

## Shear capacity of reinforced concrete columns strengthened with CFRP sheet

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**Abstract:** This paper discusses the results of tests on the shear capacity of reinforced concrete columns strengthened with carbon fiber reinforced plastic (CFRP) sheet. The shear transfer mechanism of the specimens reinforced with CFRP sheet was studied. The factors affecting the shear capacity of reinforced concrete columns strengthened with CFRP sheet were analyzed. Several suggestions such as the number of layers, width and tensile strength of the CFRP sheet are proposed for this new strengthening technique. Finally, a simple and practical design method is presented in the paper. The calculated results of the suggested method are shown to be in good agreement with the test results. The suggested design method can be used in evaluating the shear capacity of reinforced concrete columns strengthened with CFRP sheet.

**Key words:** Reinforced concrete, Carbon fiber reinforced plastic (CFRP), Column, Shear capacity  
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### INTRODUCTION

There is increased need in recent years for strengthening or rehabilitation of existing reinforced concrete structures adversely affected by overloading, construction material deterioration, seismic loads, structural deformation, etc. An effective method for increasing the shear capacity of reinforced concrete columns is the use of externally bonded carbon fiber reinforced plastic (CFRP) systems (ACI, 2002). FRP systems were first applied to reinforced concrete columns in Japan in the 1980s. The external CFRP sheet and the stirrups confine the concrete core so that the reinforced concrete column's shear capacity and deformation performance are improved. Some methods have recently been developed to predict the shear capacity of reinforced concrete columns strengthened with CFRP. It has been revealed that the shear capacity of the strengthened column is mainly influenced by the axial compression ratio, the shear span/depth ratio, the amount of CFRP sheet, etc.

A brief review of the shear capacity of rein-

forced concrete columns strengthened with FRP sheet is given below.

1. Xiao *et al.* (1999) presented a formula based on the test of three reinforced concrete cylinders strengthened with glass fabric:

$$V = V_c + V_s + V_a + V_j \quad (1)$$

where  $V_c$ ,  $V_s$ ,  $V_a$  and  $V_j$  are the shear resistance offered by reinforced concrete, stirrups, axial compression and the glass fabric respectively.

The shear resistance of glass fabric was given by the following equilibrium:

$$V_j = \frac{\pi}{2} f_{jd} t_j (D - c) \cot \theta \quad (2)$$

in which  $f_{jd}$ ,  $t_j$ ,  $D$ ,  $c$  and  $\theta$  are tensile strength of fabric, nominal thickness of fabric, diameter of column specimen, depth of compressive area and angle of truss structure, respectively.

2. The ultimate shear strength based on the test

of seven reinforced concrete columns strengthened with CFRP sheet in Ye *et al.*(2000; 2002) was specified as:

$$V_u = V_{RC} + V_{CF} \tag{3}$$

The shear strength ( $V_{RC}$ ) is defined in the Code for Design of Concrete Structures. The second item,  $V_{CF}$ , the shear resistance contribution of the CFRP sheet, is expressed as:

$$V_{CF} = \nu_{CF} \lambda_{CF} f_{CF} b h_0 \tag{4}$$

where  $\nu_{CF}$ ,  $\lambda_{CF}$ ,  $f_{CF}$ ,  $b$ , and  $h_0$  are respectively shear strengthening coefficient of the CFRP sheet, volume ratio of CFRP sheet to the concrete column, tensile strength of CFRP sheet, width and effective depth of the column cross-section.

3. The calculated shear capacity using CFRP sheet as suggested by (CEASI, 2001) is:

$$V_u = V_{RC} + V_{ccf} \tag{5}$$

The shear strength ( $V_{RC}$ ) of reinforced concrete column can be calculated according to (CDCS, 2002) and  $V_{ccf}$  is the shear resistance contribution of the CFRP sheet:

$$V_{ccf} = \phi \frac{2n_{cf} w_{cf} t_{cf}}{(s_{cf} + w_{cf})} \epsilon_{cfv} E_{cf} h_{cf} \tag{6}$$

All of the parameters in the above equation can be found in (CEASI, 2001).

This paper based on our test of five reinforced concrete columns presents a new design method for determining the shear capacity of concrete columns strengthened with CFRP sheet(s).

## TEST RESULTS

Five concrete columns strengthened with CFRP sheet(s) were tested on their shear resistance (Zhao *et al.*, 2002) in order to investigate the mechanism of their shear behavior. The strengthening conditions and the test results are given in Table 1.

### FACTORS AFFECTING THE SHEAR CAPACITY OF THE STRENGTHENED COLUMNS

Results of experiment and theoretical analysis were applied to develop this paper's simple method for calculating the shear strength as given in Eq.(7).

$$V = V_{RC} + V_{CF} \tag{7}$$

where  $V_{RC}$  is the shear strength of the reinforced concrete columns, and  $V_{CF}$  is the shear force contribution of the CFRP sheet.

#### Shear strength of reinforced concrete columns $V_{RC}$

All the shear resistance shared by the reinforced concrete column (Xiao *et al.*, 1999; Ye *et al.*, 2000; 2002) is completely determined according to the Code for Design of Concrete Structures. However, our basic experimental research (Zhao *et al.*, 2002) indicated that the axial compressive strength of the concrete column can be enhanced by its confinement by CFRP sheet, as shown in Eq.(8),

$$f'_c = (1 + 1.2\lambda')f_c \tag{8}$$

Tests showed that the shear strength of the column increases due to the CFRP confinement, especially when a large amount of CFRP sheet is used.

**Table 1 Test results**

Specimen	Strengthening condition	$f_{cu}$	Stirrup	$\lambda$	$n$	Maximum load (kN)	Failure mode
DZ-1	None	25.4	$\phi 6@100$	1.0	0.35	135.0	shear
DZ-2	One layer of high strength CFRP sheet	25.4	$\phi 6@100$	1.0	0.35	175.0	flexure-shear
DZ-3	One layer of normal strength CFRP sheet	25.4	$\phi 6@100$	1.0	0.35	165.0	flexure-shear
DZ-4	Two layers of normal strength CFRP sheet	29.2	$\phi 6@100$	1.0	0.35	218.0	flexure-shear
DZ-5	Two layers of normal strength CFRP sheet	29.2	$\phi 6@100$	1.0	0.50	224.0	flexure-shear

$f_{cu}$ =Cubic compressive strength of concrete (MPa);  $\lambda$ =Shear span/depth ratio;  $n$ =Axial compression ratio

The shear column works just like an arch structure and the external restraint of CFRP sheet can improve the compressive strength of the concrete core which controls the shear resistance of the uncracked compressive area. Various analyses showed that the shear strength of the strengthened reinforced concrete columns core can be expressed as:

$$V_{RC} = \frac{1.75}{\lambda + 1} (1 + 1.2\lambda') f_t b h_0 + f_{yv} \frac{A_{sv}}{s} h_0 + 0.07N \quad (9)$$

where  $\lambda'$  is the volume ratio of CFRP sheet to concrete column; its calculating formula is given by Zhao *et al.*(2000). Meanings of the other symbols are the same as those in CDCS (2002).

#### Shear force contribution of CFRP sheet ( $V_{CF}$ )

Test results shows that  $V_{CF}$  is affected by the axial compression ratio, the shear span/depth ratio, the layers of CFRP sheet and its width, etc. These factors will be discussed in detail later.

1. Reduction factor ( $\beta$ ) due to the amount of CFRP sheet layers

In the tests, the CFRP sheet could not reach its ultimate tensile strength even during its failure. CFRP layers greatly affect the effectiveness of the CFRP and the strengthening effectiveness was not in direct proportion to the sheet layers. As the CFRP sheet layers increased, the CFRP's average bearing capacity decreased.

For example, the ultimate strain of the CFRP sheet of our test was  $8150 \times 10^{-6}$ . Unexpected results were obtained when this kind of CFRP sheet was used to strengthen the reinforced concrete columns. When one layer of this kind of CFRP sheet was used to strengthen the reinforced concrete columns, the largest strain was only about  $6000 \times 10^{-6}$ , equivalent to 2/3 of the ultimate strain, even until sheet tensile failure. When two CFRP layers were used, the largest strain was  $4000 \times 10^{-6}$ , just 1/2 of the ultimate strain. Ye *et al.*(2002) also proved the above-mentioned conclusions with the ultimate strain of CFRP used in the test was  $14890 \times 10^{-6}$ .

However, in our specimen CS20-1-15, the failure strain of the sheet was  $9100 \times 10^{-6}$  and in specimen CS25-1-35 was  $7400 \times 10^{-6}$ . The statistical data of all the tests indicated  $\beta=2/3$ , when one layer of CFRP sheet was used;  $\beta=1/2$ , when two layers of CFRP

sheet were used. Use of three or more layers is not recommended.

2. Width coefficient ( $\gamma$ ) of CFRP sheet

With the same amount of CFRP, the strip-strengthening program is better than the whole-strengthening program (Zhao *et al.*, 2002), which reveals that the width effect can influence the CFRP confinement of the reinforced concrete column. The wider the CFRP sheet is, the less uniform the CFRP stress will be. Without doubt, in the region of high stress the sheet will be locally destroyed and then completely ripped. To make best use of the CFRP sheet, the strip width and column cross-section must be controlled. Furthermore, the width effect has relations with the strip-width/column-section-height ratio ( $w/h$ ). When  $w/h$  is less than 0.5, the strip width rarely affects the tension and shear performance of the CFRP. However, when  $w/h$  is greater than 0.5, the strip width will greatly influence the CFRP shear behavior. Generally speaking, the width effect coefficient ( $\gamma$ ) can be decided as follows:

$$\begin{cases} \gamma = 1.0, & 0.2 \leq \frac{w}{h} \leq 0.5 \\ \gamma = 1.0 - \frac{0.6}{e^{h/w}}, & \frac{w}{h} > 0.5 \end{cases} \quad (10)$$

When  $w/h$  is less than 0.2, the CFRP sheet performance might be unstable due to the manufacture process, construction procedure and many other uncontrolled factors. So, this formula is not recommended when  $w/h$  is less than 0.2.

3. Reduction factor ( $\alpha_{CF}$ ) of tensile strength of CFRP sheet

Tests showed that the CFRP sheet does not reach its maximum tensile stress even though the reinforced concrete column has reached its ultimate stress. The potential of the CFRP sheet will still be fully utilized beyond the peak shear stress of the reinforced concrete column. In other words, mechanical hysteresis delays the full use of CFRP until the reinforced concrete column has surpassed its ultimate strength. This point was also be confirmed by Ye *et al.*(2000). Therefore, the total shear capacity of the strengthened reinforced concrete column is not simply equal to the shear strength of the reinforced concrete plus the shear strength of the CFRP. There is a reduction factor ( $\alpha_{CF}$ ) between them. It can be seen from tests that

$\alpha_{CF}$  is related to the axial compression ratio, shear span/depth ratio and the amount of CFRP sheet used, etc.

According to the aforementioned analysis, the shear force contribution of the CFRP sheet can be expressed as:

$$V_{CF} = \alpha_{CF} V_{CFmax} \tag{11}$$

where  $V_{CFmax}$  is the maximum shear contribution of CFRP sheets and

$$V_{CFmax} = \beta\gamma \frac{2n_{CF}wt}{(s_{CF} + w)} f_{CF} h_0 \tag{12}$$

$n_{CF}$  represents the layer of CFRP sheet;  $w$  the width of the CFRP sheet strip;  $t$  the nominal thickness of the CFRP sheet;  $f_{CF}$  the ultimate tensile stress and  $s_{CF}$  the spacing of CFRP strips.

After obtaining the general value of  $V_{CF}$  in the test,  $\alpha_{CF}$  can be obtained from Eq.(13). The value of  $\alpha_{CF}$  is given in Table 2.

$$\alpha_{CF} = V_{CF} / V_{CFmax} \tag{13}$$

Many factors such as axial compression ratio, shear span/depth ratio, amount of CFRP affect  $\alpha_{CF}$ . These factors are discussed in detail afterwards.

(1) Axial compression ratio ( $n$ )

Both  $\alpha_{CF}$  and the shear force of the CFRP sheet are mainly affected by  $n$ . The relationship between  $n$  and  $\alpha_{CF}$  is shown in Fig.1.  $\alpha_{CF}$  decreases with increasing  $n$ . In other words, the shear strength provided by the CFRP sheet decreases when  $n$  decreases. The axial force not only delays the emergence of the diagonal shear cracks, but also reduces their inclination angle, width and length. The greater the axial force is,

the larger the uncracked concrete area will be. Moreover, the arch structure mechanism of the reinforced concrete column will play a more important role in transferring shear stress. Accordingly, the shear force distributed to the CFRP sheet will be comparatively smaller. That is why  $\alpha_{CF}$  decreases with rising  $n$ .

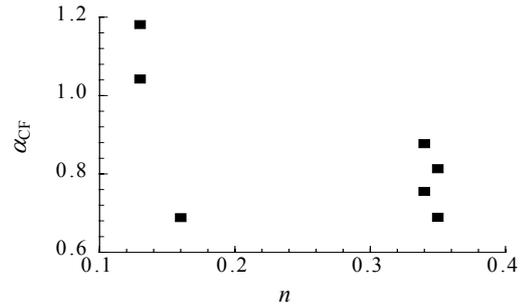


Fig.1  $n$  vs  $\alpha_{CF}$  relationship

(2) Shear span/depth ratio ( $\lambda$ )

Experiments showed that the shear span/depth ratio ( $\lambda$ ) is also an important factor influencing  $\alpha_{CF}$  and the shear capacity of the strengthened column. Fig.2 shows the relationship between  $\lambda$  and  $\alpha_{CF}$ , and that  $\alpha_{CF}$  increases with increasing  $\lambda$  and that the role of the CFRP becomes increasingly remarkable. This is because  $\lambda$  affects the length and inclination angle of

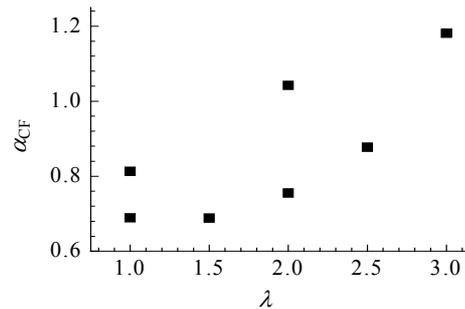


Fig.2  $\lambda$  vs  $\alpha_{CF}$  relationship

Table 2 Calculating results of  $\alpha_{CF}$

Specimen No.	$\lambda$	$n$	$\lambda'$	$V_{CFmax}$	$V_{CF}$	$\alpha_{CF}$
DZ-2	1.0	0.35	2.473	58.05	40.00	0.689
DZ-1	1.0	0.35	1.208	36.87	30.00	0.813
CS20-1-15	2.0	0.13	0.389	25.90	26.99	1.042
CS20-2-35	2.0	0.34	0.778	41.27	31.17	0.755
CS15-3-15	1.5	0.16	1.461	71.68	49.33	0.688
CS25-2-35	2.5	0.34	1.778	41.27	36.20	0.877
CS35-1-15	3.0	0.13	0.389	25.90	30.61	1.181

$\lambda$ =Shear span/depth ratio;  $n$ =Axial compression ratio;  $\lambda'$ =the volume ratio of CFRP sheet to concrete

the arch structure. The bigger the shear span/depth ratio ( $\lambda$ ) is, the longer the arch-structure will be. Accordingly, the area and the angle of the arch section will be smaller, which will decrease the shear resistance of the reinforced concrete. So that the shear force distributed to the CFRP sheet will rise as  $\lambda$  increase.

(3) Amount of the CFRP sheet

The test showed that the shear capacity of the strengthened reinforced concrete column is not in direct proportion to the amount of the CFRP sheet wrapped on the column. To some extent, the shear capacity increases with increasing amount of the CFRP. When the CFRP amount is considerable, the enhancement becomes not apparent and the shear capacity hardly rises with the increasing CFRP quantity, as shown in Fig.3.  $\alpha_{CF}$  is much bigger when  $\lambda$  is less than 0.8. Accordingly, the shear contribution of CFRP becomes larger. When  $\lambda$  is more than 0.8,  $\alpha_{CF}$  changes little and is fixed around a constant, especially when  $\lambda$  is more than 1.6. On one hand, the CFRP sheet and the reinforced concrete column make up a truss structure. When the concrete is crushed, the CFRP sheet will lose its role. Hence the shear capacity of the strengthened reinforced concrete column cannot be enhanced infinitely. On the other hand, the effectiveness of the CFRP sheet is so unsteady that the shear capacity of the CFRP sheet is not in direct proportion to the utilized CFRP quantity.

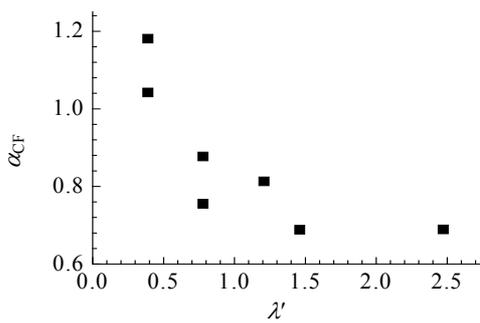


Fig.3  $\lambda$  vs  $\alpha_{CF}$  relationship

In the end, a suitable value 0.688 for  $\alpha_{CF}$  is put forward after comprehensive analysis. The reasons are listed below.

- i) 0.688 is the lower limit among all the test data;
- ii) With decreasing shear span/depth ratio,  $\alpha_{CF}$  decreases until the ratio reaches 1.5.  $\alpha_{CF}$  will gradually approach a constant 0.688. 0.688 is reliable and

safe for a column with shear span/depth ratio larger than 1.5;

iii)  $\alpha_{CF}$  will decrease with increase of axial compression ratio. Moreover  $\alpha_{CF}$  will hardly change and tend to be a constant value 0.688 gradually, when the axial compression ratio equals 0.35;

iv)  $\alpha_{CF}$  decreases with the increasing amount of CFRP, changes little actually, and tends to be the constant 0.688 little by little when  $\lambda$  becomes larger than 1.461.

SHEAR CAPACITY OF THE REINFORCED CONCRETE COLUMN WITH CFRP SHEET

From the comprehensive analysis above, the shear capacity of the reinforced concrete column strengthened with CFRP sheet can be decided by:

$$V = V_{RC} + V_{CF} \tag{7}$$

where:

$$V_{RC} = V_c + V_s + V_N = \frac{1.75}{\lambda + 1} (1 + 1.2\lambda) f_t b h_0 + f_{yv} \frac{A_{sv}}{s} h_0 + 0.07N \tag{9}$$

$$V_{CF} = \alpha_{CF} V_{CFmax} = \alpha_{CF} \beta \gamma \frac{2n_{CF} w t}{(s_{CF} + w)} f_{CF} h_0 \tag{11}$$

The meanings of the symbols are the same as mentioned previously.

Experimental data (including the results in Ye et al.(2002)) and the calculated results according to Eq.(1) are compared in Table 3. The numerical results have good convergence, agree well with the test results, satisfy practical engineering requirements and have considerably safety factor.

CONCLUSION

Based on the experiments, this paper focuses on the shear capacity of reinforced concrete columns strengthened with CFRP sheet. It can be seen from the analysis that the axial compression ratio, the shear span/depth ratio and the utilized CFRP sheet amount are main factors affecting the shear capacity of reinforced concrete columns confined by CFRP sheet. Results of experiment and theoretical analysis were

**Table 3 Comparison of numerical results and experimental data**

No.	$V_c$ (kN)	$V_s$ (kN)	$V_N$ (kN)	$V_{CF}$ (kN)	$V$ (kN)	$V_t$ (kN)	$V_t/V$
DZ-1	60.7	39.1	16.2	–	116.0	135.0	1.164
DZ-2	72.5	39.1	16.2	55.1	182.9	175.0	0.957
DZ-3	66.3	39.1	16.2	25.4	147.0	165.0	1.122
DZ-4	77.0	39.1	18.6	38.0	172.7	218.0	1.262
DZ-5	77.0	39.1	18.6	38.0	172.7	224.0	1.297
CS20-0-15	65.7	18.0	13.3	–	97.0	147.3	1.519
CS20-1-15	67.2	18.0	13.3	17.8	116.3	177.5	1.526
CS20-2-35	68.0	18.0	34.8	28.4	149.2	181.5	1.216
CS15-3-15	67.7	18.0	11.0	49.3	146.0	195.5	1.339
CS25-2-35	58.3	18.0	34.8	28.4	139.5	164.1	1.176
CS30-1-15	50.4	18.0	13.0	17.8	99.2	126.3	1.273

The average value of  $V_t/V$  is 1.259, the mean square deviation is 0.158 and the coefficient of variation is 0.125

applied to derive this paper's simple formula for calculating the shear capacity of reinforced concrete columns strengthened with CFRP sheet.

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