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On extending the concept of double negativity to acoustic waves*

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Abstract: The realization of double negative electromagnetic wave media, sometimes called left-handed materials (LHMs) or metamaterials, have drawn considerable attention in the past few years. We will examine the possibility of extending the concept to acoustic waves. We will see that acoustic metamaterials require both the effective density and bulk modulus to be simultaneously negative in the sense of an effective medium. If we can find a double negative (negative density and bulk modulus) acoustic medium, it will be an acoustic analogue of Veselago's medium in electromagnetism, and share many novel consequences such as negative refractive index and backward wave characteristics. We will give one example of such a medium.

Key words: Double negativity, Metamaterials, Acoustic
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INTRODUCTION

At this moment, novel concepts of "negative" media, having negative refraction and/or "double negativity" and their physical consequences and plausible applications are drawing considerable attention from the science and engineering community. In the past few years, considerable progress has been made in electromagnetic (EM) waves. It is our wish to generalize such concepts to acoustic waves. Since the concept of negative refraction, negative constitutive relationships and "double negativity" are more familiar in the area of electromagnetics, we will use EM waves to qualify these concepts before discussing acoustic waves.

Using EM waves as the example, we know that electric permittivity (ε) and magnetic permeability (μ) describe the response of a medium to external EM fields and collectively govern the propagation of EM waves. In particular, the refractive index (n) is given by $n=\sqrt{\varepsilon\mu}$. If either ε or μ is negative, then n becomes imaginary and the wave cannot propagate. If however, both ε and μ are simultaneously negative (double negativity) (Veselago, 1968), then waves can propagate through the media, but with a negative effective refractive index and hence the phenomenon of negative refraction. Many amazing effects, such as Doppler shifts with reversed signs, backward Cherenkov radiation and superlensing effects (Pendry, 2000) are consequences of double negativity. These "double-negative" media are characterized by the phenomenon that the Poynting vector and the wavevector are in opposite directions ($\boldsymbol{S}\cdot\boldsymbol{k}<0$). For EM waves, negative ε can be found in naturally occurring materials, but negative μ has to be made artificially. The realization of negative effective μ using "split ring" type resonators (Pendry *et al*., 1999) leads to realization of "double negativity" in EM wave experimentally (Shelby *et al*., 2001), with those materials being frequently called "metamaterials". This is currently a very active field of research. For the particular case of EM waves, these "Veselago" media are sometimes called left-handed media, but the term "double negative" medium is more informative.

If we want to extend the concept to acoustic waves, we need to examine the corresponding wave

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equations. In acoustic waves, the continuity condition and Newton's 2nd law (with harmonic field dependence $e^{-i\omega t}$) can be expressed respectively as

$$\nabla\cdot\boldsymbol{v}-\frac{i\omega}{\kappa}p=0,\qquad(1)$$
$$\nabla p-i\omega\rho\boldsymbol{v}=0,$$

where p is the pressure field and $\boldsymbol{v}$ is the velocity field. The density, ρ, and the modulus, κ, are position dependent in general. By considering a plane wave solution with wave vector $\boldsymbol{k}$ inside a homogeneous medium, the refractive index n should be defined by

$$k=|n|\omega/c,\qquad(2)$$

where $n^2=\rho/\kappa$. Therefore, in order to have propagating plane waves inside the medium, we should have either both positive ρ and κ or both negative ρ and κ. Moreover, the Poynting vector for a propagating plane wave is given by

$$\boldsymbol{S}=\frac{i}{2\omega\rho}p\nabla p^*=\frac{|p|^2\boldsymbol{k}}{2\omega\rho}.\qquad(3)$$

Comparing with the case of EM wave in which $n^2=\varepsilon\mu$ and $\boldsymbol{S}=|\boldsymbol{E}|^2\boldsymbol{k}/(2\omega\mu)$, a "negative" medium in acoustic waves will require the density and modulus to be negative at the same time. We note in particular that negative effective ρ means that $\boldsymbol{S}$ and $\boldsymbol{k}$ should point in opposite directions. And simultaneous negativity in bulk modulus and density ensures the existence of propagating waves. Although the analogy between acoustic and EM waves can be made, the case of "negative" acoustic wave is in fact more challenging than EM waves. For EM waves, at least negative ε can be found in nature. It is negative μ that has to be made artificially. For acoustic waves, neither negative ρ nor κ can be found in naturally occurring materials. They have to be derived from artificial resonances. Physically, this means that the medium displays anomalous response at some frequencies such that it expands upon compression (negative modulus) and moves to the left when being pushed to the right (negative density). It sounds very difficult, but we will see that it is possible with the illustration of a very simple model.

RESULTS

A simple model

Let us consider a 1D spring-mass model that is used frequently in solid state textbooks. This system is illustrated schematically in Fig.1.

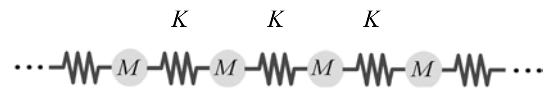

Fig.1 Schematic structure of the 1D spring-mass model

The solution of the eigenfrequency of the simple ball-and-spring model is given by $\omega^2=4KM^{-1}\sin^2(ka/2)$. This can be regarded as a dispersion relation, analogous to the EM wave case given by $k^2=(\varepsilon\mu)(\varepsilon_0\mu_0)\omega^2$. In order to have "double negativity", M and K need to be negative. Of course for a normal system, we expect both M and K to be positive. How can we make them negative? We can use resonance structures. For example, let us go to a more complex structure, as illustrated in Fig.2, which contains locally resonant building blocks. A core with mass m is connected internally, through two springs with spring constant G, to a shell with mass M. We call this shell-core pair a resonant unit. The resonant units are connected together by spring with spring constant K. The dispersion relation for such a chain of resonant units has the analytical form $\omega^2=4KM_{\text{eff}}^{-1}\sin^2(ka/2)$, which is formally equal to the simple chain except that the mass M is now replaced by the effective mass $M_{\text{eff}}=M+m\omega_0^2/(\omega_0^2-\omega^2)$, where $\omega_0=\sqrt{2G/m}$. Near the resonance frequency ω_0, the effective mass becomes negative, meaning that the response is out of phase with the input force. So, we see that resonance structure can give rise to negative constitutive parameters.

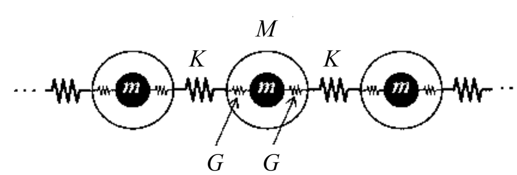

Fig.2 Schematic structure of the 1D spring-mass model with internal structure resonating at $\omega_0=\sqrt{2G/m}$

If we go back to the more complex case of "negative" acoustic materials, we need to show that negative effective density and negative effective modulus are "allowable", at least mathematically, and we also need to give explicit configurations and perform calculations to show that they indeed have the "negative" backward wave properties. This is not an easy task. Natural materials have neither a negative density nor a negative bulk modulus. Even for composite materials, the effective bulk modulus and density are normally bounded by the Hashin and Shtrikman bounds (Hashin and Shtrikman, 1962). Therefore, we still expect positive bulk modulus and density. For instance, let us examine the prototypical case of spherical particles dispersed in a fluid. The filling ratio is given by f. In the long wavelength limit, the effective bulk modulus, κ_{eff}, and effective density, ρ_{eff}, in the limit of small filling ratio (Berryman, 1980) are governed by

$$\begin{cases} \dfrac{1}{\kappa_{\text{eff}}} = \dfrac{f}{\kappa_{\text{s}}} + \dfrac{1-f}{\kappa_0}, \\ \dfrac{\rho_{\text{eff}} - \rho_0}{2\rho_{\text{eff}} + \rho_0} = f\,\dfrac{\rho_{\text{s}} - \rho_0}{2\rho_{\text{s}} + \rho_0}, \end{cases} \tag{4}$$

where the subscripts "s" and "0" denote respectively the properties for the sphere and the background fluid. It can be shown from the formulae that κ_{eff} and ρ_{eff} are positive definitely for natural materials. However, the above effective medium formulae and the traditional bounds on the effective parameters do not apply if there are low frequency resonances. As our example in the ball-and-spring model suggests, we need to employ resonances to achieve negative effective parameters. Standard homogenizations assume that the wavelengths in each local region are all much larger than the average distance between particles. At or near the resonance frequency, the wavelength within the sphere is now comparable to the size of it although the wavelength in the background material remains much larger than the average distance between particles in order to have valid effective medium description. Under such a condition, the standard effective medium theories must be extended. After some mathematics, we find that

$$\begin{cases} -1+\dfrac{\kappa_0}{\kappa_{\text{eff}}} = \dfrac{3f}{\mathrm{i}(k_0 r)^3}\dfrac{D_0}{1+D_0}, \\ \dfrac{\rho_{\text{eff}} - \rho_0}{2\rho_{\text{eff}} + \rho_0} = \dfrac{3f}{\mathrm{i}(k_0 r)^3}\dfrac{D_1}{1+D_1}, \end{cases} \tag{5}$$

where D_l (l=0, 1) is the scattering coefficient of angular momentum, l, k_0 is the wave number in the background fluid, and r is the radius of the particle. When there are resonances, D_0 and D_1 can have large magnitudes and negative signs. In that case, κ_{eff} and ρ_{eff} can be negative. We thus see that it is possible, at least in the mathematical sense, to achieve negative effective bulk modulus and negative effective density through resonance behavior in D_0 and D_1, which are functions of frequency.

So, within the context of effective medium theories, negative κ_{eff} and ρ_{eff} are mathematically "allowed" at finite but low frequencies. The question then is to find physical systems (with explicit configurations) to realize them. One possibility is to create strong Mie-type resonances. That can in principle be achieved by finding two components that have very different sound speeds.

Acoustic waves

From now on, we will give one example of an acoustic double negativity material, realized as a composite of soft rubber spheres suspending in water. We choose to use soft rubber where sound waves travel much slower in it than in water. Then, the Mie resonances (monopolar and dipolar) can be brought to very low frequency due to the high contrast of sound speed between rubber and water. Let us consider with a system of rubber spherical particles suspended in water of a volume filling ratio of 0.1 where Eq.(5) is reasonably accurate. We have ignored the shear wave within the rubber spheres due to the high velocity contrast (Kafeski and Economou, 1999) between the rubber and water for simplicity, and we emphasize the main features stay the same if we also include the shear wave within the particles[1]. The spheres are assumed to be made of a kind of silicone rubber (Liu *et al*., 2000). The effective medium result using the generalized effective medium formulae is shown in Fig.3. From Fig.3, the effective bulk modulus and

[1] If shear is included, we find additional sharp resonances but the main resonances giving rise to negative ρ and κ are essentially the same

density near the static limit are positive as predicted by Eq.(4). The monopolar resonance creates a negative bulk modulus above the normalized frequency at about 0.035 while the dipolar resonance creates a negative density above the normalized frequency at about 0.04. Here, a is the lattice constant if the spheres are arranged in an FCC lattice. Hence, there is a narrow frequency range where we have both negative bulk modulus and density. The imaginary part of the effective parameters is due to the diffusive scattering loss.

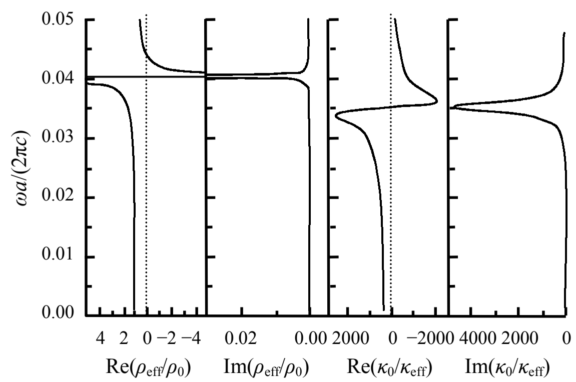

Fig.3 Effective density and bulk modulus (using Eq.(5)) for rubber (ρ=1300 kg/m^3, κ=6.27×10^5 Pa) spheres of filling ratio 0.1 within water (ρ=1000 kg/m^3, κ=2.15×10^9 Pa)

DISCUSSION

The Mie resonances at low frequency in acoustics are the analogues of the resonances created by split-rings and wires in electromagnetic left-handed medium. In EM left-handed medium, the wires and split-rings create negative electric dipolar and magnetic dipolar response by two different mechanisms while in our case a single structure gives rise to two kinds of resonances to achieve double negativity. The monopolar resonance creates a negative response such that the volume dilation of a single particle is out of phase with a hydrostatic pressure field. The dipolar resonance creates a negative response such that the motion of the centre of mass of the particle is out of phase with an incident directional pressure field. If these negative responses are large enough to compensate for the background fluid, we can have both negative effective bulk modulus and negative effective density. So, we have one example in which the effective density and modulus are negative at the same time and thus achieving double negativity.

Last but not the least, we would like to emphasize that the double negative acoustic medium is not the same as a "phononic crystal". To distinguish the two, it helps first to distinguish between a photonic crystal and a double negative EM wave medium ("Veselago"-type medium), and the comparison is given in Table 1. We are aiming here at a "Veselago"-type medium, which derives the negative refraction from "double negativity" (negative modulus and effective density). "Double negativities" are typically resonance based and are rather different from negative refraction observed in phononic crystals (Yang *et al*., 2004; Zhang and Liu, 2004), in which the mechanism is derived from Bragg scattering. A "double negativity" in constitutive relationships will lead to negative refraction, with "double negativity" referring to negative constitutive parameters. For an inhomogeneous system, the wavelength must be long compared with the embedded inhomogeneity before we can meaningfully employ effective constitutive parameters (after proper homogenization) to describe the response of the medium to incident waves. The "double-negative" medium should be viewed within the context of an effective

Table 1 The table compares photonic crystal with a double negative electromagnetic medium. The concept of photonic crystal has an analogue in acoustic waves in the form of phononic crystals, which are crystals with periodic variation of elastic constants/density. This article is aimed at examining the analogue of Veselago medium in acoustic waves, which is expected to be a system containing sub-wavelength resonators that permits a long-wavelength description with double negative constitutive relationships

Photonic crystal	Metamaterial
$\lambda \approx a$	$\lambda >> a$
Negative group velocity originating from band folding, can get negative refraction (n<0, although n may not even be well defined)	Double negative constitutive parameters (ε, μ<0) Double negativity implies negative refraction, subwavelength imaging…
Bragg scattering	Resonance
Band structure description	Effective medium description

medium. However, if we are interested in the phenomenon of negative refraction only, it can also be derived from other mechanisms. One example is that we have a highly anisotropic medium and that the principal axis is "misaligned" with another medium. We can observe negative refraction for some angles (but not all angles) in the interface, but many novel phenomena (such as the superlens effect) cannot occur. All-angle negative refraction can also be achieved by using band structure effects (arising from Bragg scattering) in periodic structures. In photonic (Luo *et al*., 2002) and phononic (Zhang and Liu, 2004) band gap systems, these have been demonstrated. While an "effective parameter" description is not necessarily meaningful in those cases, some novel effects, such as point source imaging with near-field subwavelength resolution, are also possible.

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