



Floating-roof steel tanks under harmonic settlement: FE parametric study and design criterion*

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Abstract: Large vertical steel tanks for fluid storage are usually constructed on soft foundations, so it is not surprising that the tank wall will settle unevenly with the settlement of the foundation, thus inducing deformations and stresses in the tank. This work investigates the linear static behavior of floating-roof tanks under harmonic settlement through finite element (FE) analyses. The influences of the radius-to-thickness ratio, the height-to-radius ratio and the wind girder stiffness on the structural behavior are first analyzed. Comparisons between the circumferential stresses in the wind girder and the vertical stresses in the tank bottom are then made. The displacement and the stress along the tank height are also discussed, and the concept of tank division along its height is presented. Finally, a design approximation for the radial displacement at the tank top is developed based on FE results, and a settlement criterion based on the top radial displacement is proposed which can be used in practical design.

Key words: Steel tanks, Floating-roof, Differential settlement, Harmonic settlement, Finite element analysis, Design criterion

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INTRODUCTION

Vertical cylindrical steel tanks widely used for fluid storage generally consist of a thin bottom plate, a cylindrical shell, and a fixed or floating roof. Large tanks constructed on soft foundations are susceptible to various types of settlement deflections. The settlement components are: uniform settlement, planar tilt and differential settlement. The uniform settlement and planar tilt cause rigid-body deflection or rotation of the tank, and so, are of relatively little importance (Marr *et al.*, 1982; Kamyab and Palmer, 1989; Palmer and Ceng, 1992). The differential settlement is usually minimal but leads to serious consequence for the tank structure (Marr *et al.*, 1982; Jonaidi and Ansourian, 1996; 1998). Even minimal differential settlement under the tank wall will induce

large distortion along the tank top and high stresses at the tank bottom or in the top wind girder. The consequences caused by the differential settlement beneath the tank wall are usually as follows: (a) Tank top ovality which impedes free motion of the floating roof and may cause the buckling of the top part of the tank (Malik *et al.*, 1977; Marr *et al.*, 1982; Kamyab and Palmer, 1989; Palmer and Ceng, 1992); (b) High vertical local compressive stresses developing at the tank bottom that induce local compressive buckling (Godoy and Sosa, 2003); (c) High circumferential stresses developing in the primary wind girder that induce buckling (even yielding) of the wind girder; (d) High stresses developing in the tank bottom that induce rupture of the lap-welded bottom plate (Godoy and Sosa, 2003); (e) Plasticity which may occur in parts of the tank wall.

To analyze the effect of differential settlement on the structural behavior of the tank, the settlement beneath the tank wall is often expressed as a Fourier series in harmonics:

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$$u = u_0 + \sum_{n=1}^k u_n \cos(n\varphi + \phi_n) \quad (0 < \varphi < 2\pi). \quad (1)$$

Each harmonic settlement beneath the tank wall is considered to obtain the displacements and the stresses of the tank. The displacements and stresses resulting from each harmonic component can then be superimposed to obtain the complete solution.

In order to establish the expression for displacements and stresses (mainly radial displacements at the top and vertical stresses at the bottom) of the floating-roof tank under harmonic settlement, three shell theories ranging from inextensional theory, membrane theory to modified Donnell theory were applied. Kamyab and Palmer (1989) derived three analytical solutions based on these theories and discussed the validity of each solution. They presented three critical wavenumbers n_1, n_2, n_3 ($n_1 = [36(1 - \nu^2)]^{1/8} \times (r/h)^{1/2} (r/t)^{1/4} (1/I_{ratio})^{1/8}$, $n_2 = [48(1 - \nu^2)]^{1/8} (r/h)^{1/2} (r/t)^{1/4}$, $n_3 = 1.59 I_{ratio}^{1/8} n_1$), and all three solutions are only applicable to a certain range of harmonic wavenumbers. Palmer (1994) carried out a simple parametric analysis based on the membrane solution. Jonaidi and Ansourian (1996; 1998) made comparisons between the analytical solutions and the FE solutions for uniform and tapered wall thickness. They concluded that the circumferential stresses at the top wind girder govern the tank design.

It has long been recognized that the differential settlement beneath the tank wall is harmful to the tank structure and is the main cause for the collapse of practical tanks. The many criteria proposed for the differential settlement of large steel tanks were summarized by Marr *et al.* (1982) as follows: $\Delta S < (r/2h)[w]_{allow}$ suggested by Lambe and Penman; $\Delta S \leq L/450$ by Debeer; $S_i \leq (L^2/10hr)[w]_{allow}$ by Langeveld; $\Delta S \leq (L^2/2hr)[w]_{allow}$ by Malik *et al.*; $S_i \leq 11f_y L^2/Eh$ by Marr *et al.* In the above expressions, S_i is the out of plane settlement of the datum point i ; $\Delta S_i = S_i - 0.5(S_{i+1} + S_{i-1})$; $L = 4\pi r/k$ is the arc distance between successive data points $i-1, i$, and $i+1$. All these settlement criteria for the maximum acceptable settlement of steel tanks were proposed by geotechnical engineers by empirical or semi-empirical methods, or were derived from analogy with other structural criteria. Most of the criteria are independent of the tank geometry and therefore seemed unsatisfactory from the viewpoint of the structural engi-

neering. A reasonable criterion should be the one that incorporates the tank geometry and based on the structural response.

This work investigates the linear static behavior of floating-roof tanks under harmonic settlement through finite element analyses. The influences of the harmonic wavenumber and various geometric parameters (including radius-to-thickness ratio, height-to-radius ratio and the bending rigidity of the wind girder) were analyzed. A design equation for estimating the radial displacement at the tank top was developed by regression based on the FE results, and a simple criterion based on harmonic settlement analyses was proposed.

FINITE ELEMENT MODEL

All analyses in this study were carried out using the general purpose finite element package ANSYS. For a tank under harmonic settlement, both the complete model retaining the whole tank shell and the half-wave sector model retaining only a circumferential central angle π/n of the shell (Fig.1, n is the harmonic wavenumber) may be adopted. Both models lead to identical results for linear elastic analysis, and the half wave model was used in this study due to its high efficiency.

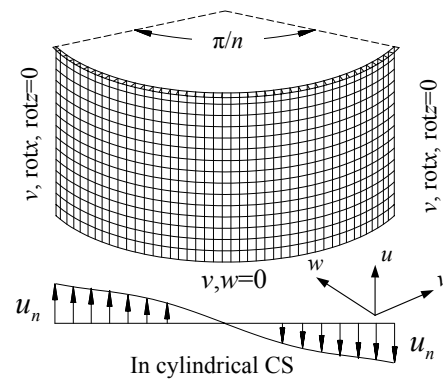


Fig.1 Half-wave FE model of cylindrical tanks

A wind girder is often provided at the tank top to strengthen the open top. A wind girder in the form of an annular plate with width h_g and height t_g is considered in this study. The circumferential bending rigidity of the wind girder is then expressed as $I_g = h_g^3 t_g / 12$. A dimensionless parameter I_{ratio} may be

defined as the ratio of I_g to the bending stiffness of the shell $D(=ht^3/[12(1-\nu^2)])$.

For the tank with a floating roof, a flexible seal is usually provided in the space between the tank shell and the edge of the roof, so that the roof provides little restraint on the tank shell, and the influence of the roof on the tank wall can be ignored. The n th harmonic settlement $u=u_n\cos(n\varphi)$ ($0\leq\varphi\leq\pi/n$) is imposed beneath the tank wall. The bottom edge is simply supported, and symmetric boundary conditions are applied on the cut edge.

Thin shell element SHELL63 which has three displacement and three rotational degrees-of-freedom (DOF) is used in the shell and the wind girder meshing. The material of the tank is assumed to have properties typical of steel: an elastic modulus E of 210 GPa and a Poisson's ratio ν of 0.3.

PARAMETRIC ANALYSES

This section presents the results of a large FE parametric study on the linear static response of floating-roof tanks under harmonic settlement. Harmonic vertical translations in range $n=2\sim 24$ were imposed. The parametric analyses covered a wide range of tank geometry: radius-to-thickness ratio $r/t=500\sim 2000$, height-to-radius ratio $h/r=0.5\sim 2$, and wind girder stiffness $I_{\text{ratio}}=5\sim 60$.

Radius-to-thickness (r/t) ratios

Fig.2 shows the variations of the radial displacement with the harmonic number n for tanks of $h/r=1$, $I_{\text{ratio}}=20$ and various r/t ratios. Three critical wavenumbers were $n_1=6$, $n_2=9$ and $n_3=14$ for $r/t=1000$, and are $n_1=7$, $n_2=11$ and $n_3=16$ for $r/t=2000$. The radial displacement at the tank top increased with increasing n and reached the maximum value at the first critical wavenumber n_1 , then fell rapidly when $n>n_1$, and tended to vanish when $n>n_3$ (Fig.2a). It was observed that the maximum radial displacement did not always occur at the tank top. For $n<n_1$, the maximum radial displacement of the tank shell usually occurred in the region near the tank top, but usually occurred in the middle or lower region when $n>n_1$. For all wavenumbers, the radial displacement of the tank top reached the maximum value at n_1 while that of the whole tank shell reached

the maximum value at n_3 , with the two values differing much. For a typical geometry of $r/t=1000$, $h/r=1$ and $I_{\text{ratio}}=20$, $w_{n_3,\text{max}}$ of the tank shell is about 1.5 times w_{n_1} at the tank top. With increasing n , transfer of the axial translations beneath the tank wall to the tank top will become more and more difficult. On the other hand, due to the radial restraints from the bottom plate, induction of a large radial displacement in the low region of the tank is unlikely, and the maximum radial displacement of the whole tank shell will not always increase with the increasing n . When $n>n_3$, the maximum radial displacement occurring in the region near the tank bottom begins to decrease (Fig.2b).

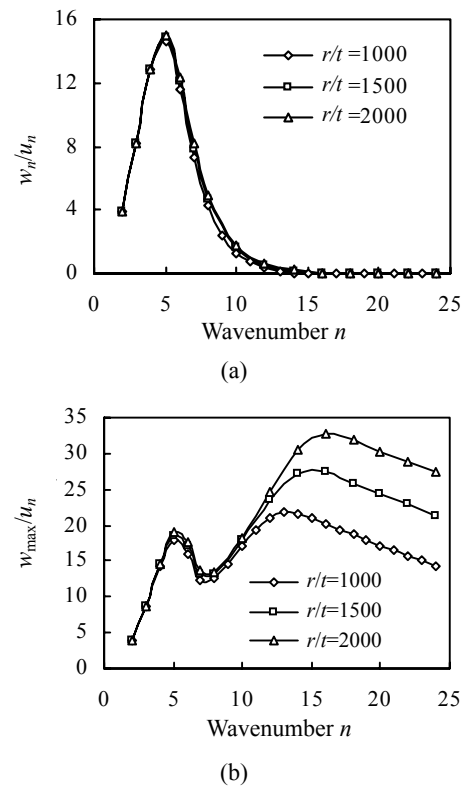


Fig.2 Radial displacement for tanks with various r/t ratios ($h/r=1$, $I_{\text{ratio}}=20$). (a) At the tank top; (b) On the tank wall

Fig.3 shows the variations of the axial stress with n for tanks with various r/t ratios ($h/r=1$, $I_{\text{ratio}}=20$). The axial stress at the tank bottom increases with the increasing n , but is usually very low for $n<n_1$. The axial stress reaches the maximum value in the region near the tank top for $n<n_1$. For harmonic numbers n_1 to n_2 , the maximum axial stress usually occurs in the middle region of the tank shell. When $n>n_3$, the axial

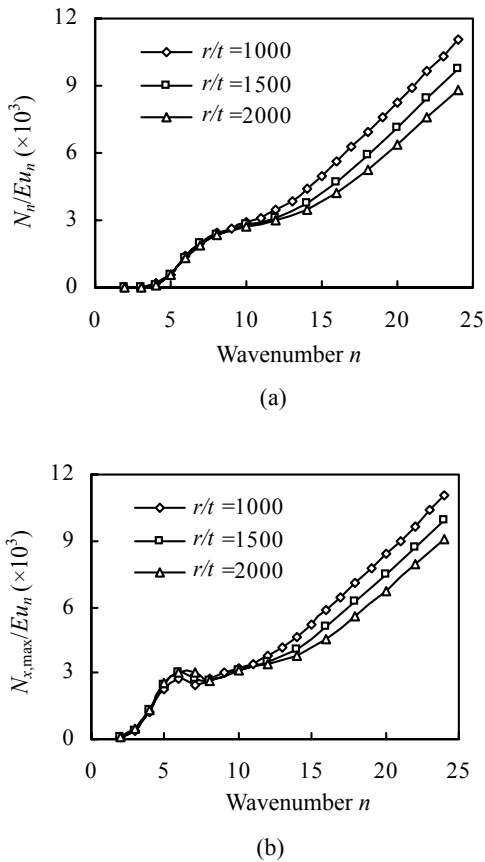


Fig.3 Axial stress for tanks with various r/t ratios ($h/r=1$, $I_{ratio}=20$). (a) At the tank bottom; (b) On the tank wall

stress reaches the maximum value at the tank bottom. For floating-roof tanks under harmonic settlement, the radial displacement at the top and the vertical stress at the bottom are two important structural responses. Variations of these two responses with the r/t ratio are plotted in Figs.4 and 5 respectively for two selected harmonic numbers (Fig.4 for a small number $n=4$ and Fig.5 for a relatively large number $n=8$). Four sets of wind girder stiffness I_{ratio} are included to cover a wide range of tank geometry. With the increase of the r/t ratio, the radial displacement at the top increases and the vertical stress at the bottom decrease at both harmonic numbers. For a tank geometry of $h/r=1$ and $I_{ratio}=5$, the top radial displacement for $r/t=2000$ is about 1.2 times that for $r/t=1000$ when $n=4$, and this value becomes as much as 6 when $n=8$. The bottom vertical stress for $r/t=500$ is about 7.5 times that for $r/t=2000$ when $n=4$, and 1.6 times when $n=8$. This indicates that the tank behavior is quite sensitive to the r/t ratio.

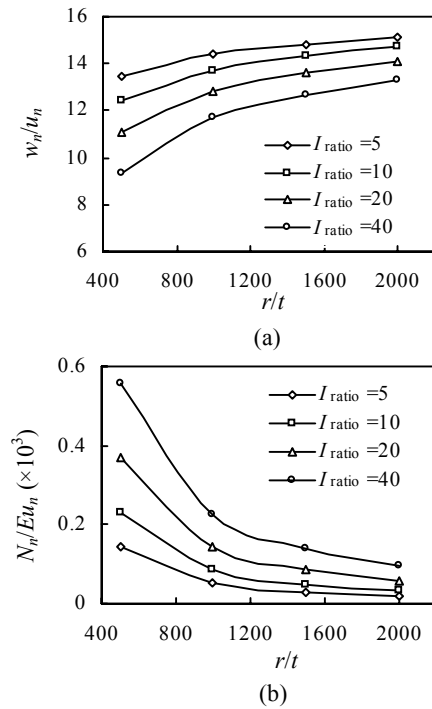


Fig.4 Variations of displacement and stress with r/t ratios for $n=4$. (a) Radial displacement at the top; (b) Vertical stress at the bottom

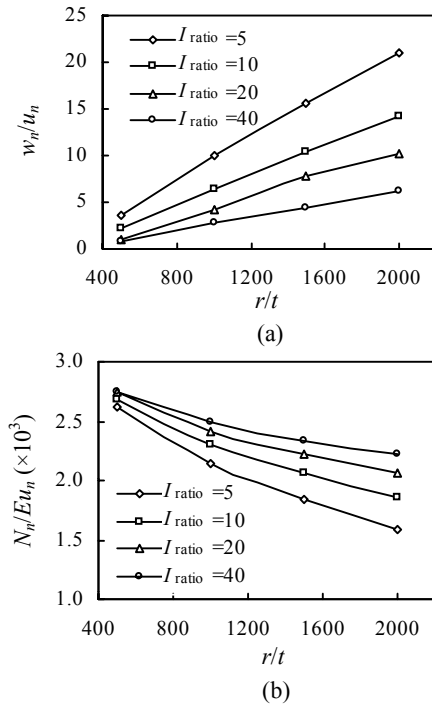


Fig.5 Variations of displacement and stress with r/t ratios for $n=8$. (a) Radial displacement at the top; (b) Vertical stress at the bottom

Height-to-radius (h/r) ratios

The influence of the height-to-radius (h/r) ratio is more complicated. Generally, for two different height tanks, the taller tank induces larger top radial displacement and larger bottom vertical stress for n below n_1 of the taller tank, while inducing smaller displacement and smaller stress for n above n_1 of the shallower tank. For n larger than n_3 , both the displacement and the stress are almost independent of the tank height.

Variations of the top radial displacement and the bottom vertical stress with the h/r ratio are plotted in Figs.6 and 7 respectively for $n=4$ and $n=8$ ($r/t=1000$). For $n=4$ (Fig.6), both the displacement and the stress increase obviously as the h/r ratio increases. For $n=8$ (Fig.7), the displacement decreases significantly as the h/r ratio increases, while the stress increases with the increasing h/r ratio and soon reaches its maximum value, then falls significantly, because stresses or deformations are governed by not only the h/r ratio but also the harmonic number and r/t ratio. For tank geometry of $r/t=1000$ and $I_{ratio}=10$, the top radial displacement for $h/r=2$ is about 2.6 times that for $h/r=0.5$ when $n=4$, and this value becomes as small as 0.07 when $n=8$. The vertical stress at the bottom for $h/r=2$ is about 25 times that for $h/r=0.5$ when $n=4$. This indicates that the tank behavior is also sensitively affected by the h/r ratio.

Bending rigidity of the wind girder (I_{ratio})

In general, for wavenumbers $n < n_3$, the smaller circumferential stiffness of the wind girder induces larger top radial displacement and smaller bottom vertical stress; while for $n > n_3$, both the displacement and the stress are almost independent of the stiffness of the wind girder.

Variations of the top radial displacement and the bottom vertical stress with the wind girder stiffness I_{ratio} are plotted in Figs.8 and 9 respectively for $n=4$ and $n=8$ ($h/r=1$). Four sets of r/t ratios are included to cover a wide range of tank geometry. At both harmonic numbers, the radial displacement at the top decreases and the vertical stress at the bottom increases as the I_{ratio} increases, but both the displacement and the stress tend to a constant for the I_{ratio} larger than a certain value, particularly when $n=8$. This indicates that, for a relatively small I_{ratio} , the variation of the wind girder stiffness leads to signifi-

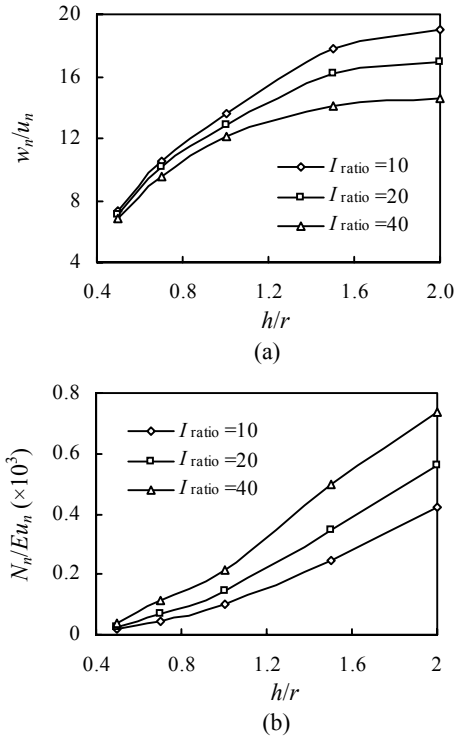


Fig.6 Variations of displacement and stress with h/r ratios for $n=4$. (a) Radial displacement at the top; (b) Vertical stress at the bottom

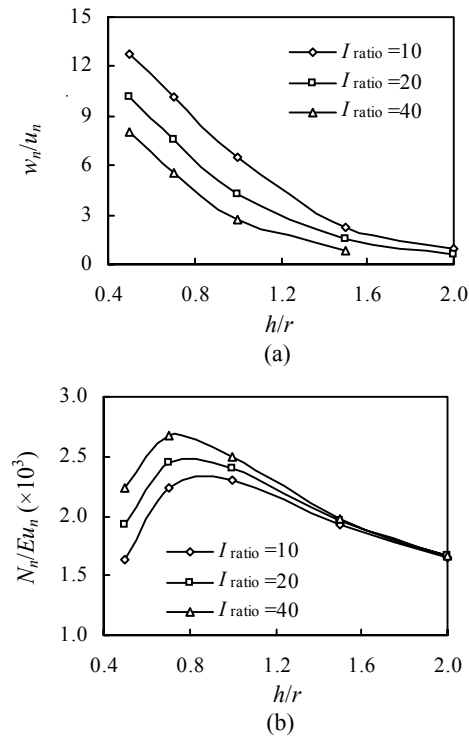


Fig.7 Variations of displacement and stress with h/r ratios for $n=8$. (a) Radial displacement at the top; (b) Vertical stress at the bottom

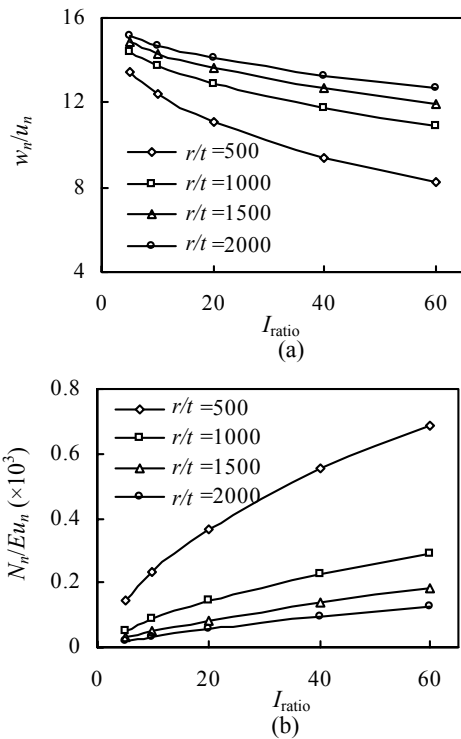


Fig.8 Variations of displacement and stress with I_{ratio} for $n=4$. (a) Radial displacement at the top; (b) Vertical stress at the bottom

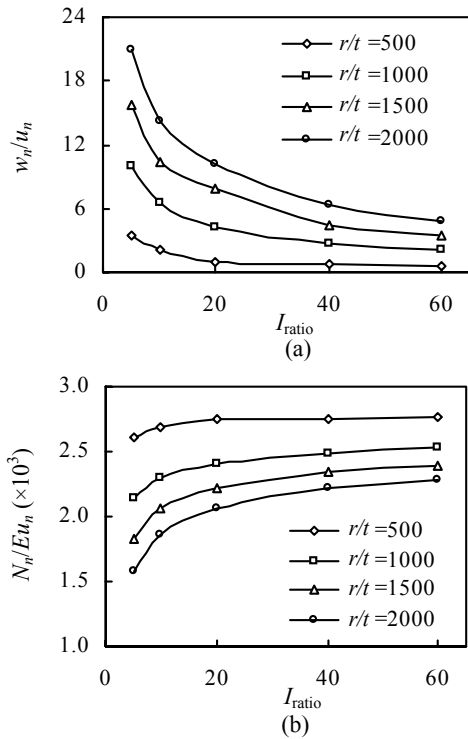


Fig.9 Variations of displacement and stress with I_{ratio} for $n=8$. (a) Radial displacement at the top; (b) Vertical stresses at the bottom

cant change in the structural behavior; while for a quite large I_{ratio} , which means that a rigid wind girder is provided, the displacement and the stress are almost independent of the wind girder stiffness.

CIRCUMFERENTIAL STRESS IN WIND GIRDER

The analytical solution of the maximum circumferential stress in the primary wind girder given by Palmer and Ceng (1992) is

$$\sigma_y = \frac{E(n^2 - 1)d}{r^2} w_n, \tag{2}$$

where, d is the distance of the outer edge from the centroid of the annular wind girder ($d=0.5h_g$ here). For a typical geometry of $r/t=1000$, $h/r=1$ and $I_{ratio}=20$, if the thickness of the annular plate $t_g=t$, then its width $h_g=28.01t$. Fig.10 shows the comparison between the FE solution and the analytical solution. The analytical solution agrees well with the FE solution excluding $n=12\sim 16$.

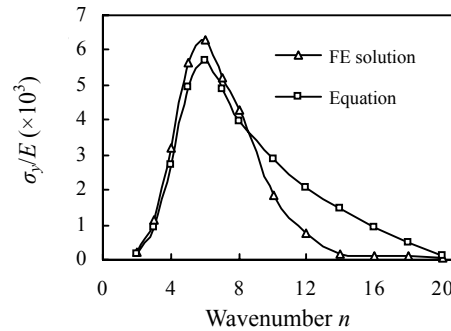


Fig.10 Comparison of circumferential stress in wind girder between FE and analytical solutions

Comparisons between the vertical stress at the tank bottom and the circumferential stress in the wind girder were also made for the above typical geometry (Fig. 11). It was found that the circumferential stresses in the wind girder are much higher than the vertical stresses at the tank bottom for small wavenumbers. So the circumferential stresses in the wind girder will govern the tank design. When n become larger, the circumferential stresses in the wind girder tend to vanish, but the vertical stresses at the tank bottom increase rapidly, in which case the vertical stresses at the tank bottom govern the design. The vertical

compressive stresses at the tank bottom may be high enough to cause local buckling of the tank.

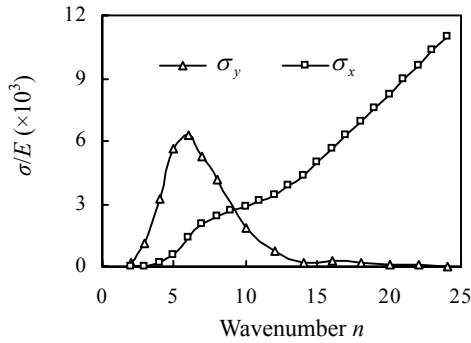


Fig.11 Comparison between vertical stress at tank bottom and circumferential stress in wind girder

TANK EFFECTS ALONG THE HEIGHT

As mentioned above, the maximum radial displacement of the tank shell does not always occur at the top, and the maximum vertical stress does not always occur at the bottom. The maximum displacement for each harmonic number occurs in a certain location for a given tank geometry. For a typical tank geometry of $r/t=1000$, $h/r=1$ and $I_{ratio}=20$, three critical wavenumbers are $n_1=6$, $n_2=9$ and $n_3=14$. Figs.12a and 12b show the radial displacement and the vertical stress along the tank height at the three critical harmonic numbers.

For the three critical harmonic numbers, the maximum radial displacements occur in three critical heights of the tank, which are the first critical height h_{n_1} , the second critical height h_{n_2} and the third critical height h_{n_3} . Four areas along the tank height according to these critical heights can then be defined, which are the upper area between the top and h_{n_1} , the middle-upper area between h_{n_1} and h_{n_2} , the middle-lower area between h_{n_2} and h_{n_3} , and the lower area between h_{n_3} and the bottom (Fig.13). For the tank of $r/t=1000$, $h/r=1$ and $I_{ratio}=20$, three critical heights obtained from FE analysis are $h_{n_1} = 0.94h$, $h_{n_2} = 0.47h$, $h_{n_3} = 0.33h$.

The maximum radial displacement occurs in the upper area when $n < n_1$, in the middle-upper area when $n_1 < n < n_2$, in the middle-lower area when $n_2 < n < n_3$, and

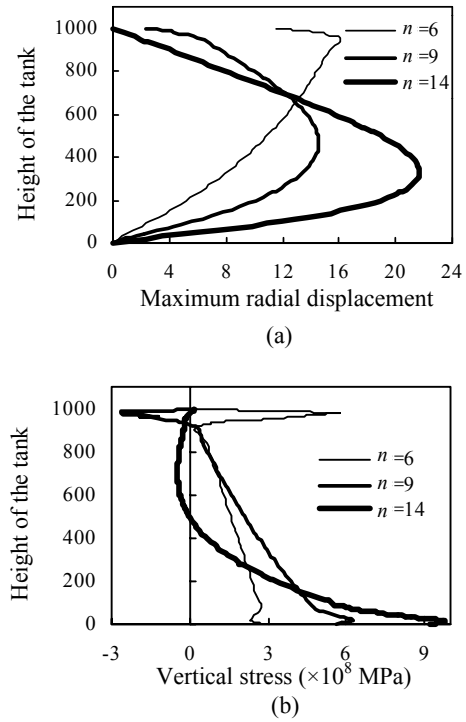


Fig.12 Radial displacement (a) and vertical stress (b) along tank height

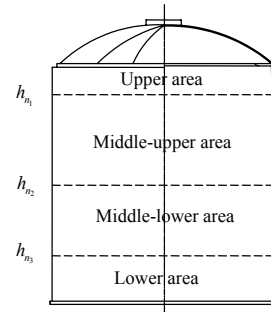


Fig.13 Divisions of the tank shell

occurs in the lower area when $n > n_3$. As mentioned above, the radial displacement reaches the maximum value in the tank shell when $n=n_3$, that is, the maximum displacement occurs at h_{n_3} . A second wind girder can thus be applied at the height h_{n_3} to strengthen the tank. For example, for tank geometry of $r/t=1000$, $h/r=1$ and $I_{ratio}=20$, when a secondary wind girder of $I_{ratio}=20$ is applied at h_{n_3} , the maximum radial displacement reduces to only 38.66% of that without a secondary wind girder.

When $n < n_1$, the axial stress at the tank bottom is usually very small, and the maximum vertical stress occurs at the local region near the tank top which may

lead to buckling of this region. When $n_1 < n < n_3$, the maximum axial stress usually occurs in the middle region of the tank shell. When $n > n_3$, the maximum axial stress occurs at the tank bottom, and rapidly increases with increasing n . The stress for large n is usually high enough to lead to local compressive buckling or yielding of the lower area of the tank.

Therefore, both the radial deformation and the vertical stress at the tank top are mainly affected by the low harmonic settlement, and the vertical stress and the displacement in the region near the tank bottom are mainly affected by the high harmonic settlement, although large radial displacements are unlikely to exist in the region near the tank bottom due to the radial restraint from the bottom plate.

The concept of the tank division along its height is of much significance for practical tanks. Harmonic settlements from Fourier decomposition of the measured settlement for the tank can be obtained for predicting the displacement and the stress in the tank wall. One can then pre-estimate the most serious location in the tank shell, and some measures can be taken for local strengthening of the tank.

SETTLEMENT CRITERION

Design equation based on the FE analyses

The analytical solutions in (Kamyab and Palmer, 1989) seem lengthy and inconvenient for use. A relatively simple design approximation for estimating the radial displacement at the tank top is developed by the regression analysis on the FE results. The fitted curve is given by:

$$\frac{w_n}{u_n} = 9(1 - \nu^2)e^{-0.075n^2 + 0.76n} \left(\frac{h}{r}\right)^{0.5} \left(\frac{t}{r}\right)^{0.15} \left(\frac{1}{1 + 0.02I_{ratio}}\right) \tag{3}$$

The above equation contains the important geometry parameters including r/t , h/r and I_{ratio} . Fig.14 compares the fitted curve and the FE results for various typical tank geometries. The FE results are close to and generally below the design curve, indicating that the design approximation is appropriate and conservative and can be used for determining the top radial displacement for floating-roof tanks under harmonic settlement.

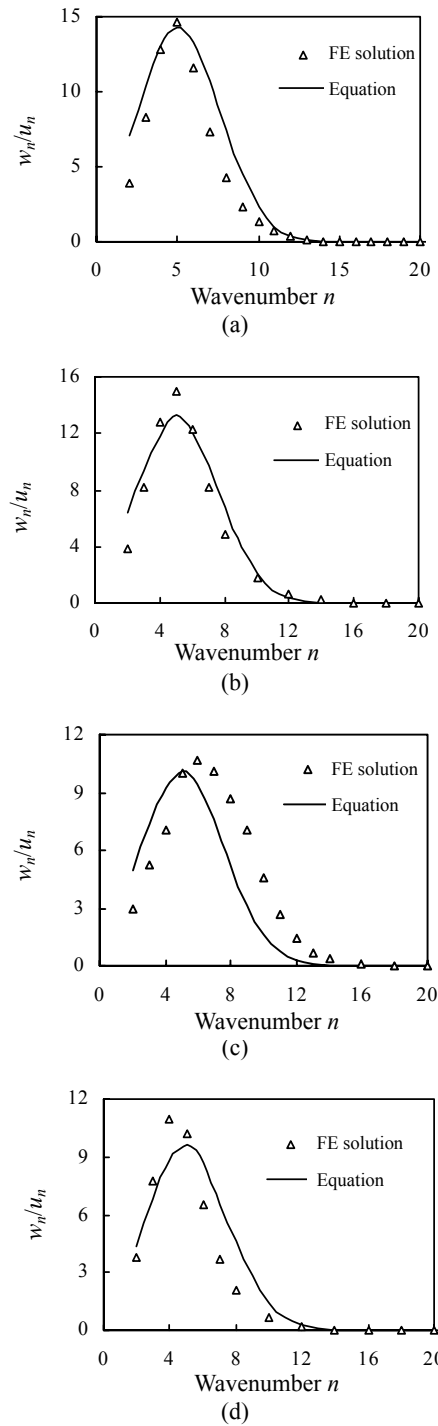


Fig.14 Comparison between the fitted curve and the FE results for various tank geometries

- (a) $r/t=1000, h/r=1, I_{ratio}=20$; (b) $r/t=2000, h/r=1, I_{ratio}=20$;
- (c) $r/t=1000, h/r=0.5, I_{ratio}=20$; (d) $r/t=1000, h/r=1, I_{ratio}=60$

Settlement criterion based on top radial displacement

For a floating-roof tank, the radial displacement

at the tank top should satisfy:

$$\lambda[w] < [w]_{\text{allow}}, \quad (4)$$

where, λ is a safety factor >1 ; $[w]$ is the equivalent radial displacement on the tank top; $[w]_{\text{allow}}$ is the allowable space between the edge of the roof and the tank shell for a floating-roof tank, which is specified as ± 100 mm in the Chinese code (GB 50341, 2003).

The top radial displacement under each harmonic settlement can be superimposed at the phase angle $\phi = \phi_k$, in which k is the harmonic number corresponding to the largest amplitude of the harmonic settlement, so $[w]$ is given by

$$[w] = w_k + \sum_{i=2}^n \cos(\phi_i - i\phi_k) w_i, \quad i \neq k. \quad (5)$$

After substituting Eqs.(3) and (5) into Eq.(4), the settlement criterion may be expressed by the amplitude and the corresponding phase angle of each harmonic settlement:

$$\lambda \left(\eta_k u_k + \sum_{i=2}^n \eta_i u_i \right) < [w]_{\text{allow}} (r/h)^{0.5} (r/t)^{0.15} \times (1 + 0.02 I_{\text{ratio}}) / 9(1 - \nu^2). \quad (6)$$

where, η_i is the combined coefficient for settlement effects, $\eta_i = e^{-0.075i^2 + 0.76i} \cos(\phi_i - i\phi_k)$, $i=2,3,\dots,n$.

Another settlement criterion may be developed from the superimposed critical condition of the von Mises equivalent stresses at the tank bottom, which is much more complicated and is not given here.

CONCLUSION

This work investigated the linear static behavior of floating-roof steel tanks under harmonic settlement. The following conclusions can be drawn from this study:

1. The deformation and the stress of the tank vary with the tank geometries including the radius-to-thickness ratio, the height-to-radius ratio and the wind girder stiffness.
2. For harmonic settlement with small wavenumbers, the circumferential stress in the wind girder governs the tank design; while for high wavenumbers, the vertical stress at the tank bottom governs the design.

3. The maximum radial displacement usually occurs in the region near the tank top for small wavenumbers. With the increase of n , to transfer the settlements beneath the tank wall to the top of the tank is difficult. So the radial displacement reaches the maximum in the region near the bottom for high wavenumbers.

4. The concept of the tank division along its height is of much significance for practical design of tanks. One can pre-estimate the most serious location in the tank shell, so that some measures can be taken for local strengthening of the tank.

5. A design approximation for the radial displacement at the tank top is developed based on FE results, and a simple settlement criterion based on the top radial displacement is proposed.

6. Further study should be carried out on the buckling behavior of steel tanks under differential settlement. The effects of nonlinearity and initial imperfections should be taken into account.

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