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A new optimization algorithm based on chaos*

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Abstract: In this article, some methods are proposed for enhancing the converging velocity of the COA (chaos optimization algorithm) based on using carrier wave two times, which can greatly increase the speed and efficiency of the first carrier wave's search for the optimal point in implementing the sophisticated searching during the second carrier wave is faster and more accurate. In addition, the concept of using the carrier wave three times is proposed and put into practice to tackle the multi-variables optimization problems, where the searching for the optimal point of the last several variables is frequently worse than the first several ones.

Key words: Chaos optimization algorithm (COA), Carrier wave two times, Multi-variables optimization, Carrier wave triple frequency

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INTRODUCTION

Chaos (Wang *et al*., 2002) is a common nonlinear phenomenon, whose action is complex and similar to that of randomness. The characteristics of chaos being highly sensitive to the initial value of chaos make a world of differences due to the ergodic property of the phase space—chaos can go through all states in certain ranges without repetition; the inherent randomness of the system—means that chaos behavior is similar to randomness which is disorderly; but at the same time it is generated according to the deterministic iteration formula. With these characteristics of chaos, we can apply it in optimization calculation.

At present there are mainly two methods that apply chaos for optimization. One is the algorithm using carrier wave two times (Li and Jiang, 1997); the other is the algorithm of variable dimension (Zhang *et al*., 1999). But their basic principles are the same. Both are based on the ergodic property orbit generated by deterministic iteration formula. While certain stopping conditions are satisfied, we think that the best state during searching is close to the problem's optimal solution (So long as the orbit is long enough, this situation can always be realized), and makes this the next starting point (Choi and Lee, 1998) of the sophisticated search. The carrier wave algorithm realizes the search through chaos variables whose ergodic boundary is very small and close to this optimal point, while the variable dimension algorithm realizes accurate search through narrowing the new solution space produced constantly and by producing the disturbance near the optimal point. The two algorithms used in the author's numerical experiment are very useful although the speed of searching for the optimum point has little differences in different problems, but the first algorithm can be controlled more easily in the optimum problem of multi-variables within large range. So in this article we will discuss the first algorithm only.

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CHAOS OPTIMUM ALGORITHM BASED ON USING CARRIER WAVE TWO TIMES AND ITS IMPROVEMENT

Question description

Chaos optimum of this article is mainly the solution of the continuous variable non-linear constraint question. The question can be described as follows:

$$\begin{aligned} &\min f(\boldsymbol{X}), \\ \text{s.t. } &g_i(\boldsymbol{X}) \le 0,\ i=1,2,\ldots,C_n, \\ &h_j(\boldsymbol{X}) = 0,\ j=1,2,\ldots,C_e. \end{aligned} \tag{1}$$

Among them, $\boldsymbol{X}\in\mathbb{R}^n$ is the independent variable and $a_i \le X_i \le b_i$ (i=1,2,…,n) where a_i, b_i are respectively the lower and upper limits of the ith variable. $f(\boldsymbol{X})$ is the objective function to be optimized. Two formulas after s.t. are equality constraint and inequality constraint respectively. C_n and C_e quantify the equality and inequality constraints respectively. Obviously, besides satisfying the objective function optimum, the given constraint condition must also be satisfied. There may be some unfeasible solutions in the solution set. We can adopt two methods for treating this problem. One requires that the new optimum point searched by the algorithm at each time must satisfy the given constraint; the other must allow inclusion of unfeasible solutions but add penalty function and penalize them through objective function. In order to simplify realization of the algorithm, we generally adopt the second method. We can revise the objective function as follows:

$$f_N(\boldsymbol{X}) = f(\boldsymbol{X}) + \sum_{i=1}^{m} R_i\{\max[0, g_i(\boldsymbol{X})]\}^2 + \sum_{j=1}^{n} R_j[h_j(\boldsymbol{X})]^2, \tag{2}$$

where R_i denotes the penalty function with the right part having greater number of ith inequality constraints, R_j denotes the penalty function with the right part having greater number of jth inequality restraints. Generally R_i and R_j may be far higher than the function value of the objective in terms of quantity grade. In this article, R_i and R_j are in the same value set as that of R.

Improvement of optimized algorithm

The chaos optimization is realized through the chaos variable. There are many methods for producing chaos variable. We select the Logistic Mapping method (Li and Jiang, 1997) which is used extensively. Its equation is as follows:

$$z_{k+1} = \mu z_k(1 - z_k), \tag{3}$$

where μ is a control parameter. It is easy to prove that when μ=4, Eq.(3) is totally in chaos state. The mathematical explanation is that all values between 0 and 1 except the fixed point (0.25, 0.5, 0.75) are produced randomly by iteration. Utilizing the chaos characteristic, which is sensitive to the initial value and setting n different initial values between 0 and 1 (except the fixed point) to z_k in Eq.(3), we can get n chaos variables of different orbits. Generally the basic steps of chaos optimization are based on using carrier wave two times (Zhang *et al*., 1999).

Careful investigation on the above-mentioned algorithms showed that the optimization result of this method depends on the quality of the approximate optimum point. An effective method is to combine large scope random search with local meticulous search. The above algorithm actually only searches a single side of the interim optimum. In this paper, some improvements are made on the algorithm:

$$x_{i,k'+1} = x_i^* + \alpha_i(2z_{i,k'+1} - 1). \tag{4}$$

In Eq.(4), α_i can be regarded as the radius of the neighborhood and 0.1 that of the given value range of the corresponding variable, so we can make sure that the search is focused on the neighborhood of the approximate optimum point. The formula can also be applied for local meticulous search for the first and second time by using carrier wave (Zhang *et al*., 1999).

The course of iteration is described below, where X_{k+1} is the next optimal point, X^* is the present optimal point.

If the above solution is accepted;

$$X_{k+1} = X^* + 0.1\alpha_i(2.0z_{i,k+1} - 1); \tag{5}$$

Else

Continue to search in large scope.

In short, the basic idea is that if the previous solution is feasible, then the solution is searched locally in small scope, the center of which is 0.1 that of the current neighborhood until no better solution appears. Then the search is resumed in large scope, so that the speed of achieving the optimum solution is increased.

In the neighborhood areas searching method used in the annealing simulation, the radius α_i of the neighborhood area for carrying out accurate search after the second time, the carrier wave should be decreased a little, which can be done by subtracting it by some small number or by multiplying it by some number which is less than 1.0 during every search time. In this paper, the latter method is used; the formula is:

$$\alpha_{i,k+1} = \lambda\alpha_{i,k+1-N_c}. \tag{6}$$

In the formula, λ is 0 to1, and can be set to 0.99. The subscript 'k+1−N_c' denotes decreasing the neighborhood radius when no better solution can be found at every N_c (≤10 in this paper) step. The lower limit must be fixed in the decreasing course of the neighborhood, which can be stopped when the radius is ≤1.0E−10.

During the numerical value experiment, it was found that the last several variables were always more different from the true optimum than the first several variables and that the nearer the variable was to the end of the variables sequence, the worse the precision was, so the method of conversed adjustment is aimed at this situation, which is:

for (j=n−1; j>n−1−n/3; j−−)
  adjusting $X_{j,k}$ as Step 5.

The numerical value experiment showed that such strategy is effective.

EXAMPLES AND ANALYSIS

The calculation procedure worked out in C++ language according to the algorithm of this paper. To examine the effectiveness of the algorithm we carried out optimization calculation using the following functions. Among them, as function F_5 has too many variables, it is required to use carrier wave three times. Table 1 lists the testing results of the F_1~F_4 functions by the optimization method in this paper. Table 2 lists the results of using carrier wave three times aiming at the F_5 function. It should be pointed out that the results in the tables are from the first of the writer's continuous 10 times of optimization calculation results and not the average one. It was found that all of the ten times optimizations were successful and that the precisions were surprisingly high. We consider optimization successful when the difference between the best objective function values obtained by optimization and the true optimum one was less than 0.1%. Some control parameters chosen during calculation are given as mentioned above.

$$F_1=100(x_2-x_1{}^2)^2+(1-x_1)^2,$$
$$-2.084 \le x_i \le 2.084,\ i=1,2;$$
$$F_2=(x_1^2+x_2^2)^{0.25}\left[\sin^2\left[50(x_1^2+x_2^2)^{0.1}\right]+0.1\right],$$
$$-100 \le x_i \le 100,\ i=1,2;$$
$$F_3=0.5+\frac{\sin^2\sqrt{x_1^2+x_2^2}-0.5}{\left[1.0+0.001(x_1^2+x_2^2)\right]^2},$$
$$-100 \le x_i \le 100,\ i=1,2;$$
$$F_4=-\left[1.0+8.0x_1-7x^2+\frac{7}{3}x_1^3-\frac{1}{4}x_1^4\right](x_2^2\mathrm{e}^{-x_2})(x_3\mathrm{e}^{-(x_3+1)}),$$
$$0 \le x_i \le 10,\ i=1,2,3;$$

Table 1 Calculation results of improved COA based on using carrier wave two times

Function	F_1	F_2	F_3	F_4
The real optimum point	(1,1)	(0,0)	(0,0)	(4,2,1)
Optimum point calculated	(1,1)	(−4.21E−10,1.39E−9)	(2.87E−6,6.06E−6)	(4,2,1)
The real optimum value	0	0	0	−0.464
Optimum value calculated	2.33E−21	5.98E−5	5.23E−11	−0.464
Time consumed (s)	<1	<2	<6	<2
Success rate	100%	100%	100%	100%

Table 2 The results of F_5 by carrier wave three times

Optimum seeking stage	Calculated optimum point	Calculated optimum objective function value	Consuming time from the former stage (s)
The 1st time to use carrier wave	(3.7415,1.85777,0.884278,−1.90331,2.22711, 1.32795,0.329938, −0.512907,−2.23689,1.98294)	31746	<6
The 2nd time to use carrier wave	(0.999999,0.999998,0.999997,0.999994,0.999987, 0.999974,0.999949,0.999898,0.999795,0.999589)	5.61948E−8	148~149
The 3rd time to use carrier wave	(0.999999,0.999998,0.999997,0.999994,0.999987, 0.999974,0.999957,0.999919,0.999839,0.999678)	4.43281E−8	<10

$$F_5=\sum_{i=1}^{9}[100(x_{i+1}-x_i^2)^2+(1-x_i^2)^2],$$
$$-10\le x_i\le 10,\quad i=1,2,...,10.$$

Table 1 shows that using this algorithm in this paper, the speed and precision of the search for optimum value are very excellent.

From Table 2, we can see that the optimum points obtained by using carrier wave two times are worse when the variable is nearer to the end of the variable sequence. After using carrier wave three times we improved the precision of the last several variables. We may not see very obviously from only one time of calculation, but it indicates that the optimum points obtained by using carrier wave two times are worse than the result of calculation using carrier wave three times. Obviously, it is necessary for the last 1/3 of variables to use carrier wave three times in the multi-dimensional optimization problems.

CONCLUSION AND SUGGESTION

The chaos optimization algorithm adopts chaos variable to search and the searching course goes on according to its chaos regularity. The iteration formulae are easy to understand and program. Chaos variable's traversal property ensures that a true optimum solution can be found if allowed to run for sufficient time. Even if the optimization calculation time is limited we can get approximate solution with extremely good precision. Improving the method in this paper can largely improve the optimum seeking speed by using carrier wave two times. For the multi-dimension variable situation, we can use carrier wave three times to find the better approximately optimum solution and improve the poor accuracy of the latter variables by using carrier wave three times.

According to the authors' experience, the optimization result drawn from the chaos optimization is generally superior to that of GA algorithm and AS algorithm (Tokuda *et al*., 1998) and the optimization procedure is easier than that of the other two methods. We can combine and use this method with other methods during practical application and get a series of new methods which can be applied to other large-scale optimization problems.

On the whole, because the optimization procedure is based on chaos searching which is similar to randomness, we should see that chaos optimization requires much more iteration steps and much more time when there are more parameters.

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