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Analytical solution for electromagnetic scattering from a sphere of uniaxial left-handed material*

GENG You-lin (耿友林)[1,2], HE Sailing (何赛灵)[3,4]

([1]Institute of Antenna and Microwaves, Hangzhou Dianzi University, Hangzhou 310018, China)
([2]School of Electrical Engineering, Xidian University, Xi'an 710071, China)
([3]Centre for Optical and Electromagnetic Research, Center of Electromagnetic Research, Zhejiang University, Hangzhou 310027, China)
([4]Division of Electromagnetic Theory, Alfven Laboratory, Royal Institute of Technology, Stockholm S-100 44, Sweden)
E-mail: gengyoulin@yahoo.com.cn; Sailing@zju.edu.cn

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Abstract: Based on the analytical solution of electromagnetic scattering by a uniaxial anisotropic sphere in the spectral domain, an analytical solution to the electromagnetic scattering by a uniaxial left-handed materials (LHMs) sphere is obtained in terms of spherical vector wave functions in a uniaxial anisotropic LHM medium. The expression of the analytical solution contains only some one-dimensional integral which can be calculated easily. Numerical results show that Mie series of plane wave scattering by an isotropic LHM sphere is a special case of the present method. Some numerical results of electromagnetic scattering of a uniaxial anisotropic sphere by a plane wave are given.

Key words: Electromagnetic scattering, Uniaxial anisotropic LHM, Spherical vector wave functions, Radar cross section (RCS)
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INTRODUCTION

The concept of left-handed materials (LHM for short, also called negative index materials) was introduced by Veselago (1968). An LHM has both negative permittivity and negative permeability, which makes the wave vector, the electric-field vector, and the magnetic-field vector in the LHM obey the left-hand rule. Experimental verifications for the existence of an LHM were made and its use as a perfect lens was suggested (Pendry, 2000; Shelby *et al*., 2001a; 2001b; Smith *et al*., 2000; Smith and Kroll, 2000). In light of their abnormal phenomena and potential applications, isotropic media with negative refractive index have received much attention recently. Electromagnetic scattering by an object of LHM has behavior different from that of an object of ordinary right-handed material (RHM), which has both positive permittivity and positive permeability.

One of the basic problems of studying waves in LHMs is to characterize the scattering properties of an LHM object. The electromagnetic scattered fields can be calculated using traditional Mie series solution for a dielectric and magnetic sphere (Stratton, 1941), or a frequency-domain method of moments (MoM) (Medgyesi-Mitschang *et al*., 1994), and numerical results of electromagnetic scattering by an isotropic LHM sphere can be found in (Monzon *et al*., 2004). The popular LHM (Shelby *et al*., 2001a), a combination of a lattice of infinitely long parallel wires and a lattice of the so-called split-ring resonators (SRRs), is not isotropic (the lattice of infinitely long parallel wires gives an inherently uniaxial anisotropy). In the present paper, we give the analytical solution of electromagnetic fields in a uniaxial LHM sphere in terms of spherical vector wave functions, and then

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calculate the radar cross section of a uniaxial LHM sphere for a plane wave. Numerical results are presented to give more physical insight into this problem.

In the subsequent analysis, a time dependence of the form exp(−jωt) is assumed for all the electromagnetic fields but is suppressed throughout the treatment.

FORMULATION OF THE SCATTERING PROBLEM

Assume that a homogeneous, uniaxial LHM sphere of radius a is center-located in the free space in a spherical coordinate system (Fig.1). The permittivity and permeability tensors are characterized by the following two matrices (Lindell *et al*., 2001):

$$\bar{\boldsymbol{\varepsilon}} = \varepsilon_t(\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\hat{\boldsymbol{y}}) + \varepsilon_z\hat{\boldsymbol{z}}\hat{\boldsymbol{z}} = \begin{bmatrix} \varepsilon_t & 0 & 0 \\ 0 & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \quad (1a)$$

$$\bar{\boldsymbol{\mu}} = \mu_t(\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\hat{\boldsymbol{y}}) + \mu_z\hat{\boldsymbol{z}}\hat{\boldsymbol{z}} = \begin{bmatrix} \mu_t & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_z \end{bmatrix}. \quad (1b)$$

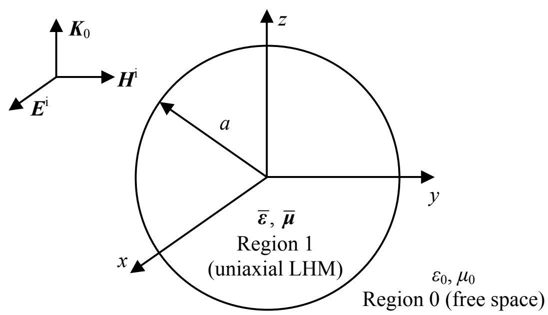

Fig.1 The geometry of scattering from a sphere of a uniaxial anisotropic LHM

In this paper, the real parts of the parameters ε_t, ε_z, μ_t and μ_z are all negative. With Fourier transform (Geng *et al*., 2004; Ren, 1993) and the plane wave expansion in terms of spherical vector wave functions in an isotropic RHM (Sarkar and Halas, 1997), the electromagnetic fields in a uniaxial LHM can be expressed as follows (Geng *et al*., 2004):

$$\boldsymbol{E}_1 = \sum_{q=1}^{2}\sum_{mn}\sum_{n'} G_{mn'q}\int_0^{\pi}[A^{\mathrm{e}}_{mnq}\boldsymbol{M}^{(1)}_{mn}(\boldsymbol{r},k_q) + B^{\mathrm{e}}_{mnq}\boldsymbol{N}^{(1)}_{mn}(\boldsymbol{r},k_q) + C^{\mathrm{e}}_{mnq}\boldsymbol{L}^{(1)}_{mn}(\boldsymbol{r},k_q)] \cdot P^m_{n'}(\cos\theta_{\boldsymbol{k}})k_q^2\sin\theta_{\boldsymbol{k}}\mathrm{d}\theta_{\boldsymbol{k}}, \quad (2a)$$

$$\boldsymbol{H}_1 = \sum_{q=1}^{2}\sum_{mn}\sum_{n'} G_{mn'q}\int_0^{\pi}[A^{\mathrm{h}}_{mnq}\boldsymbol{M}^{(1)}_{mn}(\boldsymbol{r},k_q) + B^{\mathrm{h}}_{mnq}\boldsymbol{N}^{(1)}_{mn}(\boldsymbol{r},k_q) + C^{\mathrm{h}}_{mnq}\boldsymbol{L}^{(1)}_{mn}(\boldsymbol{r},k_q)] \cdot P^m_{n'}(\cos\theta_{\boldsymbol{k}})k_q^2\sin\theta_{\boldsymbol{k}}\mathrm{d}\theta_{\boldsymbol{k}}. \quad (2b)$$

In the above two expressions, $\boldsymbol{L}^{(l)}_{mn}(\boldsymbol{r},k_q)$, $\boldsymbol{M}^{(l)}_{mn}(\boldsymbol{r},k_q)$ and $\boldsymbol{N}^{(l)}_{mn}(\boldsymbol{r},k_q)$ (here l=1) are spherical vector wave functions of the lth kind (Geng *et al*., 2004; Ren, 1993; Sarkar and Halas, 1997), n and n' are from 0 to ∞ and m is from $-n$ to n, $\boldsymbol{r}$ points to the (θ, φ)-direction and wave vector $\boldsymbol{k}_q$ (q=1,2) points to the ($\theta_{\boldsymbol{k}}$, $\varphi_{\boldsymbol{k}}$)-direction in the spherical coordinates. The expansion coefficients $G_{mn'q}$ are unknown but can be determined by the boundary condition, and A^t_{mnq}, B^t_{mnq} and C^t_{mnq} (t=e, h) are given in (Geng *et al*., 2004)

(1) When q=1 and m≥0,

$$A^{\mathrm{e}}_{mnq} = \mathrm{j}^n\frac{2n+1}{2n(n+1)}\frac{(n-m)!}{(n+m)!}[(n+m)(n-m+1) \times P^{m-1}_n(\cos\theta_{\boldsymbol{k}}) - P^{m+1}_n(\cos\theta_{\boldsymbol{k}})], \quad (3a)$$

$$B^{\mathrm{e}}_{mnq} = \mathrm{j}^n\frac{1}{2n(n+1)}\frac{(n-m)!}{(n+m)!}[(n+1)(n+m) \times (n+m-1)P^{m-1}_{n-1}(\cos\theta_{\boldsymbol{k}}) + (n+1) \times P^{m+1}_{n-1}(\cos\theta_{\boldsymbol{k}}) + n(n-m+2)(n-m+1) \times P^{m-1}_{n+1}(\cos\theta_{\boldsymbol{k}}) + nP^{m+1}_{n+1}(\cos\theta_{\boldsymbol{k}})], \quad (3b)$$

$$C^{\mathrm{e}}_{mnq} = \mathrm{j}^n\frac{1}{2k_q}\frac{(n-m)!}{(n+m)!}[(n+m)(n+m-1) \times P^{m-1}_{n-1}(\cos\theta_{\boldsymbol{k}}) + P^{m+1}_{n-1}(\cos\theta_{\boldsymbol{k}}) - (n-m+2)(n-m+1)P^{m-1}_{n+1}(\cos\theta_{\boldsymbol{k}}) - P^{m+1}_{n+1}(\cos\theta_{\boldsymbol{k}})], \quad (3c)$$

$$A^{\mathrm{h}}_{mnq} = \mathrm{j}^{n+1}\frac{k_q\sin\theta_{\boldsymbol{k}}}{\omega\mu_t}\frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\left\{(W^{(\mathrm{h})}_q(\theta_{\boldsymbol{k}})/2) \times[(n+m)(n-m+1)P^{m-1}_n(\cos\theta_{\boldsymbol{k}}) + P^{m+1}_n(\cos\theta_{\boldsymbol{k}})] + \mu m P^m_n(\cos\theta_{\boldsymbol{k}})\right\}, \quad (3d)$$

$$B_{mnq}^{\mathrm{h}} = \mathrm{j}^{n+1}\frac{k_q \sin\theta_{\boldsymbol{k}}}{\omega\mu_t}\frac{1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\Big\{(W_q^{(\mathrm{h})}(\theta_{\boldsymbol{k}})/2) \times[(n+1)(n+m)(n+m-1)P_{n-1}^{m-1}(\cos\theta_{\boldsymbol{k}}) -(n+1)P_{n-1}^{m+1}(\cos\theta_{\boldsymbol{k}})+n(n-m+2) \times(n-m+1)P_{n+1}^{m-1}(\cos\theta_{\boldsymbol{k}})-nP_{n+1}^{m+1}(\cos\theta_{\boldsymbol{k}})] +\mu[n(n-m+1)P_{n+1}^{m}(\cos\theta_{\boldsymbol{k}})-(n+1) \times(n+m)P_{n-1}^{m}(\cos\theta_{\boldsymbol{k}})]\Big\}, \tag{3e}$$

$$C_{mnq}^{\mathrm{h}} = \mathrm{j}^{n+1}\frac{\sin\theta_{\boldsymbol{k}}}{\omega\mu_t}\frac{(n-m)!}{(n+m)!}\Big\{(W_q^{(\mathrm{h})}(\theta_{\boldsymbol{k}})/2)\times[(n+m) \times(n+m-1)P_{n-1}^{m-1}(\cos\theta_{\boldsymbol{k}})-P_{n-1}^{m+1}(\cos\theta_{\boldsymbol{k}}) -(n-m+2)(n-m+1)P_{n+1}^{m-1}(\cos\theta_{\boldsymbol{k}}) +P_{n+1}^{m+1}(\cos\theta_{\boldsymbol{k}})]-(2n+1)\mu\cos\theta_{\boldsymbol{k}}P_n^m(\cos\theta_{\boldsymbol{k}})\Big\}, \tag{3f}$$

where $m>0$,

$$A_{-mnq}^{\mathrm{e}} = (-1)^m\frac{(n+m)!}{(n-m)!}A_{mnq}^{\mathrm{e}}, \tag{4a}$$

$$B_{-mnq}^{\mathrm{e}} = (-1)^{m+1}\frac{(n+m)!}{(n-m)!}B_{mnq}^{\mathrm{e}}, \tag{4b}$$

$$C_{-mnq}^{\mathrm{e}} = (-1)^{m+1}\frac{(n+m)!}{(n-m)!}C_{mnq}^{\mathrm{e}}, \tag{4c}$$

$$A_{-mnq}^{\mathrm{h}} = (-1)^{m+1}\frac{(n+m)!}{(n-m)!}A_{mnq}^{\mathrm{h}}, \tag{4d}$$

$$B_{-mnq}^{\mathrm{h}} = (-1)^{m}\frac{(n+m)!}{(n-m)!}B_{mnq}^{\mathrm{h}}, \tag{4e}$$

$$C_{-mnq}^{\mathrm{h}} = (-1)^{m}\frac{(n+m)!}{(n-m)!}C_{mnq}^{\mathrm{h}}. \tag{4f}$$

(2) When q=2 and $m\geq0$,

$$A_{mnq}^{\mathrm{e}} = \mathrm{j}^{n+1}\frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\Big\{(W_q^{(\mathrm{e})}(\theta_{\boldsymbol{k}})/2) \times[(n+m)(n-m+1)P_n^{m-1}(\cos\theta_{\boldsymbol{k}}) +P_n^{m+1}(\cos\theta_{\boldsymbol{k}})]+mP_n^m(\cos\theta_{\boldsymbol{k}})\Big\}, \tag{5a}$$

$$B_{mnq}^{\mathrm{e}} = \mathrm{j}^{n+1}\frac{1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\Big\{(W_q^{(\mathrm{e})}(\theta_{\boldsymbol{k}})/2) \times[(n+1)(n+m)(n+m-1)P_{n-1}^{m-1}(\cos\theta_{\boldsymbol{k}}) -(n+1)\times P_{n-1}^{m+1}(\cos\theta_{\boldsymbol{k}})+n(n-m+2) \times(n-m+1)P_{n+1}^{m-1}(\cos\theta_{\boldsymbol{k}})-nP_{n+1}^{m+1}(\cos\theta_{\boldsymbol{k}})] +[n(n-m+1)P_{n+1}^{m}(\cos\theta_{\boldsymbol{k}})-(n+1) \times(n+m)P_{n-1}^{m}(\cos\theta_{\boldsymbol{k}})]\Big\}, \tag{5b}$$

$$C_{mnq}^{\mathrm{e}} = \mathrm{j}^{n+1}\frac{1}{k_q}\frac{(n-m)!}{(n+m)!}\Big\{(W_q^{(\mathrm{e})}(\theta_{\boldsymbol{k}})/2)\times[(n+m) \times(n+m-1)P_{n-1}^{m-1}(\cos\theta_{\boldsymbol{k}})-P_{n-1}^{m+1}(\cos\theta_{\boldsymbol{k}}) -(n-m+2)(n-m+1)\times P_{n+1}^{m-1}(\cos\theta_{\boldsymbol{k}}) +P_{n+1}^{m+1}(\cos\theta_{\boldsymbol{k}})]-(2n+1)\times\cos\theta_{\boldsymbol{k}}P_n^m(\cos\theta_{\boldsymbol{k}})\Big\}, \tag{5c}$$

$$A_{mnq}^{\mathrm{h}} = \mathrm{j}^{n}\frac{2n+1}{2n(n+1)}\frac{(n-m)!}{(n+m)!}[(n+m)(n-m+1) \times P_n^{m-1}(\cos\theta_{\boldsymbol{k}})-P_n^{m+1}(\cos\theta_{\boldsymbol{k}})]\times W_q^{(\mathrm{h})}(\theta_{\boldsymbol{k}}), \tag{5d}$$

$$B_{mnq}^{\mathrm{h}} = \mathrm{j}^{n}\frac{1}{2n(n+1)}\frac{(n-m)!}{(n+m)!}[(n+1)(n+m)(n+m-1) \times P_{n-1}^{m-1}(\cos\theta_{\boldsymbol{k}})+(n+1)P_{n-1}^{m+1}(\cos\theta_{\boldsymbol{k}}) +n(n-m+2)(n-m+1)P_{n+1}^{m-1}(\cos\theta_{\boldsymbol{k}}) +nP_{n+1}^{m+1}(\cos\theta_{\boldsymbol{k}})]\times W_q^{(\mathrm{h})}(\theta_{\boldsymbol{k}}), \tag{5e}$$

$$C_{mnq}^{\mathrm{h}} = \mathrm{j}^{n}\frac{1}{2k_q}\frac{(n-m)!}{(n+m)!}[(n+m)(n+m-1) \times P_{n-1}^{m-1}(\cos\theta_{\boldsymbol{k}})+P_{n-1}^{m+1}(\cos\theta_{\boldsymbol{k}}) -(n-m+2)(n-m+1)P_{n+1}^{m-1}(\cos\theta_{\boldsymbol{k}}) -P_{n+1}^{m+1}(\cos\theta_{\boldsymbol{k}})]\times W_q^{(\mathrm{h})}(\theta_{\boldsymbol{k}}), \tag{5f}$$

where $m>0$,

$$A_{-mnq}^{\mathrm{e}} = (-1)^{m+1}\frac{(n+m)!}{(n-m)!}A_{mnq}^{\mathrm{e}}, \tag{6a}$$

$$B_{-mnq}^{\mathrm{e}} = (-1)^{m}\frac{(n+m)!}{(n-m)!}B_{mnq}^{\mathrm{e}}, \tag{6b}$$

$$C_{-mnq}^{\mathrm{e}} = (-1)^{m}\frac{(n+m)!}{(n-m)!}C_{mnq}^{\mathrm{e}}, \tag{6c}$$

$$A_{-mnq}^{\mathrm{h}} = (-1)^{m}\frac{(n+m)!}{(n-m)!}A_{mnq}^{\mathrm{h}}, \tag{6d}$$

$$B_{-mnq}^{\mathrm{e}} = (-1)^{m+1}\frac{(n+m)!}{(n-m)!}B_{mnq}^{\mathrm{h}}, \tag{6e}$$

$$C_{-mnq}^{\mathrm{h}} = (-1)^{m+1}\frac{(n+m)!}{(n-m)!}C_{mnq}^{\mathrm{h}}. \tag{6f}$$

From Eqs.(3a) to (6f), one obtains the following expressions for coefficients $W_q^{(t)}(\theta_{\boldsymbol{k}})$ (t=e, h):

$$W_q^{(\mathrm{e})}(\theta_{\boldsymbol{k}}) = \frac{k_q^2\sin\theta_{\boldsymbol{k}}\cos\theta_{\boldsymbol{k}}}{k_q^2\cos^2\theta_{\boldsymbol{k}}-a_1},\ q=2, \tag{7a}$$

$$W_q^{(h)} = \begin{cases} -\dfrac{\cos\theta_{\boldsymbol{k}}}{\sin\theta_{\boldsymbol{k}}}, \ q=1, \\ \dfrac{k_q^2}{\omega\mu_t}[W_q^{(e)}(\theta_{\boldsymbol{k}})\cos\theta_{\boldsymbol{k}} - \sin\theta_{\boldsymbol{k}}], \ q=2, \end{cases} \quad (7b)$$

where

$$k_1^2 = \frac{a_1}{\mu\sin^2\theta_{\boldsymbol{k}} + \cos^2\theta_{\boldsymbol{k}}}, \quad (8a)$$

$$k_2^2 = \frac{a_1 a_2}{a_1\sin^2\theta_{\boldsymbol{k}} + a_2\cos^2\theta_{\boldsymbol{k}}} \quad (8b)$$

and

$$a_1=\omega^2\mu_t\varepsilon_t, \quad (9a)$$

$$a_2=\omega^2\mu_t\varepsilon_z, \quad (9b)$$

$$\mu=\mu_t/\mu_z. \quad (9c)$$

Note that the real part of k_q (q=1, 2) must be negative in LHM (Cui *et al*., 2004; Pendry, 2000; Shelby, *et al*., 2001a; 2001b; Smith, *et al*., 2000; Smith and Kroll, 2000; Veselago, 1968; Ziolkowski and Engheta, 2003).

The expansion forms of the incident plane wave and scattered fields in terms of spherical vector wave functions in the isotropic medium (free space) can be found in (Geng *et al*., 2004; Sarkar and Halas, 1997; Wu and Wang, 1991). Applying boundary conditions at the interface between the LHM sphere and free space, the coefficients of the scattered fields can be derived and then the radar cross section (RCS) of a uniaxial LHM sphere by a plane wave can be obtained, their detailed formulae can be found in (Geng *et al*., 2004).

NUMERICAL RESULTS AND DISCUSSION

The numerical results for electromagnetic scattering of a plane wave by an LHM sphere are presented in this section. The incident field is a plane wave with unity amplitude for the electric field polarized along x-direction, and propagates in the negative z-direction. The elements of the permeability tensor are chosen as $\mu_t=\mu_z=-\mu_0$ except in Fig.2.

To check the present formulae and the Fortran code developed in this paper, we compared the calculated bistatic RCS, and compared bistatic RCS in ***E***-plane (xoz-plane of Fig.1) or ***H***-plane (yoz-plane of Fig.1) with data in (Monzon *et al*., 2004). The series in Eqs.(2a) and (2b) converge rapidly, and it is sufficient to take N=16 as the upper limit of the summation indices n and n'–as is known, the convergence rate or the upper limit of the summation depends on the electrical dimension of the sphere (with respect to the wavelength) and parameter of uniaxial LHM. In Fig.2, the RCS in ***E***- or ***H***-plane calculated with the present formulations are compared with those in (Monzon *et al*., 2004) for a special case of isotropic LHM sphere with $\varepsilon_t/\varepsilon_0=\varepsilon_z/\varepsilon_0=\mu_t/\mu_0=\mu_z/\mu_0=-1+0.001$j (the radius of the LHM sphere is 10 cm and the frequency of the incident wave is 6 GHz). Excellent agreement was found between the RCS results of the two methods. Fig.2 shows that Mie series of plane wave scattering by an isotropic LHM sphere is a special case of the present method.

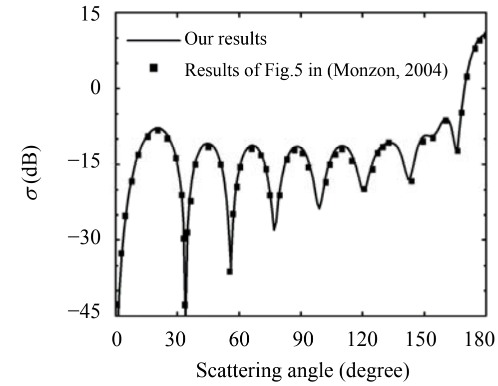

Fig.2 Radar cross sections (RCSs) versus scattering angle θ (in degrees): results of this paper (solid curve) and of Fig.5 in (Monzon, 2004). The radius of LHM sphere is a=10 cm, the frequency is f=6 GHz, and the permittivity tensor elements are assumed to be $\varepsilon_t=\varepsilon_z=(-1+0.001\text{j})\varepsilon_0$, and $\mu_t=\mu_z=(-1+0.001\text{j})\mu_0$

The RCSs for a lossless uniaxial LHM sphere are shown in Fig.3, where the permittivity tensor elements are assumed to be $\varepsilon_t/\varepsilon_0=-1$ and $\varepsilon_z/\varepsilon_0=-2$. The electrical dimension of the uniaxial LHM sphere is chosen to be $k_0a=0.5\pi$ or $k_0a=\pi$, and we chose N=6 for the upper limit of the summation.

Fig.4 shows the RCSs for a more general lossy uniaxial LHM sphere with $\varepsilon_t/\varepsilon_0=-2+0.01$j and $\varepsilon_z/\varepsilon_0=-4+0.02$j. The electrical dimension of the uniaxial LHM sphere was chosen to be $k_0a=0.75\pi$ or $k_0a=1.5\pi$.

We chose the upper limit of the summation as N=6 for the case of k_0a=0.75π and N=10 for the case of k_0a=1.5π. Figs.3 and 4 show that the bistatic RCS be comes sharper when the electrical dimension of the uniaxial LHM sphere increases.

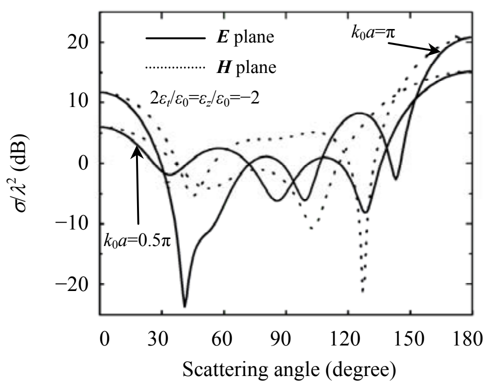

Fig.3 Radar cross sections (RCSs) of lossless uniaxial LHM sphere versus scattering angle θ (in degrees) in the *E*-plane and *H*-plane

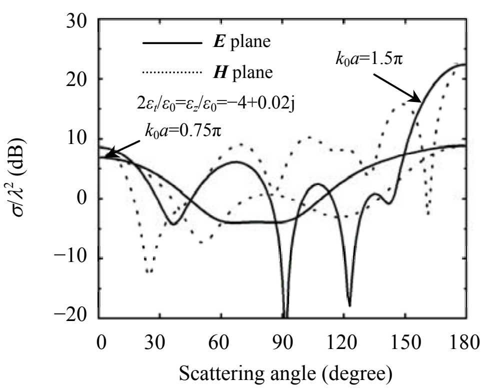

Fig.4 Radar cross sections (RCSs) of loss in uniaxial LHM sphere versus scattering angle θ (in degrees) in the *E*-plane and *H*-plane

The RCSs for a uniaxial LHM sphere with relatively large electrical dimension k_0a=3.5π are given in Fig.5 (under a plane wave incidence). The permittivity tensor elements were chosen to be $\varepsilon_t/\varepsilon_0$=−3+0.02j and $\varepsilon_z/\varepsilon_0$=−1.5+0.01j. Here we chose N=16 for the upper limit of the summation (should increase when the electrical dimension of the sphere increases).

Fig.5 shows that the RCSs vary rapidly with the scattering angle when the electrical dimension of the sphere is large.

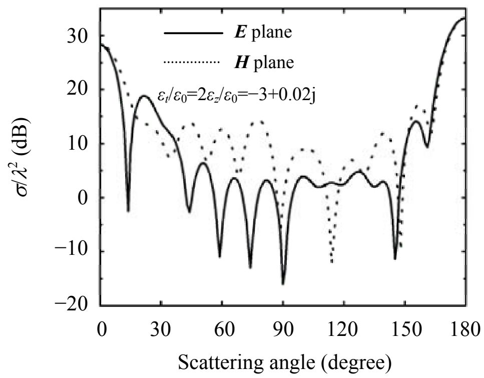

Fig.5 Radar cross sections (RCSs) of large size uniaxial LHM sphere versus scattering angle θ (in degrees) in the *E*-plane and *H*-plane

CONCLUSION

By expanding the analytical solution to source-free Maxwell's equations in a uniaxially anisotropic medium with spherical vector wave functions, an analytical solution for the scattering of a plane wave by a uniaxial LHM sphere was derived in this paper. Some numerical results of RCSs are presented. The present formulations can be generalized to some more complex cases, such as a uniaxial LHM-coated conducting sphere, a layered uniaxial LHM sphere, and multiple spheres of uniaxial LHM.

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