{U}^*, \quad (9)$$

is an $m\times m$ rational real fractional matrix.

AN EXAMPLE

The transfer function matrix of a heater system is described as

$$\boldsymbol{G}(s)=\begin{bmatrix} 1.0/(1+4s) & 0.7/(1+5s) & 0.3/(1+5s) \\ 0.6/(1+0.5s) & 1.0/(1+4s) & 0.4/(1+5s) \\ 0.4/(1+5s) & 0.3/(1+5s) & 1.0/(1+4s) \end{bmatrix}.$$

The outputs and inputs are the temperatures for three water-flow pipes and the flows of three oil pipes for ignitions, respectively.

The sampling period is chosen as 1 second and the zero-order holder is utilized, the discrete-time model for $\boldsymbol{G}(s)$ is

$$\boldsymbol{G}=\begin{bmatrix} \frac{0.22}{z-0.78} & \frac{0.126}{z-0.82} & \frac{0.054}{z-0.82} \\ \frac{0.108}{z-0.82} & \frac{0.22}{z-0.78} & \frac{0.072}{z-0.82} \\ \frac{0.072}{z-0.82} & \frac{0.054}{z-0.82} & \frac{0.22}{z-0.78} \end{bmatrix}.$$

By Algorithm 1, firstly the row characteristic function for $\boldsymbol{G}$ is chosen as $\frac{0.22}{z-0.78}$, therefore we have the following decoupling form:

$$\boldsymbol{G}=\boldsymbol{U}\boldsymbol{T}_1\boldsymbol{L},$$

where $\boldsymbol{U}=\boldsymbol{I}$, $\boldsymbol{T}_1=\mathrm{diag}\left\{\frac{0.22}{z-0.78},\frac{0.22}{z-0.78},\frac{0.22}{z-0.78}\right\}$, and $\boldsymbol{L}=$

$$\begin{bmatrix} 1 & \frac{0.57(z-0.78)}{(z-0.82)} & \frac{0.245(z-0.78)}{(z-0.82)} \\ \frac{0.49(z-0.78)}{(z-0.82)} & 1 & \frac{0.327(z-0.78)}{(z-0.82)} \\ \frac{0.327(z-0.78)}{(z-0.82)} & \frac{0.245(z-0.78)}{(z-0.82)} & 1 \end{bmatrix}$$

$\boldsymbol{L}(0)$ is selected to take the place of $\boldsymbol{L}$.

For the diagonal system $\boldsymbol{T}_1$, the set-point optimal controller is developed as

$$\boldsymbol{K}_1=\mathrm{diag}\left\{\frac{4.54(z-0.78)}{(z-1)},\frac{4.54(z-0.78)}{(z-1)},\frac{4.54(z-0.78)}{(z-1)}\right\}.$$

By Eq.(9) the robust optimal controller is obtained by

$$\boldsymbol{K}=\begin{bmatrix} 6.1579 & -3.2238 & -0.4354 \\ -2.4153 & 6.1579 & -1.3462 \\ -1.3462 & -0.4354 & 4.9886 \end{bmatrix}\left(\frac{z-0.78}{z-1}\right).$$

For set-point step inputs that change from 0°~100°, the response curves of the three water pipes temperatures are shown in Fig.1. We describe inputs as $u1$, $u2$, $u3$, and output responses as $y1$, $y2$, $y3$. The simulation result showed that excellent time domain performances were obtained for the heater control system.

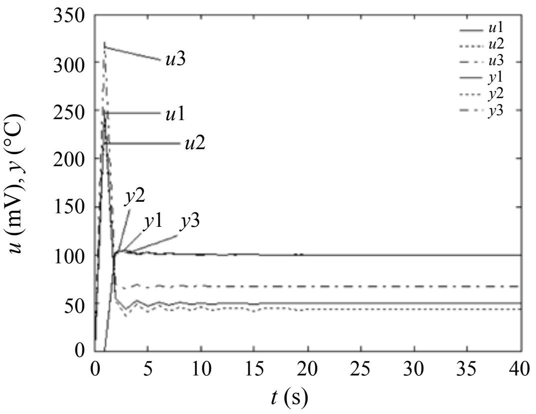

Fig.1 Temperature response curves for three water pipes of a heater

Considering the system disturbances and model uncertainties, the perturbation model can be described as $\boldsymbol{G}'(s)=\boldsymbol{G}(s)[\boldsymbol{I}+\boldsymbol{\Delta}(s)]$, we choose $\Delta(s)=\begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$, retain the same controller $\boldsymbol{K}$, give the same step inputs that change from 0°~100°, then we get the response curves of the three water pipes temperatures shown in Fig.2. Apparently the controller sufficiently ensures the robustness of the perturbation model.

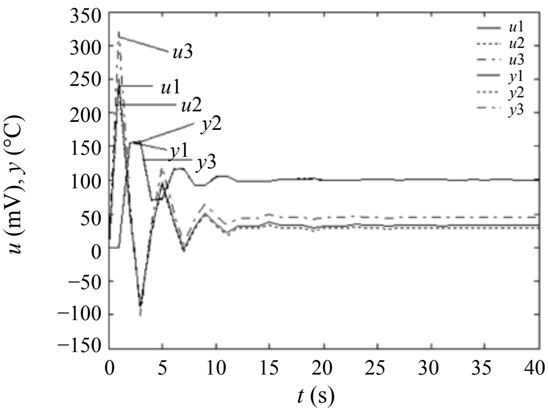

Fig.2 Temperature response curves for three water pipes of a heater with disturbance

CONCLUSION

A new robust optimal control scheme for MIMO digital square matrix systems has been presented. The key idea is the concept of row characteristic functions for robust decoupling. This scheme is intended to make the closed-loop system have both H^{∞} robust stability and optimal time-domain indexes. How to achieve a satisfactory right frame matrix is important in the controller design procedure. Reasonable simplification for the frame matrix can cut down the order of the optimal controller and enhance the controllers robustness.

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