Journal of Zhejiang University SCIENCE A
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Performance of a novel carrier frequency offset estimation algorithm for OFDM-based WLANs*

ZHAO Zhong-wei (赵忠伟)
(*Institute of Wireless Communication and Network, Department of Information and Electronic Engineering, Zhejiang Gongshang University, Hangzhou 310035, China*)
E-mail: zhaozzw@mail.hzic.edu.cn
Received Feb. 10, 2005; revision accepted Oct. 13, 2005

Abstract: This paper presents a novel carrier frequency offset estimation (CFO) algorithm for orthogonal frequency division multiplexing (OFDM)-based Wireless Local Area Networks (WLANs). Compared with previous approaches, this paper extends the whole frequency offset acquisition range by embedding a synthetic algorithm according to the preamble structure of WLANs symbols. The numerical results presented support the effectiveness of this algorithm by which the estimation error of the whole carrier frequency offset in the WLANs is effectively decreased.

Key words: Carrier frequency, OFDM, WLANs, Preamble structure, Acquisition
doi:10.1631/jzus.2006.A0321 **Document code:** A **CLC number:** TP391

INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been treated as the key transmission method in Wireless Local Area Networks (WLANs) based on IEEE802.11a protocol (IEEE Standard 802.11a- 1999). However, the sensitivity of the scheme to the frequency offset limits its development. Moose (1994) proposed maximum likelihood estimation for the carrier frequency offset by using two different received symbols, but the limitation of this method is that the acquisition range is only ±1/2 of the sub-carrier frequency spacing. Schmidl and Cox (1997) presented an algorithm to extend the frequency acquisition range by limiting the training symbols, but this algorithm could not be directly adopted by IEEE802.11a-based WLANs groups because it was performed by using a special preamble structure. Li *et al.*(2001) presented a nonlinear least squares (NLS) scheme to estimate the offset, but this proposition is only used when the frequency offset is less than the sub-carrier frequency spacing. Therefore, the objective of this paper is to estimate the entire frequency offset range with OFDM data symbols via a high performance novel estimation algorithm.

PROBLEM FORMATION

The transmitted frame structure specified by IEEE802.11a is described in Fig.1. Each frame consists of a packet preamble, signal section and data blocks. The packet preamble consists of ten identical short OFDM training symbols (each containing 16 data samples) and two long OFDM training symbols (each containing 64 data samples). Between the short training symbols and long training symbols, there is a guard interval (GI2) which constitutes the cyclic prefix of the long training symbols to reduce the ISI. We consider a multi-path and frequency selective fading

* Project supported by the National Natural Science Foundation of China (No. 2004012F33AA), and Education Foundation of Zhejiang Education Department (No. 20040125-66), China

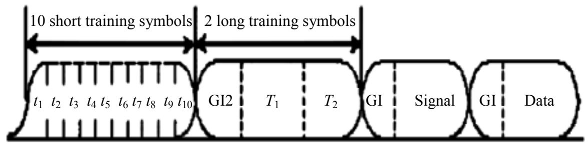

Fig.1 OFDM preamble structure

channel and let M_S (M_L) denotes the number of short (long) training symbols and N_S (N_L) denotes the samples of short (long) symbols (subscripts 'S' and 'L' represent short symbols and long symbols). According to IEEE802.11 standardization (IEEE Standard 802.11a-1999), M_S=10, M_L=2, N_S=16, N_L=64. Since the short and long training series in the preamble are only used for the signal detection, timing synchronization and frequency offset estimation, we only focus on the relationship between frequency offset and the data symbols mapped onto sub-carriers. We let $x(m,n)$ denote the nth sample of the mth noise-free OFDM symbols prior to taking N-point IFFT, and let $s(m,n)$ denote the nth sample of the mth noise-free OFDM symbols after taking N-point IFFT:

$$s(m,n)=\sum_{k=-N/2}^{N/2-1} x(m,k)\mathrm{e}^{\mathrm{j}2\pi k(n-N_\mathrm{g})/N},$$
$$m=1,2;\ k=1,2,\ldots,N;\ n=1,2,\ldots,N_{\mathrm{sym}}, \quad (1)$$

where N_{sym} is the number of samples for every data block of symbols and $N_{\mathrm{sym}}=N+N_\mathrm{g}=80$. N_g denotes the number of samples of the guard interval (GI). Meantime, let h_i and τ_i denote respectively the component of the channel impulse response and the component of the time delay, and also let $r(m,n)$ denote the nth sample of the mth received OFDM symbols at the receiver, then $r(m,n)$ is given by

$$r(m,n)=\mathrm{e}^{\mathrm{j}2\pi n\Delta fT}\left[\sum_i h_i(nT)s(m,nT-\tau_i)+w(nT)\right],$$
$$n=1,2,\ldots,N_{\mathrm{sym}}, \quad (2)$$

where T is the sample period and $w(nT)$ is the sample value of the zero mean additive white Gaussian noise (AWGN). Note that data block after taking the IFFT is only mapped onto an individual sub-carrier rather than sub-carrier pairs, the sample frequency $1/T$ is equal to sub-carrier frequency spacing $1/T_\mathrm{u}$. The received nth sample of the mth OFDM symbol after removing the cyclic prefix and demodulated by FFT is given by

$$Z(m,k)=\sum_{n=N_\mathrm{g}+1}^{N_{\mathrm{sym}}} r(m,n)\mathrm{e}^{-\mathrm{j}2\pi(n-N_\mathrm{g})k/N}$$
$$=(\mathrm{e}^{\mathrm{j}\pi\Phi_k}\mathrm{e}^{\mathrm{j}2\pi(N_{\mathrm{L,sym}}+N_\mathrm{g})\Phi_k/N})\mathrm{sin}c(\pi\Phi_k)X_{\mathrm{L},k}H_{\mathrm{L},k}$$
$$+\sum_{k',k'\neq k}(\mathrm{e}^{\mathrm{j}\pi\Phi_{k,k'}}\mathrm{e}^{\mathrm{j}2\pi(N_{\mathrm{L,sym}}+N_\mathrm{g})\Phi_k/N})\mathrm{sin}c(\pi\Phi_{k,k'})X_{\mathrm{L},k}H_{\mathrm{L},k'}+N_{\mathrm{L},k'}$$
$$(k=1,2,\ldots,N;\ n=N_\mathrm{g}+1, N_\mathrm{g}+2, \ldots, N_{\mathrm{sym}}), \quad (3)$$

where Φ denotes the phase offset from frequency offset Δf and H_k is the channel frequency response and $\mathrm{sin}c(x)=\sin x/x$ is the sample function. In Eq.(3), the first section is an available value with frequency offset, the second section is inter-carrier interference (ICI), the third is AWGN. According to (Narayanan, 2001), the maximal sub-carrier frequency offset may be given by

$$(\Delta fT_\mathrm{u})<\frac{\sqrt{3}}{\pi}\sqrt{\left(1-\frac{1}{\Delta\gamma_{\max}}\right)\frac{1}{\gamma}}, \quad (4)$$

where γ is the Signal Noise Rate (SNR) and $\Delta\gamma_{\max}$ is the maximal and allowable SNR loss because of the ICI. Apparently, $\Delta\gamma_{\max}$ is up to 1 dB and maximal frequency offset $(\Delta f)_{\max}$ must be less than 1% when the SNR is 30 dB. This algorithm has two disadvantages: One is that the precision of the frequency offset estimation will be greatly influenced when considering other factors like timing offset and phase shift; The other is that the range of the acquisition process is limited because of not considering the different offset variable. Li *et al*.(2001) presented a nonlinear least squares (NLS) scheme to estimate the offset, but this algorithm can be utilized only if the frequency offset is less than the sub-carrier frequency spacing.

Our problem of interest herein is to estimate the entire frequency offset Δf that not only includes the integral times of sub-carrier spacing, but contains the fraction part less than the sub-carrier frequency spacing.

ALGORITHM

According to the requirement of the frequency offset estimation specified by IEEE802.11 standardization, the entire procedure of estimation is divided into two processes: an acquisition process and a tracking process. We will only consider the acquisition algorithm training symbols. If the offset can be precisely obtained in the acquisition process, the residual value can be easily fixed in the tracking process. The major steps are shown in Fig.2. Fig.2 shows that the input signal $X(m,n)$ is transmitted in the channel whose time domain is $h(t)$, and received signal is $r(m,n)$. At the same time, we let the transmitter, channel, two-tap loop filter form the forward feedback circuit if we suppose the delay time unit is N_L (Zhao and Li, 2004a; 2004b; Kim *et al*., 2003). Apparently, the forward feedback circuit is responsible for achieving the integral and fractional offset in the time and frequency domain. In this scheme, we let Δf_I denote the integral offset and Δf_F denote the fractional offset, then the offset Δf in Eq.(4) is transformed into

$$\Delta f=\Delta f_I+\Delta f_F=(n_I+\Delta f'_F)T_u^{-1}, \quad (5)$$
$$\Delta f'=(\Delta f\times n_I)/T_u^{-1}+\Delta f'_F, \quad (6)$$

where n_I, $\Delta f'$, $\Delta f'_F$ respectively denote the normalized value of Δf_I, Δf, Δf_F relative to sub-carriers frequency spacing $1/T_u$. Apparently, the task of the acquisition process is to construct the orthogonality between the different sub-carriers and rapidly estimate the integral normalized value n_I as well as $\Delta f'_F$ fractional. Considering the efficiency in Fig.2, we divide the estimation into two procedures: The first objective is to estimate the integral normalized value n_I in the frequency domain by using the characteristic of the frequency shift after the FFT and the received long training symbols in the preamble structure; The second step is to obtain the fractional part $\Delta f'_F$ in the time

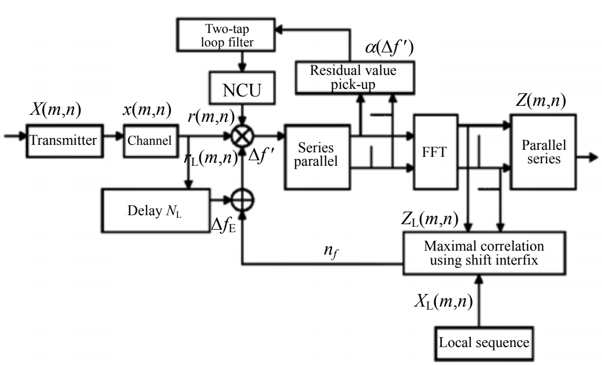

Fig.2 Diagram of the synthetic carrier frequency offset estimation algorithm

domain by utilizing the received long and short symbols in the preamble structure. Let $r_L(m,n)$ (m=1, 2,…,M_L; n=1,2,…,N_L) denote the nth samples of the mth noise-free long OFDM symbols after removing the cyclic prefix but prior to taking the FFT, any one of two received long OFDM symbols is given by

$$r_L(m,n)=\exp(j2\pi\Delta f')x_L(m,n),\ m=1,2;\ n=1,2,\ldots,N_L. \quad (7)$$

Assuming the decimal part of the offset has been compensated for, Eq.(7) can be substituted by

$$r_L(m,n)=\exp(j2\pi\Delta n_I)x_L(m,n),\ m=1,2;\ n=1,2,\ldots,N_L. \quad (8)$$

After taking the FFT of Eq.(8) and considering the characteristic of the circular shift in the frequency domain, Eq.(8) is transformed into

$$Z_L(m,n)=(X_L(m,n-n_I))N_L+N_L(m,n),$$
$$m=1,2;\ n=1,2,\ldots,N_L, \quad (9)$$

where $Z_L(m,n)$ is a representation of $r_L(m,n)$ after taking the FFT and $X_L(m,n)$ denotes $x_L(m,n)$ before taking the IFFT at the transmitter. We have obviously constructed a local long training symbol with offset coefficient i to correlate with any one long received symbol. By gradually changing the relative factor i, the normalized integral offset n_I can be approximately estimated by

$$\hat{n}_I=\max_i\left|\sum_{n=1}^{N_L}Z_L(m,n)X_L^*(m,n+i)\right|,$$
$$i=-N_L/2,\ldots,-1,0,1,2,\ldots,N_L/2. \quad (10)$$

On the other hand, let $r_S(m,n)$ (m=1,...,M_S; n=1,…,N_S) denote the nth samples of the mth noise-free short OFDM symbols after removing cyclic prefix but prior to taking the FFT. At the receiver, apart from a multiplicative complex exponential accounting carrier frequency offset, these ten short symbols are identical to each other (the first short symbol is always treated as a cyclic prefix for another nine short symbols). The complex exponential weighting for the short symbols is given by

$$r_S(m,n)=\exp(j2\pi\Delta f' mN_S)r_S(l,n),$$
$$m=2,3,\ldots,M_S;\ n=1,2,\ldots,N_S. \quad (11)$$

The phase offset for two long symbols is given by

$$r_L(m,n)=\exp(j2\pi\Delta f' mN_L)r_L(l,n),\quad m=M_L=2;\ n=1,2,\ldots,N_L. \tag{12}$$

According to the MLE (Moose, 1994; Lawrey and Kikkert, 2001; Lawrey, 1999), the normalized fractional offset is estimated by

$$\hat{\Delta} f_F' = \frac{1}{2\pi(M_S-1)}\sum_{m=2}^{M_S}\arg\left(\sum_{n=1}^{N_S} r_S(m,n)r_S^*(m-1,n)\right) + \frac{1}{2\pi}\sum_{m=1}^{M_L}\arg\left(\sum_{n=1}^{N_L} r_L(m,n)r_L^*(m-1,n)\right). \tag{13}$$

SIMULATION

We evaluated the algorithm performance of a multi-path and frequency-selective fading channel assuming that the packet timing is available, and that the Doppler shift is scheduled as 0.03. Meantime, we also suppose that the amplitude envelop of the channel impulse response h_i (i=1,2,3) follows a Rayleigh-distribution and that the phase is uniformly distributed over mean-distribution in [0,2π]. We obtain the mean-squared errors (MSEs) of the carrier frequency offset estimates corresponding to the Cramer-Rao bounds (CRBs) as a function of SNR. Fig.3 shows the MSEs and CRBs of the CFO estimates versus SNR when Δf=3.5.

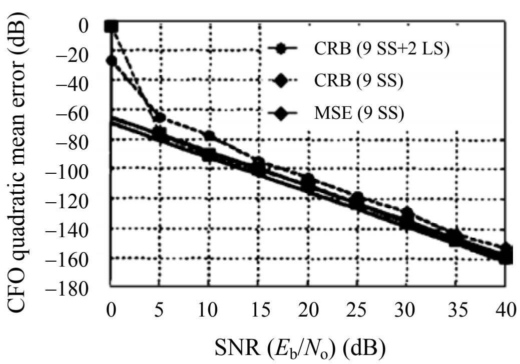

Fig.3 Comparison of MSEs with CRBs as a function of SNR for the OFDM CFO estimates with a multi-path frequency-selective fading channel when Δf=3.5

We also compared the estimate error as a function of SNR for different frequency offset. Fig.4 shows the relationship between the quadratic mean error (QME) and SNR when the offset is set differently.

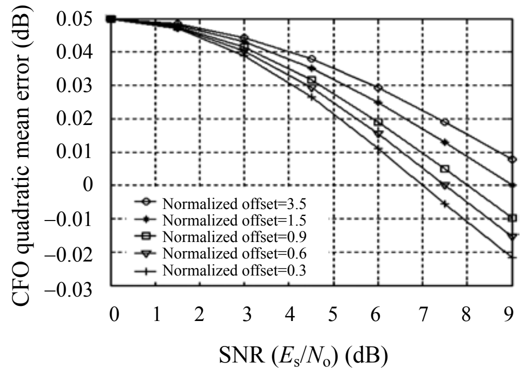

Fig.4 Comparison of the estimate error as a function SNR for different normalized frequency offset

Fig.5 shows the comparison of the CFO variance as a function SNR for different normalized frequency offset.

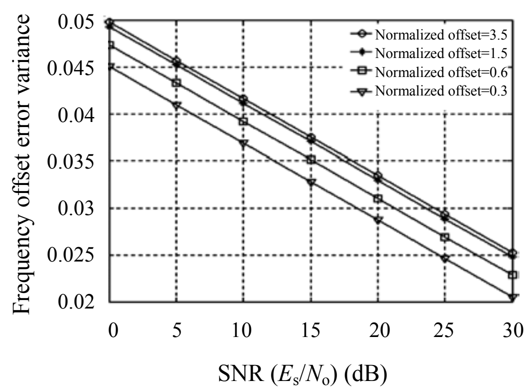

Fig.5 Comparison of the CFO variance as a function SNR for different normalized frequency offset

Figs.4 and 5 show that the variance and QME decrease with increasing SNR and different normalized frequency offset. When offset was 1.5 or more, the convergence rate of the QME and variance was not very affected as the SNR gradually decreased. Fig.6 shows the comparison of QME resulting from long and short symbols in the preamble structure between this algorithm and the approach presented by Li *et al*.(2001) when SNR was 13 dB.

In Fig.6, it is apparent that the bound of the acquisition process is greatly extended. Fig.7 shows the frequency offset tracking simulation in the tracking process. The tracking bound basically keeps balance under condition of different SNR.

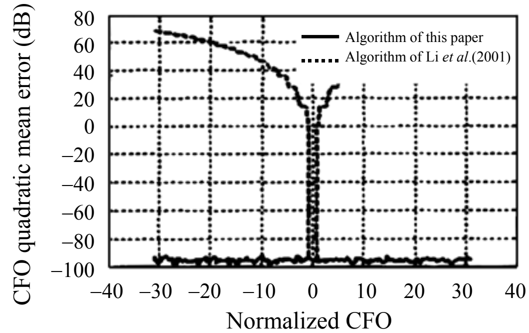

Fig.6 Comparison of QME resulting from long and short symbols in the preamble between the different algorithms when SNR was 13 dB

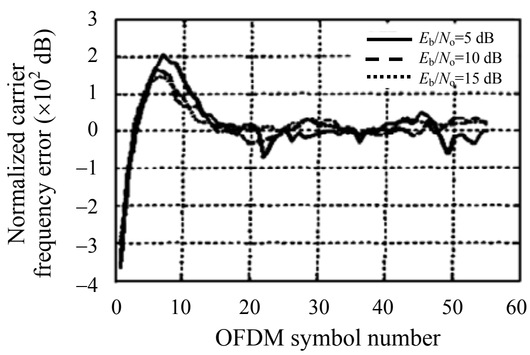

Fig.7 Post-FFT carrier frequency tracking performance

CONCLUSION

We have presented an efficient CFO estimation algorithm for OFDM-based WLANs by using a packet preamble structure and OFDM data symbol structure adopted by the IEEE802.11 standardization group. The simulated performance showed that the proposed algorithm not only has less quadratic mean error and better convergence than the current approaches, but effectively extends the acquisition range of the carrier frequency offset estimation without enhancing the estimation error. The new estimation algorithm may be considered as a component at the receiver for OFDM-based WLANs.

References

Kim, K.Y., Lee, J.Y., Choi, H.J., Kim, H., 2003. Symbol frame synchronization technique for OFDM burst mode Transmission. *IEEE Trans. Comm.*, **56**:1761-1771.

Lawrey, E., 1999. Multiuser OFDM. Proceedings of the Fifth International Symposium on Signal Processing and its Applications, ISSPA'99, Brisbane, Australia, p.761-764.

Lawrey, E., Kikkert, C.J., 2001. Maximizing signal strength for OFDM inside buildings. *IEEE Trans. on Microwave Theory and Techniques*, **49**(11):2131-2136. [doi:10.1109/22.963148]

Li, J., Liu, G., Giannakis, G.B., 2001. Carrier frequency offset estimation for OFDM-based WLANs. *IEEE Signal Processing Lett.*, **8**:80-82. [doi:10.1109/97.905946]

Moose, P.H., 1994. A technique for orthogonal frequency division multiplexing frequency offset correction. *IEEE Trans. Commun.*, **42**:2908-2914. [doi:10.1109/26.328961]

Narayanan, P.S., 2001. Acquisition in OFDM-based WLANs. www.tenet.res.in/Papers/ccs/june/29062001.pdf.

Schmidl, T.M., Cox, D.C., 1997. Robust frequency and timing synchronization for OFDM. *IEEE Trans. Commun.*, **45**:1613-1621. [doi:10.1109/26.650240]

Zhao, Z.W., Li, S.J., 2004a. A Synthetic Carrier Frequency Offset Estimation for OFDM-based WLANs. Proceeding of the Fourth IASTED International Multi-conference on Wireless and Optical Communications, Banff, Canada. p.424060-424065.

Zhao, Z.W., Li, S.J., 2004b. A Novel Blind Estimation Algorithm of Timing Offset for PCC-OFDM System. Proceeding of the Fourth IASTED International Multi-conference on Wireless and Optical Communications, Banff, Canada, p.424065-424072.