



Optimal design of the separate type heat pipe heat exchanger

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Abstract: Separate type heat pipe heat exchangers are often used for large-scale heat exchanging. The arrangement of such a heat exchanger conveniently allows heat input to and output from the heat exchanger at remote locations. The traditional method of designing an ordinary HPHE (heat pipe heat exchanger) is commonly applied in the separate type exchanger design, but the calculations have to be carried out separately, which makes it very complicated. In this work, the ε -NTU (effectiveness-Number of Transfer Units) method was applied for optimization analysis of single- or multi-level separate type heat pipe heat exchangers. An optimizing formula for single-level separate type heat pipe heat exchangers was obtained. The optimizing principles of effectiveness-NTU and heat transfer rate by the equal distribution method for multi-level separate type heat pipe heat exchanger are presented. The design of separate type heat pipe heat exchangers by the optimizing method is more convenient and faster than by the traditional method.

Key words: Separate type heat pipe heat exchanger, Optimal design, ε -NTU (effectiveness-Number of Transfer Units)

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INTRODUCTION

A heat pipe heat exchanger (HPHE), consisting of a bundle of individual heat pipes with vaporizing and condensing sections at the respective ends, cannot easily satisfy various large-scale needs in industrial applications. A separate type HPHE works according to the same basic principle, but it can achieve remote-heating exchange, and thereby insulate the hot and cold fluid streams perfectly from each other, and allows mixed counter flow and parallel flow arrangements (Jin and Chen, 1986; Chen, 1996; Flavio, 1991; Zhuang *et al.*, 1989). Thanks to these advantages, this kind of HPHE has received much attention since it was launched into industry at the beginning of the 1980's (Tu, 1989).

A single-level separate type HPHE consists of one individual vaporizing and condensing section at the respective ends. Whereas, multi-level separate type HPHE is made up of several vaporizing and condensing sections. The schematic diagram of sin-

gle- or multi-level separate type HPHE is shown in Fig.1 and Fig.2, respectively.

A review of the current literature shows that the traditional method of designing an ordinary HPHE is often used to design the separate type HPHE. But the final design will not be satisfied until several design programs have been compared with each other. So the calculation procedure is very complicated and inconvenient. In this study, a new optimal design method for heat pipe heat exchangers, called "effectiveness-Number of Transfer Units method" was used.

OPTIMAL DESIGN OF SINGLE-LEVEL SEPARATE TYPE HEAT PIPE HEAT EXCHANGERS

The objective of this study is to optimize the geometric design and operating parameters k_1F_1 , k_2F_2 for the HPHE, where k_1 , k_2 are the thermal conductivity of hot and cold fluids, F_1 , F_2 are the area of hot side and cold side of heat exchangers, respectively.

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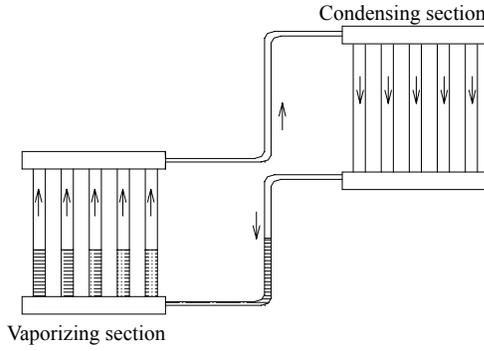


Fig.1 Single-level separate type HPHE

The effectiveness of a heat exchanger is defined as the ratio of the actual heat transfer rate to the maximum possible heat transfer rate (Yang and Tao, 1998).

$$\varepsilon = \frac{\max(t'_1 - t''_1, t'_2 - t''_2)}{(t'_1 - t'_2)} \quad (1)$$

t'_1 , t'_2 are the inlet temperature of hot and cold fluids, t''_1 , t''_2 are the outlet temperature of hot and cold fluids, respectively.

Each heat pipe is considered as being composed of a vaporizing part heated by the hot fluid and a condensing part cooled by the cold fluid, with these two parts being connected by an intermediate fluid assumed to be isothermic. Now the relation between number of transfer units, NTU , and effectiveness ε can be obtained using correlative heat transfer theory.

The number of transfer units of the hot side, NTU_1 , is:

$$NTU_1 = k_1 F_1 / G_1 c_1 \quad (2)$$

The number of transfer units of the cold side, NTU_2 is:

$$NTU_2 = k_2 F_2 / G_2 c_2 \quad (3)$$

The effectiveness of the hot side of the HPHE ε_1 is:

$$\varepsilon_1 = 1 - \exp(-NTU_1) \quad (4)$$

The effectiveness of the cold side of the HPHE ε_2 is:

$$\varepsilon_2 = 1 - \exp(-NTU_2) \quad (5)$$

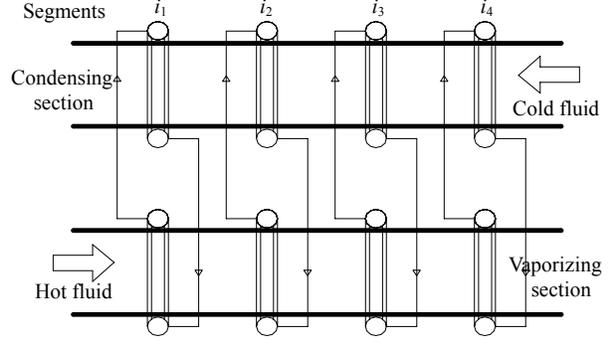


Fig.2 Multilevel separate type HPHE

The effectiveness of the HPHE ε is:

$$\varepsilon = \frac{\varepsilon_{\max} \varepsilon_{\min}}{\mu \varepsilon_{\min} + \varepsilon_{\max}} \quad (6)$$

In Eqs.(3)~(6), k is the coefficient of heat transfer ($W/(m^2 \cdot ^\circ C)$); F is the area of heat transfer (m^2); $G_1 c_1$, $G_2 c_2$ are the heat capacity rate of the hot and cold fluid, respectively ($W/^\circ C$); $\varepsilon_{\min} = \min(\varepsilon_1, \varepsilon_2)$; $\varepsilon_{\max} = \max(\varepsilon_1, \varepsilon_2)$; μ is a dimensionless parameter, $\mu = (Gc)_{\min} / (Gc)_{\max}$, where $(Gc)_{\min} = \min(G_1 c_1, G_2 c_2)$, $(Gc)_{\max} = \max(G_1 c_1, G_2 c_2)$. Therefore Eq.(6) is applicable to both counter flow and parallel flow heat exchangers.

The system would be optimised by $\min(k_1 F_1 + k_2 F_2)$ or $\min(\beta, NTU_{\min})$:

$$\begin{cases} \min : f(\beta, NTU_{\min}) = (kF)_1 + (kF)_2 \\ \quad = (1 + \beta) NTU_{\min} (Gc)_{\min} \\ \quad \varepsilon = \varepsilon_{\max} \varepsilon_{\min} / (\mu \varepsilon_{\min} + \varepsilon_{\max}) \end{cases} \quad (7)$$

The correlative parameters are defined as follows:

$$\begin{aligned} NTU_{\min} &= \min(NTU_1, NTU_2), \quad NTU_{\max} = \max(NTU_1, NTU_2) \\ (kF)_{\max} &= \max(k_1 F_1, k_2 F_2), \quad (kF)_{\min} = \min(k_1 F_1, k_2 F_2), \\ \beta &= NTU_{\max} / \mu NTU_{\min} \end{aligned} \quad (8)$$

If $\varepsilon \neq 1$, $\mu \neq 1$, the following Eq.(9) can be obtained from Eq.(7) under the condition that

$$AX_1^2 + BX_1 + C = 0 \quad (9)$$

with $X_1 = \exp(-NTU_{\min})$, $X_2 = \exp(-NTU_{\max})$. In Eq.(9),

$A=\mu(1-\mu)(\varepsilon\mu-1)$; $B=\mu(1-\mu)[\varepsilon(\varepsilon-2)(1+\mu)+2]$; $C=\mu \times (1-\mu)(1-\varepsilon)[\varepsilon(\mu+1)-1]$.

Eq.(9) can be solved to get two roots, and one of them must be put aside because it is negative, and thus

$$X_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A} = (1 - \varepsilon) \times \frac{1 - \varepsilon(\mu + 1)}{1 - \varepsilon\mu} \quad (10)$$

$$X_1 \text{ satisfies } NTU_{\min} = -\ln(X_1) > 0 \quad (11)$$

Namely $0 < X_1 < 1$, and accordingly X_1 is significant.

Fig.3 illustrates the variation range of ε and μ as X_1 satisfies the above-mentioned conditions.

Similarly,

$$X_2 = \frac{(1 - \mu)[\varepsilon(1 + \mu) - 1] + \mu(\varepsilon\mu - 1)X_1}{\varepsilon - 1} \quad (12)$$

$$NTU_{\max} = -\ln(X_2) \quad (13)$$

The minimum heat transfer rate can be calculated as

$$f = (1 + \beta)NTU_{\min}(Gc)_{\min} \quad (14)$$

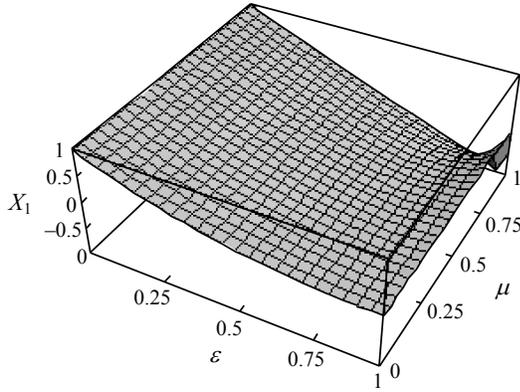


Fig.3 Relationships between X_1 and ε, μ

DISCUSSION OF OPTIMIZATION PROCEDURE

Case $\varepsilon=1$

The present result is obtained with the assumption $\varepsilon \neq 1$. For $\varepsilon=1$, extreme cases are: $X_1=0$ or $X_1=1$. While $X_1=0$, $NTU_{\min} \rightarrow \infty$. Since $(Gc)_{\min} \neq 0$, $kF \rightarrow \infty$. Such a single-level separate type HPHE does not exist. While $X_1=1$, $NTU_{\min}=0$. Since $kF \neq 0$, $(Gc)_{\min} \rightarrow \infty$, and

it is under the condition of isothermal heat exchange with phase change, thus $\varepsilon=0$, which conflicts with the supposition of $\varepsilon=1$. In conclusion, it is impossible for a single-level separate type HPHE to realize $\varepsilon=1$.

Case $\mu=0$ ($\varepsilon \neq 1$)

While $\mu=0$, X_1 can be obtained from Eq.(9), whereas both Eqs.(6) and (13) are not applicable. Since $(kF)_{\max} \neq 0$, $(Gc)_{\max} \rightarrow \infty$, the following relations arise:

$$X_1 = (1 - \varepsilon)^2 \quad (15)$$

$$\varepsilon = \frac{\beta \varepsilon_{\min} NTU_{\min}}{\varepsilon_{\min} + NTU_{\min} \beta} \quad (16)$$

Eqs.(15) and (16) can be combined with Eqs.(4) and (10) to get β .

While $\mu=1$ ($\varepsilon \neq 1$)

In Eq.(10), while $\mu=1$, then the following relation can be obtained:

$$X_1 = 1 - 2\varepsilon \quad (17)$$

Obviously: $0 < \varepsilon < 1/2$. Since $\mu=1$, $(Gc)_{\min} = (Gc)_{\max}$, which means that the temperature drop of the hot fluid is always equal to the temperature rise of the cold fluid. From Eq.(13), we can get that $\beta=1$. If $\varepsilon=1/2$, $X_1=0$, $NTU_{\min} \rightarrow \infty$. Such a system is impossible to realize, but can be approached closely.

Comparison with general heat exchangers

At the same condition of heat transfer, for counter flow general heat exchangers, the effectiveness is

$$\varepsilon = \frac{1 - \exp[-NTU(1 + \mu)]}{1 + \mu} \quad (18)$$

For parallel flow general heat exchangers, the effectiveness is

$$\varepsilon = \frac{1 - \exp[-NTU(1 - \mu)]}{1 - \mu \exp[-NTU(1 - \mu)]} \quad (19)$$

For the effectiveness of a separate type HPHE,

the fluid direction is irrelevant, which is very different from general heat exchangers whose effectiveness is greatest for counter flow. It is smallest for parallel flow. The relation between a single-level separate type HPHE's effectiveness and that of a general heat exchangers (parallel flow) is $\varepsilon < 1/(1+\mu)$, which is illustrated in Fig.3 as the intersection of X_1 and plan $\varepsilon=0$. Obviously, the effectiveness of a single-level separate type HPHE is smaller than that of counter flow general heat exchangers. This property is bad for heat-transfer, so multi-level separate type heat pipe heat exchangers are usually adopted to make their effectiveness close to that of counter flow condition.

OPTIMIZATION OF MULTI-LEVEL SEPARATE TYPE HEAT PIPE HEAT EXCHANGERS

A separate type HPHE can be separated into several segments to enhance its effectiveness. This is similar to arranging small separate type heat pipe heat exchangers in series on condition that the heat capacity rate of the hot and cold streams is the same, namely $\mu=\mu_i$. Now our purpose is to study how to confirm the effectiveness ε_i of each segment, on the premise that ε is given, to reach an optimization, which can be realized as ε_i of each segment must obviously be minimal.

$$\begin{cases} \min : \sum_{i=1}^n \varepsilon_i \\ \varepsilon = \text{const} \end{cases} \quad (20)$$

Supposing that the inlet temperature of a segment ($i+1$) is equal to the outlet temperature of the last one (i), the auxiliary effectiveness ε'_i of each segment is defined as follows

$$\varepsilon'_i = \frac{\max[(t'_{1i} - t'_{1(i+1)}), (t'_{2(i+1)} - t'_{2i})]}{\Delta t'_i} \quad (21)$$

The definition of ε'_i is different from ε in that the denominator of Eq.(21) is inlet temperature difference of hot fluid, and it satisfies the condition $\varepsilon'_i \geq 0$. The relation between ε'_i and ε_i is as follows:

1. For the counter current HPHE, as the hot fluid possesses the minimum heat capacity rate:

$$\varepsilon'_i = \varepsilon_i / (1 - \mu\varepsilon_i) \quad (22)$$

2. For the counter current HPHE, as the cold fluid possesses the minimum heat capacity rate:

$$\varepsilon'_i = \varepsilon_i / (1 - \varepsilon_i) \quad (23)$$

3. For the parallel flow HPHE:

$$\varepsilon'_i = \varepsilon_i \quad (24)$$

Eqs.(22)~(24) indicate that the auxiliary effectiveness ε'_i is a function of effectiveness ε_i . For the condition of parallel flow, they are equal.

Similarly, the total auxiliary effectiveness is defined as follows:

$$\varepsilon' = \frac{\max[(t'_1 - t'_1''), (t'_2'' - t'_2')]}{\Delta t'} \quad (25)$$

The relation between the total auxiliary effectiveness ε' and the total effectiveness ε is as follows:

1. For the counter flow HPHE, as the hot fluid possesses the minimum heat capacity rate:

$$\varepsilon' = \varepsilon / (1 - \mu\varepsilon) \quad (26)$$

2. For the counter flow HPHE, as the cold fluid possesses the minimum heat capacity rate:

$$\varepsilon' = \varepsilon / (1 - \varepsilon) \quad (27)$$

3. For the parallel flow HPHE:

$$\varepsilon' = \varepsilon \quad (28)$$

The relation between ε' and ε_i of each segment can be described as follows ($b \neq 0$):

$$\varepsilon' = \frac{\prod_{i=1}^n (1 + b\varepsilon'_i) - 1}{b} \quad (29)$$

As the hot fluid possesses the minimum heat capacity rate for the counter current HPHE, $b=\mu-1$; as the cold fluid possesses the minimum heat capacity rate, $b=1-\mu$. For the parallel flow HPHE, $b=1+\mu$.

According to the relation between ε and ε' , Eq.(20) can be rewritten as:

$$\left\{ \begin{array}{l} \min : \sum_{i=1}^n \varepsilon'_i \\ \varepsilon' = \frac{\prod_{i=1}^n (1 + b\varepsilon'_i) - 1}{b} = \text{const} \end{array} \right. \quad (30)$$

The following equation can be obtained from Eq.(30) by using the Lagrange variational method.

$$\frac{\partial \sum \varepsilon'_i}{\partial \varepsilon'_k} - \lambda \frac{\partial \varepsilon'}{\partial \varepsilon'_k} = 1 - \lambda \prod_{i=1, i \neq k}^n (1 + b\varepsilon'_i) = 0 \quad (k=1, 2, \dots, n) \quad (31)$$

with which it follows that $\varepsilon'_1 = \varepsilon'_2 = \dots = \varepsilon'_i = \dots = \varepsilon'_n$.

From Eqs.(22)~(24), we can get the following expression $\varepsilon_1 = \varepsilon_2 = \dots = \varepsilon_i = \dots = \varepsilon_n$.

The result indicates that the summation of the effectiveness ε_i of each segment can reach its minimum, as the effectiveness ε_i of each segment is equal to each other. We call this conclusion the “optimization principle of equal effectiveness distribution”. Both the effectiveness ε_c and the auxiliary effectiveness ε'_c of each segment are constant, and thus the total auxiliary effectiveness can be rewritten as follows:

$$\varepsilon' = \frac{(1 + b\varepsilon'_c)^n - 1}{b} \quad (32)$$

$$\varepsilon'_c = \frac{\sqrt[n]{1 + b\varepsilon' - 1}}{b} \quad (33)$$

After confirming the optimal auxiliary effectiveness of each segment, we can implement an optimal design of each segment with the theory of single-level separate type HPHE. According to the conclusion deduced in Section 2, that $(NTU_i)_{\max}$ and $(NTU_i)_{\min}$ is confirmed at given ε'_c and μ , the NTU of each segment should be equal, namely:

$$(NTU_i)_{\min} = (NTU_c)_{\min} = \text{const} \quad (34)$$

$$(NTU_i)_{\max} = (NTU_c)_{\max} = \text{const} \quad (35)$$

We call the set of Eqs.(34) and (35) “optimiza-

tion principle of equal NTU distribution”.

Moreover, $[(Gc)_i]_{\max} = (Gc)_{\max}$, $[(Gc)_i]_{\min} = (Gc)_{\min}$.

The following relations can be obtained:

$$[(kF)_i]_{\min} = [(kF)_c]_{\min} = \text{const} \quad (36)$$

$$[(kF)_i]_{\max} = [(kF)_c]_{\max} = \text{const} \quad (37)$$

We call the set of Eqs.(36) and (37) “optimization principle of equal heat transfer flux distribution”.

AN EXAMPLE

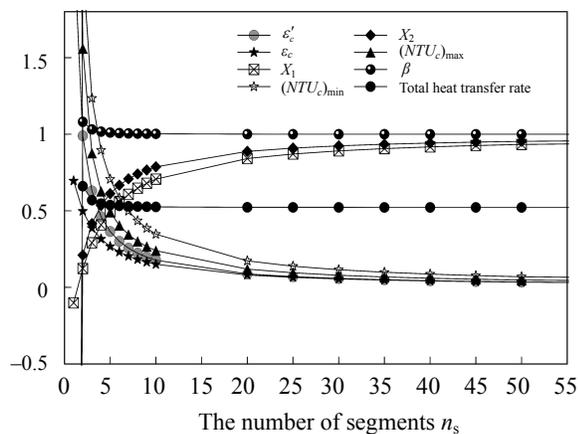
Based on the above analysis, we can conveniently implement optimal design of a separate type HPHE. Giving an example: In a steelwork, blast-furnace gas after being dealt with needs to be preheated with outlet air. Considering safety and field conditions, a separate type HPHE is adopted. Some correlative parameters are listed below:

Outlet airflow is $G_1 = 3.3 \times 10^5 \text{ Nm}^3/\text{h}$, and its inlet and outlet temperature is $t'_1 = 250 \text{ }^\circ\text{C}$, $t''_1 = 120 \text{ }^\circ\text{C}$ respectively; blast-furnace gas flow is $G_2 = 2.0 \times 10^5 \text{ Nm}^3/\text{h}$, and its inlet and outlet temperature is $t'_2 = 20 \text{ }^\circ\text{C}$, $t''_2 = 140 \text{ }^\circ\text{C}$, respectively; heat transfer rate $Q = 12.1 \text{ MW}$.

As the blast-furnace gas possesses the minimum heat capacity rate, the following parameters can be obtained from Eq.(25): $\varepsilon' = 2.286$, $\mu = 0.6875$, $(Gc)_{\min} = 75625 \text{ W}/^\circ\text{C}$, $(Gc)_{\max} = 110000 \text{ W}/^\circ\text{C}$, $\varepsilon = 0.6957$. Since $t'_1 > t''_2$, HPHE cannot be realized with single-level or multi-level parallel flow arrangement, so multi-level counter flow arrangement has to be applied, while $b = 1 - \mu = 0.3125$. The calculated optimization results are listed in Table 1. As shown in Table 1 the number of segments is 10, $\beta = 1.003 \approx 1$. And thus more segments are meaningless; they only result in more complicated structure. The relationship of the parameters listed in Table 1 as a function of number of segments n_s is presented in Fig.4. It can be concluded that the optimal solution can be obtained as long as $n_s > 1$, the total heat transfer rate approaches a constant as $n_s > 3$. In fact, usually n_s is between 5 and 10 in industrial application as an empirical result. Therefore we can select $n_s = 5$ in this example.

Table 1 Calculated optimization results for multilevel separate type HPHE

Parameters	$n=5$	$n=10$	$n \rightarrow \infty$
ε'_c	0.3643	0.1772	$\rightarrow 0$
ε_c	0.2670	0.1506	
$(X_1)_c$	0.4933	0.7067	1
X_2	0.6120	0.7872	1
$(NTU_c)_{\min}$	0.7066	0.3471	$\rightarrow 0$
$(NTU_c)_{\max}$	0.4910	0.2393	$\rightarrow 0$
β	1.011	1.003	$\rightarrow 1$
$(kF_c)_{\min}$ (W/°C)	53437	26249	
$(kF_c)_{\max}$ (W/°C)	54010	26343	

**Fig.4** Relationships of different parameters as a function of number of segments

CONCLUSION

The ε -NTU (effectiveness-Number of Transfer Units) method is applied for optimizing analysis of single-level or multi-level separate type heat pipe heat exchangers. Three optimization principles, equal effectiveness, equal NTU and equal heat-transfer rate distribution are brought forward, which provide a theoretical basis for optimal design of a separate type HPHE. A series of optimizing formulas is presented which can be used directly in projects. The optimal design method mentioned in this paper is more convenient and faster than the traditional one.

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