Journal of Zhejiang University SCIENCE A
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Modelling spatial vagueness based on type-2 fuzzy set

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Received Dec. 23, 2004; revision accepted Apr. 4, 2005

Abstract: The modelling and formal characterization of spatial vagueness plays an increasingly important role in the implementation of Geographic Information System (GIS). The concepts involved in spatial objects of GIS have been investigated and acknowledged as being vague and ambiguous. Models and methods which describe and handle fuzzy or vague (rather than crisp or determinate) spatial objects, will be more necessary in GIS. This paper proposes a new method for modelling spatial vagueness based on type-2 fuzzy set, which is distinguished from the traditional type-1 fuzzy methods and more suitable for describing and implementing the vague concepts and objects in GIS.

Key words: Fuzzy theory, Type-2 fuzzy set, Spatial vagueness, Geographic Information System (GIS), Membership function (MF)

doi:10.1631/jzus.2006.A0250 **Document code:** A **CLC number:** TP391

INTRODUCTION

In current Geographic Information System (GIS), spatial objects such as points, lines, and regions that are kept in the database, are supposed to be precisely defined, sharp objects. Examples of these sharp objects are cities, roads, and regions. Their boundaries are sharp. But firstly, as we know, some spatial objects are not sharp by themselves, e.g., forest cannot always be sharply defined. Moreover, errors can happen during measurement. We cannot find the exact boundary from such measurement. Thus, the boundary cannot always be defined precisely, and is vague. Therefore, vagueness can be part of spatial information. Secondly, human spatial cognition also includes some spatial concepts, which always have vague features, e.g., near river and around playground, etc. Spatial vagueness always exits in the implement of GIS. In this paper only vague spatial objects and vague spatial concepts are considered.

Fisher (2000) explained the two principal views of vagueness. The boundary of spatial object is not sharp by itself. We cannot find the exact boundary. The vagueness is viewed as a matter of degree that is more or less certain. The boundary that clear for any individual at any time is a moving boundary because there are different opinions among people. For example, one defines a place 1 km away to be a nearby place but another one may define a place 1.5 km away to be a nearby place. Type-2 fuzzy set is the solution to both views. This thesis considers the vagueness as an inherent property of an object.

The concept of type-2 fuzzy set was introduced by Zadeh (1975). Type-2 fuzzy set is a further extension of the classic fuzzy set that is also called type-1 fuzzy set. Compared to type-1 fuzzy set, type-2 fuzzy set has a fuzzy membership function that represents the uncertainty about the degrees of membership (Mizumoto and Tanaka, 1976). Type-2 fuzzy set is useful in circumstances where it is difficult to determine the exact membership function for a fuzzy set. As we know, once type-1 fuzzy membership function has been defined the set becomes a crisp set and all the vagueness disappears. However, type-2 fuzzy membership function is fuzzy itself. Our aim is to use the type-2 fuzzy set to express and handle uncertainties and vagueness in the GIS.

TYPE-2 FUZZY SET PRELIMINARIES

Type-1 and type-2 fuzzy sets are defined as follows.

Definition 1 Given a universe of discourse U, a type-1 fuzzy set A is a fuzzy set whose membership function is characterized by μ_A:$U\rightarrow[0,1]$, and denoted by $A=\{u, \mu_A(u)\}$, $u\in U$.

Definition 2 Given a universe of discourse X, a type-2 fuzzy set $\tilde{A}$ is characterized by a membership function $\mu_{\tilde{A}}$, such that $\mu_{\tilde{A}}(x)=\{v_x|v_x:[0,1]\rightarrow[0,1]\}$ or $\mu_{\tilde{A}}: X\rightarrow[0,1]^{[0,1]}$.

A type-2 membership grade can be any subset in [0,1]—the primary membership; and corresponding to each primary membership, there is a secondary membership (which can also be in [0,1]) that defines the possibilities of the primary membership. A type-1 fuzzy set is a special case of a type-2 fuzzy set; its secondary membership function is a subset with only one element-unity (Liang and Mendel, 2000). Of course, type-n fuzzy set whose membership function ranges over fuzzy sets of type-($n-1$) is infinite when n is growing, but it is always sufficient to talk about type-2 fuzzy set as it allows us to handle spatial vagueness and linguistic uncertainties. The simplest type-2 fuzzy set is in the interval type-2 set whose elements' degree of membership are intervals with secondary membership degree of 1 or 0, and this is the case studied in this paper.

Examples of type-2 fuzzy sets

As we know, Gaussian or normal distribution is common in natural or social phenomena. Many random variables such as stature of adult people in some region, result of estimating of something, crop of one province, etc., obey or almost obey Gaussian distribution. Here we consider the case of a fuzzy set characterized by a Gaussian membership function. According to the definition of the type-2 fuzzy set, three Gaussian type-2 fuzzy membership function shapes based on Gaussian distribution are defined as follows.

Definition 3 Given a universe of discourse X, a type-2 fuzzy set $\tilde{A}$ is characterized by a membership function $\mu_{\tilde{A}}$, such that $\mu_{\tilde{A}}(x)=\exp(-(x-a)^2/(2\sigma^2))$, where a is the mean or expected value of the Gaussian function. σ is a deviation, and σ is a random value between $[\sigma_1,\sigma_2]$.

According to Definition 3 we can find that different value of σ leads to different membership grade (Fig.1). So corresponding to each value of x (except $x=a$), the membership grade can be any possible number depending on the value of σ, in other words, the Gaussian type-2 fuzzy set membership grade is not a single or crisp number, it is a fuzzy set.

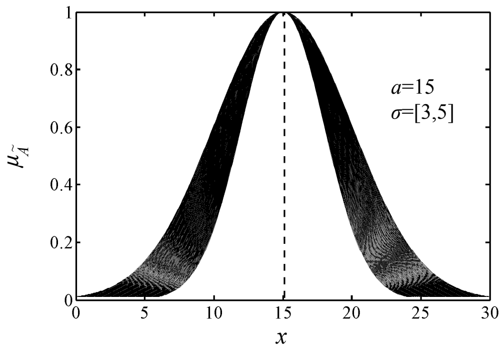

Fig.1 Type-2 fuzzy set

Definition 4 Given a universe of discourse X, a type-2 fuzzy set $\tilde{A}$ is characterized by a membership function $\mu_{\tilde{A}}$, such that $\mu_{\tilde{A}}(x)=\exp(-(x-a)^2/(2\sigma^2))$, in which a is the mean or expected value of the Gaussian function, and a is a random value between $[a_1, a_2]$. σ is deviation.

Definition 4 shows a case of Gaussian function with uncertain mean. In this case $\mu_{\tilde{A}}$ is still a fuzzy set corresponding to a certain x depending a random value of a (Fig.2).

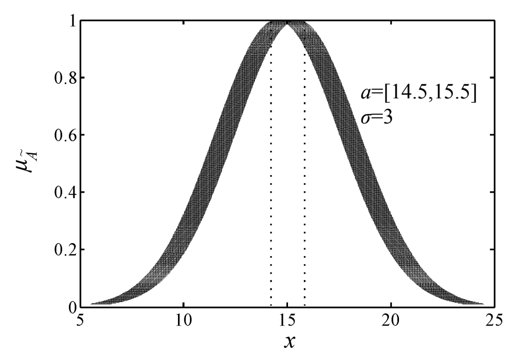

Fig.2 Type-2 fuzzy set

Definition 5 Given a universe of discourse X, a type-2 fuzzy set $\tilde{A}$ is characterized by a membership function $\mu_{\tilde{A}}$, such that $\mu_{\tilde{A}}(x)=\exp(-(x-a)^2/(2\sigma^2))$, in which a is the mean or expected value of the Gaussian function, and a is a random value between $[a_1, a_2]$. σ is deviation, and σ takes a random value between $[\sigma_1,\sigma_2]$.

In the case of Definition 5 both a and σ are random or uncertain, and can describe more vague condition (Fig.3).

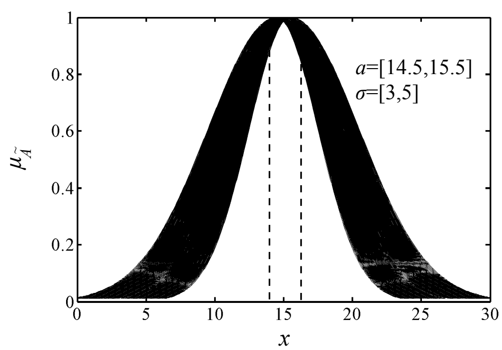

Fig.3 Type-2 fuzzy set

With further study we can define the random value of a or σ as some possible distribution such as Gaussian distribution. Corresponding membership grade for each point in the set of a Gaussian type-1 fuzzy set, different standard deviation of Gaussian distribution for a or σ can be defined. For example the standard deviations of these Gaussians decrease as the difference between 0.5 and the membership grade for each point in the set of a Gaussian type-1 fuzzy set increases (Fig.4), i.e., nearby the 0.5 we always get more uncertainty or possibility, which is shown by the thinner line at the bottom of Fig.4.

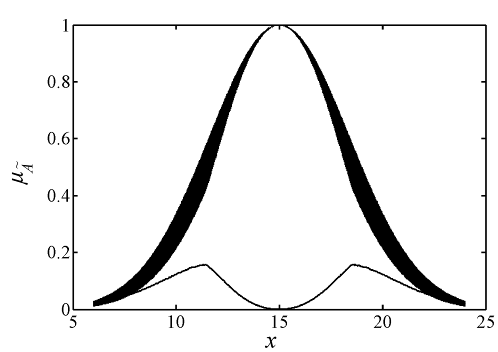

Fig.4 Type-2 fuzzy set

The level-sets of type-2 fuzzy set

Definition 6 Let $\tilde{A}$ be type-2 fuzzy subset of X, and for each $x\in X$, $\mu_{\tilde{A}}(x)$ is convex type-1 fuzzy subset of [0,1], such that, for each $a\in[0,1]$, the α-level sets of the type-2 fuzzy membership functions $\mu_{\tilde{A}}(x)$ are interval-valued membership function $\mu_{\tilde{A}}^{\alpha}(x)$.

According to Definition 6, the α-level sets of type-2 fuzzy set are different from the α-level sets of type-1 fuzzy set, in other words, the α-level sets of type-1 are crisp subsets, but the α-level sets of type-2 are still an interval-valued membership functions, which mean they are still fuzzy subsets. Figs.5a and 5b show the 0.5-level sets and 0.8-level sets of the type-2 fuzzy set in Fig.3.

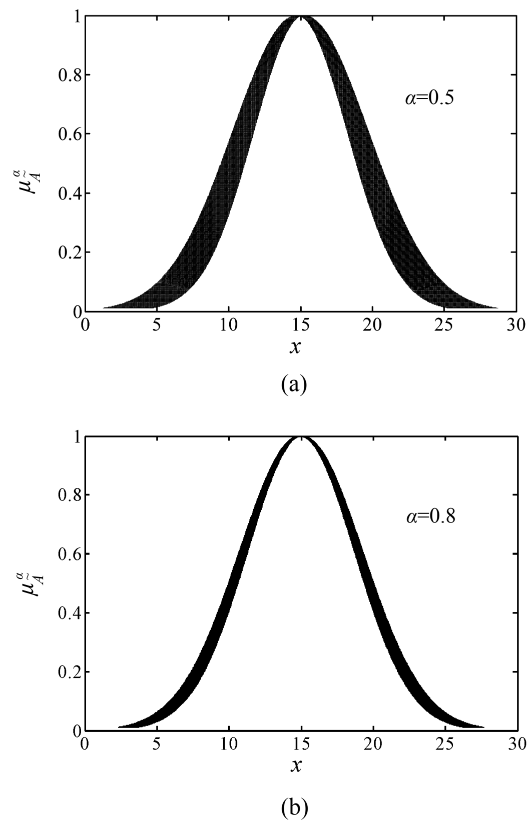

Fig.5 Level sets of type-2 fuzzy sets
(a) 0.5-level sets; (b) 0.8-level sets

MODELLING SPATIAL VAGUENESS

Concept of spatial vagueness

Increasingly, researchers are beginning to realize that the current mapping of spatial phenomena of the real world to exclusively crisp spatial objects is an insufficient abstraction process for many spatial applications and that spatial vagueness or spatial indeterminacy are features of many geographic data (Burrough and Frank, 1996). Many spatial objects in reality have no sharp boundaries or its boundaries cannot be precisely determined. Examples are natural, social, or cultural phenomena like land features with continuously changing properties (as population density, soil quality, vegetation, pollution, temperature, air pressure), oceans, deserts, English speaking areas, or mountains and valleys. The transition between a valley and a mountain usually cannot be exactly ascertained so that the two spatial objects valley and mountain cannot be precisely separated and defined in a crisp way. We will designate this kind of entities as vague spatial objects (Schneider, 1999). On the other hand, vagueness also exits in the cognition of the human or language expressions for spatial objects, such as nearby school, along railway, and one central region, etc. Vague spatial objects always separated into vague points, vague lines and vague regions. The following section will define the vague objects based on type-2 fuzzy set respectively.

Models of spatial vagueness

(1) Vague points

A vague point can be viewed as a point in two-dimensional Euclidean space with a membership value greater than 0, since 0 documents the non-existence of a point. A vague point $\tilde{p}$ at (a,b) in R^2, written $\tilde{p}(a,b)$, is defined by

$$\mu_{\tilde{p}(a,b)}(x,y)=\{v_{(x,y)}\mid v_{(x,y)}:[0,1]\times[0,1]\rightarrow[0,1]\},$$

where $\mu_{\tilde{p}(a,b)}(x,y)$ means the two-dimensional type-2 fuzzy set membership grade.

Take Gaussian type-2 fuzzy set as example, and $\mu_{\tilde{p}(a,b)}(x,y)$ can be defined as follows

$$\mu_{\tilde{p}(a,b)}(x,y)=\mathrm{e}^{-\frac{1}{2}\left[\frac{(x-a)^2}{\sigma_x^{\,2}}+\frac{(y-b)^2}{\sigma_y^{\,2}}\right]},$$

where $\sigma_x\in[\sigma_x^1,\sigma_x^2]$, $\sigma_y\in[\sigma_y^1,\sigma_y^2]$, $a\in[a_1,a_2]$ and $b\in[b_1,b_2]$.

Fig.6a shows the membership of the vague point, and in Fig.6b thicker points represent higher membership values.

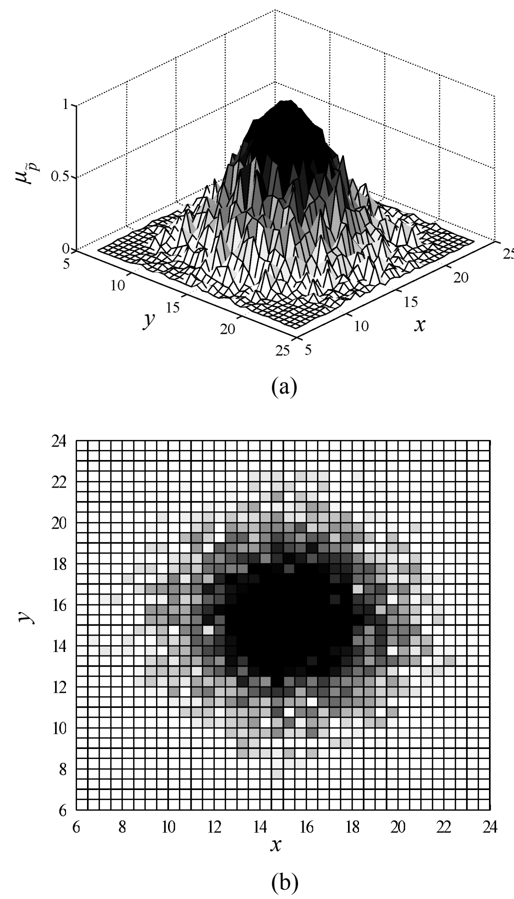

Fig.6 Vague point
(a) Type-2 fuzzy set membership grade; (b) Vague point

(2) Vague lines

A vague line can be represented with a series of vague points, and any point of the line has some certainty of degree of belonging to the line. In two-dimensional Euclidean space a crisp line l can be defined as l:y_l=$f(x_l)$, and then a vague line $\tilde{l}$ can be defined as

$$\mu_{\tilde{l}(x_l,y_l)}(x,y)=\{v_{(x,y)}\mid v_{(x,y)}:[0,1]\times[0,1]\rightarrow[0,1]\},$$

where $\mu_{\tilde{l}(x_l,y_l)}(x,y)$ represents the two-dimensional type-2 fuzzy set membership grade.

Take Gaussian type-2 fuzzy set as example, and $\mu_{\tilde{l}(x_l,y_l)}(x,y)$ can be defined as follows

$$\mu_{\tilde{l}(x_l,y_l)}(x,y)=\mathrm{e}^{-\frac{1}{2}\left[\frac{(x-x_l)^2}{\sigma_x^2}+\frac{(y-y_l)^2}{\sigma_y^2}\right]},$$

where $\sigma_x\in[\sigma_x^1,\sigma_x^2]$, $\sigma_y\in[\sigma_y^1,\sigma_y^2]$, $x_l\in[x_l^1,x_l^2]$ and $y_l\in[y_l^1,y_l^2]$. For each point (x,y), there must exist a point of line l, (x_l,y_l), which minimizes $\sqrt{(x-x_l)^2+(y-y_l)^2}$.

Fig.7a shows the membership of the vague line, and in Fig.7b thicker points represent higher membership values in the vague line.

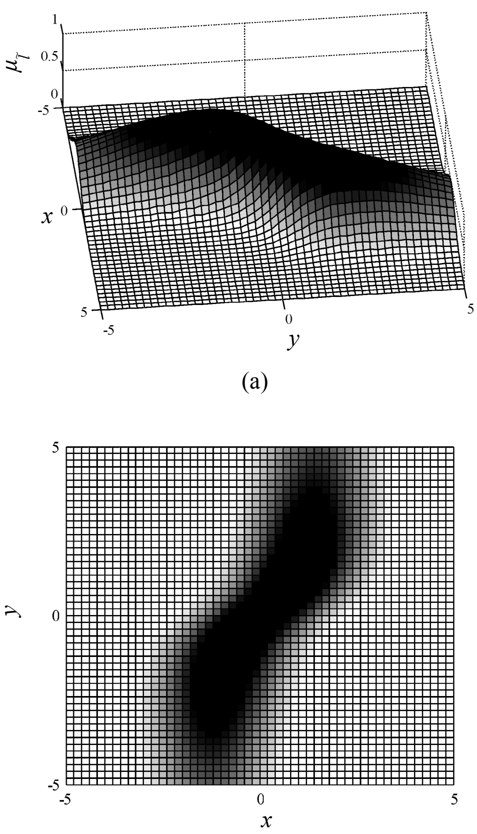

Fig.7 Vague line
(a) Type-2 fuzzy set membership grade; (b) Vague line

(3) Vague regions

A vague region is a region with an uncertain or vague boundary, such that points in the vague boundary have different membership grades, which represent the degree of belonging to the region. A vague region can be considered as one flat area with vague core or vague boundary. In two-dimensional Euclidean space a crisp region r can be defined as $r(x_r,y_r)$, and a vague region $\tilde{r}$ can be defined as

$$\mu_{\tilde{r}(x_r,y_r)}(x,y)=\{v_{(x,y)}\mid v_{(x,y)}:[0,1]\times[0,1]\rightarrow[0,1]\},$$

where $\mu_{\tilde{r}(x_r,y_r)}(x,y)$ represents the two-dimensional type-2 fuzzy set membership grade.

Take Gaussian type-2 fuzzy set as example, and $\mu_{\tilde{r}(x_r,y_r)}(x,y)$ can be defined as follows

$$\mu_{\tilde{r}(x_r,y_r)}(x,y)=\begin{cases}\mathrm{e}^{-\frac{1}{2}\left[\frac{(x-x_r)^2}{\sigma_x^2}+\frac{(y-y_r)^2}{\sigma_y^2}\right]}, & (x,y)\notin r(x_r,y_r)\\ 1, & (x,y)\in r(x_r,y_r)\end{cases}$$

where $\sigma_x\in[\sigma_x^1,\sigma_x^2]$, $\sigma_y\in[\sigma_y^1,\sigma_y^2]$, $x_r\in[x_r^1,x_r^2]$ and $y_r\in[y_r^1,y_r^2]$. When $(x,y)\notin(x_r,y_r)$, for each point (x,y), there must exist a point of region r, (x_r,y_r), which minimizes $\sqrt{(x-x_r)^2+(y-y_r)^2}$.

Fig.8a shows the membership of the vague region, and in Fig.8b thicker points represent higher membership values in the vague region.

Features of the vague spatial models

Type-2 fuzzy set, a fuzzy set with fuzzy membership function, is the extension of the type-1 fuzzy set, so the vague spatial models based on the type-2 fuzzy set have more advanced features than the models based on type-1 fuzzy set. This section mainly discusses the advantage of type-2 fuzzy set based on the level-sets, and here only vague points based on two kinds of fuzzy sets are concerned. The same goes for the line and polygon.

In order to show the different effect on vague point between the type-2 and type-1 fuzzy sets, type-1 fuzzy membership function is the same as the primary membership function. Fig.9 and Fig.10 show the vague points and their level-sets. Obviously, level-sets of the vague points in Fig.9c are crisp sets, but level-sets of the vague points in Fig.10c are still fuzzy sets. In fact, the case of Fig.10 is more compatible to describe the real spatial world than the case in Fig.9.

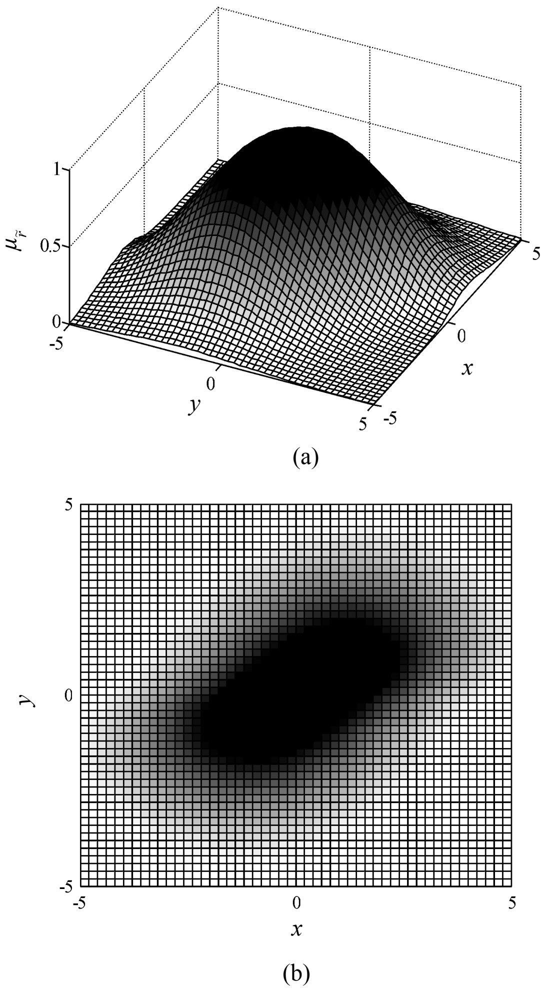

Fig.8 Vague region
(a) Type-2 fuzzy set membership grade; (b) Vague region

CONCLUSION AND FURTHER WORK

This paper lays the conceptual and formal foundation for the treatment of spatial objects blurred by the feature of type-2 fuzziness. It is also a contribution to bridge the gap between the fuzziness and probabilities of spatial phenomena. The paper focuses on the modelling of basic vague spatial objects leading to three type-2 fuzzy spatial data types for vague points, vague lines, and vague regions whose structure and semantics is formally defined. The characteristic feature of the design is the modelling of smoothness and flexibility, which is inherent to the objects themselves and to the transitions between

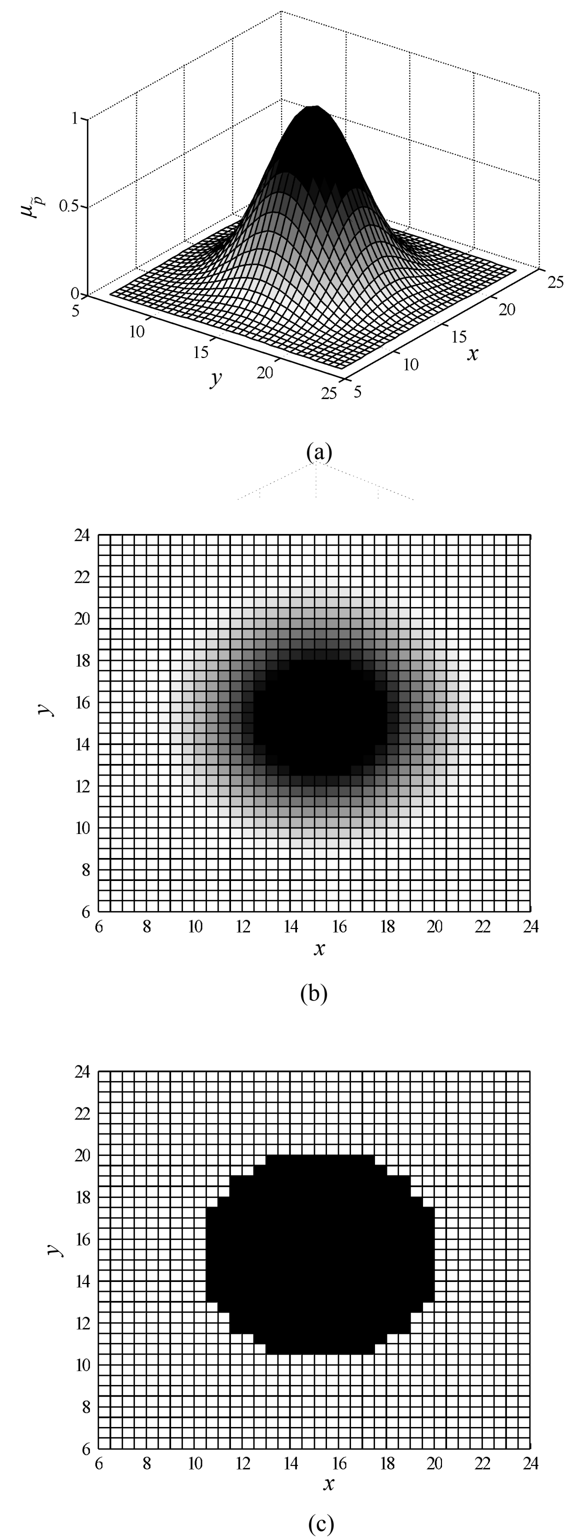

Fig.9 Vague point based on type-1 fuzzy set
(a) Type-1 fuzzy set membership grade; (b) Vague point; (c) Type-1 level-sets (0.25-level sets)

different vague objects. This is achieved by the framework of type-2 fuzzy set theory which allows partial and multiple membership and hence different membership degrees of an element in sets. Different structured views of vague regions as special collections of crisp regions enable us to obtain a better understanding of their nature and to decrease their complexity. Further work will have to deal with the formal definition of vague spatial operations and vague spatial relations between vague spatial objects, with the integration of vague spatial data types into human query languages.

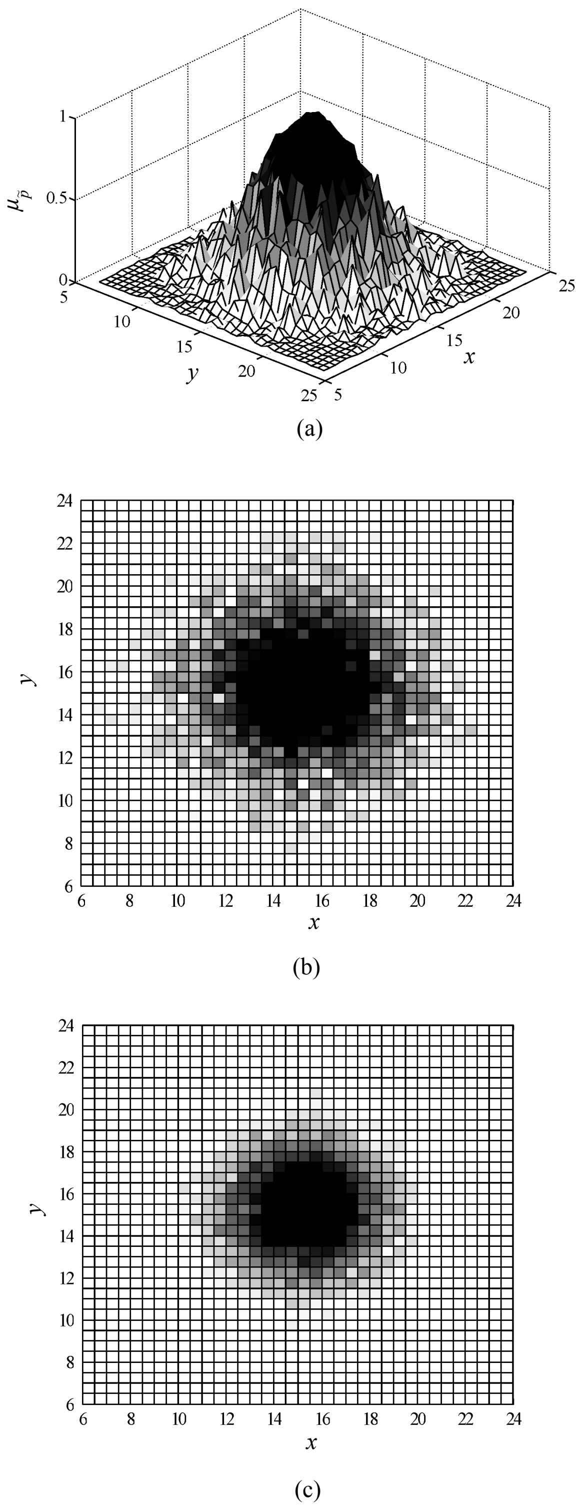

Fig.10 Vague point based on type-2 fuzzy set
(a) Type-2 fuzzy set membership grade; (b) Vague point; (c) Type-2 level-sets (0.25-level sets)

References

Burrough, P.A., Frank, A.U., 1996. Geographic Objects with Indeterminate Boundaries. GISDATA Series, Vol. 2. Taylor & Francis.

Fisher, P., 2000. Sorites paradox and vague geographies. *Fuzzy Sets and Systems*, **113**(1):7-18. [doi:10.1016/S0165-0114(99)00009-3]

Liang, Q.L., Mendel, J.M., 2000. Interval type-2 fuzzy logic system: theory and design. *IEEE Transaction on Fuzzy System*, **8**(5):535-550. [doi:10.1109/91.873577]

Mizumoto, M., Tanaka, K., 1976. Some properties of fuzzy sets of type-2. *Information and Control*, **31**(4):312-340. [doi:10.1016/S0019-9958(76)80011-3]

Schneider, M., 1999. Uncertainty Management for Spatial Data in Databases: Fuzzy Spatial Data Types. SSD'99, LNCS 1651, p.330-351.

Zadeh, L., 1975. The concept of a linguistic variable and its application to approximate reasoning-I. *Information Sciences*, **8**(3):199-249. [doi:10.1016/0020-0255(75)90036-5]