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Modelling and stability analysis of emergent behavior of scalable swarm system*

CHEN Shi-ming†[1,2], FANG Hua-jing[1]

([1]*Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China*)

([2]*School of Electrical and Electronic Engineering, East China JiaoTong University, Nanchang 330013, China*)

†E-mail: c1977318@hotmail.com

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Abstract: In this paper we propose a two-layer emergent model for scalable swarm system. The first layer describes the individual flocking behavior to the local goal position (the center of minimal circumcircle decided by the neighbors in the positive visual set of individuals) resulting from the individual motion to one or two farthest neighbors in its positive visual set; the second layer describes the emergent aggregating swarm behavior resulting from the individual motion to its local goal position. The scale of the swarm will not be limited because only local individual information is used for modelling in the two-layer topology. We study the stability properties of the swarm emergent behavior based on Lyapunov stability theory. Simulations showed that the swarm system can converge to goal regions while maintaining cohesiveness.

Key words: Two-layer emergent model, Stability, Emergent behavior, Swarm
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INTRODUCTION

There is increasing interest in the dynamics and control of group behavior of intelligent vehicular swarms, for example, systems of multiple unmanned air vehicles (UAVs). Research activity on this topic has benefited from the participation of experts from areas as diverse as biology, physics and engineering. Familiar examples of group behavior in biology, such as herds of animals, flocks of birds, and swarms of insects, have served as an important motivating factor in this research theme. There is great interest to learn from these naturally occurring examples so that engineered group behavior can be achieved that has some desirable properties exhibited in nature, such as emergent behavior, which means global goal-oriented behavior resulting from simple local individual behavior rules.

Much progress in modelling and exploring the collective dynamics has been made in biology, physics and engineering (Aldana and Huepe, 2003; Breder, 1954; Chu *et al*., 2003; Couzin *et al*., 2002; Erdmann and Ebeling, 2003; Gazi and Passino, 2003; 2004; Grunbaum and Okubo, 1994; Jadbabaie *et al*., 2003; Liu *et al*., 2005; Liu and Passino, 2004; Mogilner *et al*., 2003; Okubo, 1986; Parrish *et al*., 2003; Shi *et al*., 2004; Topaz and Bertozzi, 2004; Warburton and Lazarus, 1991). Breder (1954) proposed a simple model composed of constant attraction and repulsion inversely proportional to the square of the inter-individual distance. Warburton and Lazarus (1991) studied the effect on cohesion of a family of attraction/repulsion functions. Okubo (1986) and Grunbaum & Okubo (1994) provided a good background and review of the swarm modelling concepts. Gazi and Passino (2003; 2004) contributed significantly to mathematical modelling and stability analysis of swarm systems focusing on modelling of swarms in n-dimensional space. Their results showed that individuals converge to more favorable regions and diverge from unfavorable regions while forming

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a cohesive swarm. One drawback of the model is that all the individuals need to know the exact relative positions of all other individuals. This is not biologically very realistic, and although in engineering it may be overcome with technology like global positioning system, the production cost will increase exponentially along with the scale of the swarm. In order to overcome this problem, Chen and Fang (2005) proposed a minimal circumcircle method for modelling static aggregating swarm, which is not based on the individual characteristics but (neighbor) sub-swarm characteristics. Chen and Fang (2006) extended the minimal circumcircle method to dynamic foraging swarm with bounded attraction/repulsion function.

In this paper we propose a two-layer emergent topology for scalable swarm system. The first layer describes the emergent characteristics of individual flocking behavior to the local goal position (the center of minimal circumcircle which is decided by the neighbors in the positive visual set of individual) resulting from the individual motion to one or two farthest neighbors in its positive visual set, which is defined as neighbor layer. The second layer describes the emergent aggregating characteristic of swarm resulting from the individual motion to its local goal position (this motion is the emergent result of neighbor layer), which is defined as swarm layer. The scale of the swarm will not be limited by the two-layer topology, and we will give the explanations in the next section.

EMERGENT MODEL IN SWARM LAYER

We first put forward an assumption and several definitions.

Assumption 1 Consider a swarm of m individuals in 2D Euclidean space. We model the individuals as points and ignore their dimensions. The position of individual i of the swarm is described by $\boldsymbol{x}^i \in \mathbb{R}^2$. It is assumed that all individuals know the global goal position and have the same observation ability. That is, individual i can find the exact position of individual j if individual j can find the exact position of individual i. We assume synchronous motion and no time delays.

Definition 1 The "visual set" composed of all individuals in the visual range of individual i is defined as $A_i=\{\boldsymbol{x}^j: \|\boldsymbol{x}^i-\boldsymbol{x}^j\|\le\varepsilon, \forall j\in S, j\ne i\}$, where $S=\{1, 2, \ldots, m\}$ is composed of all individuals in the swarm. ε denotes the visual range of all individuals. And we use 2-norm $\|\boldsymbol{x}^i-\boldsymbol{x}^j\|=\sqrt{(\boldsymbol{x}^i-\boldsymbol{x}^j)^{\mathrm{T}}(\boldsymbol{x}^i-\boldsymbol{x}^j)}$ to describe the distance between individuals i and j.

Definition 2 The "repulsive set" composed of all individuals in repulsive range of individual i is defined as $R_i=\{\boldsymbol{x}^j: \rho<\|\boldsymbol{x}^i-\boldsymbol{x}^j\|<r, \forall j\in S, j\ne i\}$, where r denotes the farthest distance within which repulsion function between individuals can have effect, and the minimal safe distance between individuals is described by ρ.

Definition 3 The "minimal circumcircle" problem is maximum-minimum problem. The mathematical description of minimal circumcircle problem is as follows: note the set composed of all individuals as $S'=\{1, 2, \ldots, n\}$, consider all individuals $\boldsymbol{x}^j$ ($j\in S'$) in the visual range of individual i, and the minimal circumcircle problem is to find a point $\bar{\boldsymbol{x}}_{io}$ to make $\min\max_{j\in S'}(\|\bar{\boldsymbol{x}}_{io}-\boldsymbol{x}^j\|)$ exist.

Definition 4 $A_i^+=\{j: -\pi/2<[\angle(\boldsymbol{x}^i,\boldsymbol{x}^j)-\angle(\boldsymbol{x}^i,\boldsymbol{x}_{\text{goal}})]<\pi/2, j\in A_i\}$ denotes the "positive visual set" of individual i, where $\angle(\boldsymbol{x}^i,\boldsymbol{x}^j)$ denotes the angle between vectors $\boldsymbol{x}^i$ and $\boldsymbol{x}^j$. $\boldsymbol{x}_{\text{goal}}$ denotes the global goal position of the swarm. We select the point $\boldsymbol{x}^i$ as the origin of the coordinate system, the vertical axis is in the direction from $\boldsymbol{x}^i$ to $\boldsymbol{x}_{\text{goal}}$ and the horizontal axis is in the direction 90° clockwise from the vertical axis.

Definition 5 "Mutual observability" of individuals is defined as: for two arbitrary individuals i and j, if there are some individuals k, l, …, $n\in S$ to make $\|\boldsymbol{x}^i-\boldsymbol{x}^k\|\le\varepsilon$, $\|\boldsymbol{x}^k-\boldsymbol{x}^l\|\le\varepsilon$, …, $\|\boldsymbol{x}^n-\boldsymbol{x}^m\|\le\varepsilon$ simultaneously exist or $\|\boldsymbol{x}^i-\boldsymbol{x}^j\|\le\varepsilon$ directly exist, we identify the individuals i and j with mutual observability and refer to the set $S''=\{k,l,\ldots,n\}$ (k, l, …, $n\in S$) as mutually observable chain of individuals i and j.

If some individuals are not mutually observable in the initial distribution of the swarm, the stability analysis of social foraging behavior for the swarm is meaningless because the swarm can be separated into several irrelevant sub-swarms. So, similar to the initial distribution of swarm in (Chen and Fang, 2005; 2006), we will only consider the situation that there exists a mutually observable chain between arbitrary two individuals in the initial distribution of the swarm. With this kind of initial distribution, the swarm layer can be considered as a composition of several related neighbor layers as shown in Fig.1, so we call it a scalable swarm system.

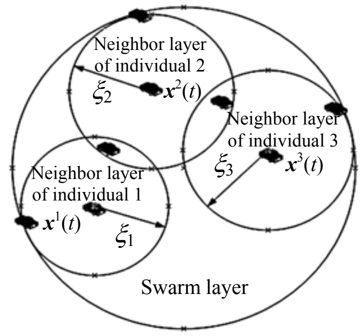

Fig.1 Relationship of swarm layer and neighbor layer

The emergent model of swarm layer is given in this part when Assumption 1 is satisfied. It is assumed that individual i discretely calculates the minimal circumcircle center composed by all individuals in A_i^+ at every sampling time, and keep the center invariable between every two sampling times (it is reasonable because information on the position of other individuals are updated at every sampling time). The motion equation for individual i is given by

$$\dot{\boldsymbol{x}}^i = g_{\mathrm{a}}(\overline{\boldsymbol{x}}_{io+}) + g_{\mathrm{r}}(\min_{j\in R_i} \|\boldsymbol{x}^i - \boldsymbol{x}^j\|),\ i=1,\ 2,\ \ldots,\ m \quad (1)$$

where the attractive function $g_{\mathrm{a}}(\overline{\boldsymbol{x}}_{io+})$ determines the attraction to the local or global object position and takes the following form

$$g_{\mathrm{a}}(\overline{\boldsymbol{x}}_{io+}) = -a(\boldsymbol{x}^i - \overline{\boldsymbol{x}}_{io+}),\ a>0, \quad (2)$$

the repulsive functions $g_{\mathrm{r}}(\boldsymbol{x}^i,\boldsymbol{x}^j)$ determine the repulsion from the other individuals and take the following form

$$g_{\mathrm{r}}(\boldsymbol{x}^i,\boldsymbol{x}^j) = \begin{cases} 0, & r \le \|\boldsymbol{x}^i - \boldsymbol{x}^j\|, \\ b\dfrac{r-\|\boldsymbol{x}^i - \boldsymbol{x}^j\|}{(r-\rho)\|\boldsymbol{x}^i - \boldsymbol{x}^j\|}(\boldsymbol{x}^i - \boldsymbol{x}^j), & \rho < \|\boldsymbol{x}^i - \boldsymbol{x}^j\| < r, \\ b\dfrac{\boldsymbol{x}^i - \boldsymbol{x}^j}{\|\boldsymbol{x}^i - \boldsymbol{x}^j\|}, & \|\boldsymbol{x}^i - \boldsymbol{x}^j\| \le \rho, \end{cases} \quad (3)$$

where $b>0$ (a, b are parameters for the single swarm layer), $\overline{\boldsymbol{x}}_{io+} \in \mathbb{R}^n$ denotes the minimal circumcircle center composed by all individuals in A_i^+; r denotes the farthest distance within which repulsion function between individuals can have effect, and the minimal safe distance between individuals is described by ρ; the model is based on the local individual information because all terms in the model are either predefined or decided by the neighbors in the visual range of individual i. The negative sign before a in attractive function means that individual i moves in the direction to reduce the distance from global object position ($\boldsymbol{x}_{\mathrm{goal}}$) or local object position ($\overline{\boldsymbol{x}}_{io+}$) to individual i when the attractive function has effect. The value of constant a is limited by top speed of real agent. For example, $\max\|\boldsymbol{x}^i - \overline{\boldsymbol{x}}_{io}\|=\varepsilon/2$ exists due to Definition 3, so the value of a should satisfy $a\le 2v_{\max}/\varepsilon$ when we assume that the top speed of individual i is $v_{\max}$. If the global object position is not in the visual range of individual i, individual i will follow the local goal position in A_i^+ to maintain the swarm aggregated; if $A_i^+=\{i\}$ exists or the global object position is in the visual range of individual i, we can consider the global object as a virtual neighbor of individual i and add it into the set A_i^+. Constant b in the repulsive function is used to adjust the equilibrium of individual i when attractive and repulsive functions have effect at the same time, and its influence on equilibrium will be analyzed later.

Stability analysis of emergent behavior in swarm layer

The emergent behavior stability in the swarm layer is specified in Theorem 1.

Theorem 1 Consider individual i described by Eq.(1) with an attractive/repulsive function given in Eqs.(2) and (3). Individual j is in the line between $\boldsymbol{x}^i$ and $\overline{\boldsymbol{x}}_{io+}$. As time goes on between every two sampling time, the individuals will converge to a hyperball

$$B_\theta(\boldsymbol{x}^i,\overline{\boldsymbol{x}}_{io+}) = \{\|\boldsymbol{x}^i - \overline{\boldsymbol{x}}_{io+}\| \le \theta,\ \forall i\in S\},$$

where $\theta=b/a$, individual j is the nearest neighbor in the repulsive range of individual i.

Proof (Chen and Fang, 2006) From the definition of the swarm model, the minimal circumcircle center is invariable between every two sampling time, so we know that $\dot{\bar{\boldsymbol{x}}}_{io+}(t)$=0 between every two sampling time.

Let $\boldsymbol{e}^i=\boldsymbol{x}^i-\bar{\boldsymbol{x}}_{io+}$ and $V_i=(1/2)\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{e}_i$. Therefore, we have

$$\begin{aligned}\dot{V}_i&=(\dot{\boldsymbol{e}}_i)^{\mathrm{T}}\boldsymbol{e}_i=(\dot{\boldsymbol{x}}^i-\dot{\bar{\boldsymbol{x}}}_{io+})^{\mathrm{T}}(\boldsymbol{x}^i-\bar{\boldsymbol{x}}_{io+})\\&=-a(\boldsymbol{x}^i-\bar{\boldsymbol{x}}_{io+})^{\mathrm{T}}(\boldsymbol{x}^i-\bar{\boldsymbol{x}}_{io+})+(g_{\mathrm{r}}(\boldsymbol{x}^i,\boldsymbol{x}^j))^{\mathrm{T}}(\boldsymbol{x}^i-\bar{\boldsymbol{x}}_{io+}).\end{aligned} \tag{4}$$

From the precondition that individual j is in the line between $\boldsymbol{x}^i$ and $\bar{\boldsymbol{x}}_{io+}$. $\dfrac{\boldsymbol{x}^i-\bar{\boldsymbol{x}}_{io+}}{\|\boldsymbol{x}^i-\bar{\boldsymbol{x}}_{io+}\|}$ and $\dfrac{\boldsymbol{x}^i-\boldsymbol{x}^j}{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|}$ denote the same unit vector, so the repulsion force defined in Eq.(3) is bounded by

$$g_{\mathrm{r}}(\boldsymbol{x}^i,\boldsymbol{x}^j)\le b\frac{\boldsymbol{x}^i-\boldsymbol{x}^j}{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|}=b\frac{(\boldsymbol{x}^i-\bar{\boldsymbol{x}}_{io+})}{\|\boldsymbol{x}^i-\bar{\boldsymbol{x}}_{io+}\|},$$

so we have

$$\dot{V}_i\le-a\|\boldsymbol{e}^i\|^2+(b\boldsymbol{e}^i/\|\boldsymbol{e}^i\|)^{\mathrm{T}}\boldsymbol{e}^i=-a\|\boldsymbol{e}^i\|(\|\boldsymbol{e}^i\|-b/a).$$

Therefore, if $\|\boldsymbol{e}^i\|>b/a$, then we have $\dot{V}_i<0$.

Remark 1 We know from Eq.(4) that if there is not any individual in the repulsive range of the individual, then the repulsion function second term in Eq.(4) will not have effect. In this case, the emergent behavior stability is obvious for any positive number θ.

Remark 2 The swarm system can be considered as a kind of hybrid system. Theorem 1 describes the stability characteristic of the continuous state between every two sampling time. The hybrid swarm system convergence based on the discretely switching rule (discretely calculate the minimal circumcircle center at every sampling time) is satisfied because the radius of the circle is decreasing while searching for the characteristic point, and we will give the clear explanation of minimal circumcircle method in the next subsection.

From the item $\theta=b/a$, we know that the repulsive constant b decides the dimension of the swarm whether we consider bounded repulsion force or not. The swarm enlarges when b increases and vice versa.

Introduction of minimal circumcircle method (MCM)

Assume that the swarm initially distributes as shown in Fig.2, in which the solid-line circle describes the visual range of individual 1, and the dashed-line circle describes the minimal circumcircle composed of all individuals in the visual range of individual 1.

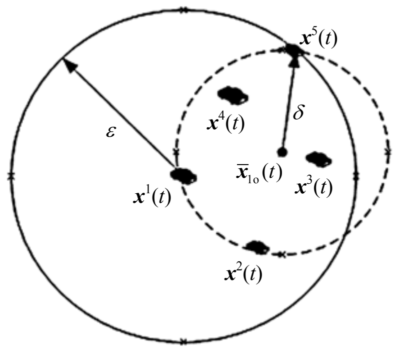

Fig.2 Description of minimal circumcircle method

Let us take the motion of individual 1 as example to illustrate the maintenance of the mutual observability. In Fig.2 we can find $\|\boldsymbol{x}^1-\boldsymbol{x}^5\|=\max\limits_{j\in S'}(\|\boldsymbol{x}^1-\boldsymbol{x}^j\|)$ and $\|\bar{\boldsymbol{x}}_{1o}-\boldsymbol{x}^5\|=\max\limits_{j\in S'}(\|\bar{\boldsymbol{x}}_{1o}-\boldsymbol{x}^j\|)$, S'={1, 2, 3, 4, 5}. From the definition of "minimal circumcircle", we have $\|\bar{\boldsymbol{x}}_{1o}-\boldsymbol{x}^5\|\le\|\boldsymbol{x}^1-\boldsymbol{x}^5\|$. That is, $\delta\le\varepsilon$, which means individual 1 will shorten the distance from itself to its farthest neighbor when the individual is attracted to the center of minimal circumcircle composed by all individuals in its visual range. So we can conclude that individuals can maintain the mutual observability when minimal circumcircle method is used in deriving the individual motion equation.

The algorithm used to solve minimal circumcircle center proposed by Ge and Tang (1996) can be described as follows:

We first need to measure an arbitrary circumcircle that includes m individuals (we can select the centroid of all individuals as the circumcircle center, and take the farthest distance from the centroid to an arbitrary farthest neighbor individual as radius of the circumcircle) as shown in Fig.3.

In Fig.3, O is the center of the arbitrary circumcircle including all m individuals, P_1 is the individual

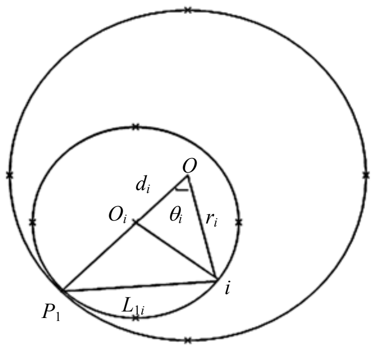

Fig.3 Description of looking for the second characteristic point

farthest from the center (so the circle which takes OP_1 as the radius includes all individuals). Let us move O on the line from O to P_1 (to decrease the radius ϑ). Assume that the distance from O to O_i is d_i when any point i ($i \neq P_1$) touches the circle, we can find the second characteristic point P_2.

Then, if P_1P_2 is the diameter of the arbitrary circle, the algorithm is terminated. Or we will move the circle center to the midpoint of line P_1P_2, just as shown in Fig.4, which means the radius of the circle is decreasing, and we can see that the third characteristic point P_3 we are looking for is not in the arc P_1BP_2, but in P_1AP_2.

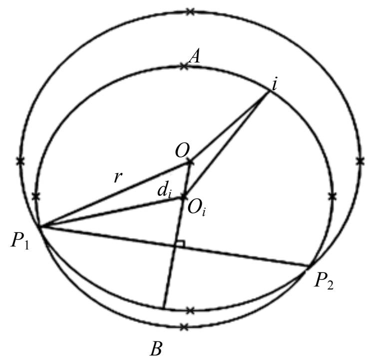

Fig.4 Description of looking for the third characteristic point

At last, we need to discuss the attribute of the triangle composed by the three characteristic points. If it is an obtuse triangle, we will abandon the obtuse point. Then the other two points will be evaluated as P_1 and P_2, and go to the second step until we can find the third characteristic point that together with P_1 and P_2, does not form an obtuse triangle. Because the radius of the circle decreases during searching for the characteristic point, the algorithm is necessarily convergent.

EMERGENT MODEL IN NEIGHBOR LAYER AND STABILITY ANALYSIS

Definition 6 The "positive farthest neighbor" set composed of the farthest individuals in the positive visual range of individual i is defined as

$$O_i^+ = \{1,2,...,k:\ \|\boldsymbol{x}^i - \boldsymbol{x}^k\| = \xi,\ k \in A_i^+,\ \rho < \xi \le \varepsilon\},$$

where ξ denotes the distance between individual i and its farthest neighbors at sampling time (e.g., ξ_1, ξ_2, ξ_3 as shown in Fig.1).

Similarly, the emergent model of neighbor layer is given in this part when Assumption 1 is satisfied. It is assumed that individual i discretely calculates the distance to its farthest neighbors at every sampling time, and that the global goal position is invariable between every two sampling time. The motion equation for individual i is given by

$$\dot{\boldsymbol{x}}^i = \sum_{p=1}^{q} f(\boldsymbol{x}^i - \boldsymbol{x}^p) + f\left(\min_{j \in R_i}(\|\boldsymbol{x}^i - \boldsymbol{x}^j\|)\right), \quad q = 1,\ 2;\ i = 1,\ 2,\ ...,\ m \tag{5}$$

where $\boldsymbol{x}^p \in O_i^+$, q=1 when there is only one element in O_i^+, q=2 when there are two or more elements in O_i^+ and we select the farthest two as $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$.

Remark The two farthest neighbors $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$ are the first and second characteristic points in the minimal circumcircle algorithm described above, and the stability analysis in neighbor layer is defined as the stability of the individual's motion to approach the minimal circumcircle center based on the characteristic points.

The repulsion/attraction function takes the following form

$$f(y) = -y[a_t - b_t \exp(-\|y\|^2 / c_t)], \tag{6}$$

where a_t, b_t, c_t, parameters for two-layer emergent topology, are positive constants and $b_t > a_t$.

Note that this function is attractive for large distances and repulsive for small distances. By equating $f(y)=0$, we can find that the function switches sign at the set of points defined as $\kappa=\{y=0$ or $\|y\|=\delta=\sqrt{c_t \ln(b_t/a_t)}\}$, where δ is the balance distance between the attraction force and repulsion force.

Our first result is that there is not any neighbor in the repulsive range of individual i, which we call a free individual. The emergent behavior stability in the neighbor layer will be specified in Theorem 2.

Theorem 2 Consider individual i described by the model in Eq.(5) with an attractive/repulsive function given in Eq.(6) as a free individual. As time goes on, the individual will converge to a hyperball

$$B_\delta(\boldsymbol{x}^p)=\left\{\boldsymbol{x}^i:\ \left\|\boldsymbol{x}^i-\left(\sum_{p=1}^{q}\boldsymbol{x}^p\right)\Big/q\right\|\le\delta\right\},\ q=1 \text{ or } 2,$$

where $\delta=\sqrt{c_t\ln(b_t/a_t)}$ and $\boldsymbol{x}^p\in O_i^+$.

Proof Let $\boldsymbol{e}^i=\boldsymbol{x}^i-\left(\sum_{p=1}^{q}\boldsymbol{x}^p\right)\Big/q$ be error function and $V_i=(1/2)\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{e}_i$ be the corresponding Lyapunov function.

$$\dot{V}_i=(\dot{\boldsymbol{e}}_i)^{\mathrm{T}}\boldsymbol{e}_i=(\dot{\boldsymbol{x}}^i)^{\mathrm{T}}\left[\boldsymbol{x}^i-\left(\sum_{p=1}^{q}\boldsymbol{x}^p\right)\Big/q\right].$$

(1) If $q=1$, we have

$$\begin{aligned}\dot{V}_i&=\{-(\boldsymbol{x}^i-\boldsymbol{x}^1)[a_t-b_t\exp(-\|\boldsymbol{x}^i-\boldsymbol{x}^1\|^2/c_t)]\}^{\mathrm{T}}(\boldsymbol{x}^i-\boldsymbol{x}^1)\\&=-[a_t-b_t\exp(-\|\boldsymbol{x}^i-\boldsymbol{x}^1\|^2/c_t)]\|\boldsymbol{x}^i-\boldsymbol{x}^1\|^2,\end{aligned}$$

when $\|\boldsymbol{e}^i\|=\|\boldsymbol{x}^i-\boldsymbol{x}^1\|\ge\delta$, we have $\dot{V}_i<0$.

(2) If $q=2$, we have

$$\begin{aligned}\dot{V}_i&=\sum_{p=1}^{2}-\{[a_t-b_t\exp(-\|\boldsymbol{x}^i-\boldsymbol{x}^p\|^2/c_t)]\\&\quad\cdot(\boldsymbol{x}^i-\boldsymbol{x}^p)^{\mathrm{T}}[\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2]\}\\&=-2a_t\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|^2+\sum_{p=1}^{2}b_t\exp(-\|\boldsymbol{x}^i\\&\quad-\boldsymbol{x}^p\|^2/c_t)(\boldsymbol{x}^i-\boldsymbol{x}^p)^{\mathrm{T}}[\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2]\\&\le-2a_t\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|^2+\sum_{p=1}^{2}b_t\exp(-\|\boldsymbol{x}^i\\&\quad-\boldsymbol{x}^p\|^2/c_t)\|\boldsymbol{x}^i-\boldsymbol{x}^p\|\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|.\end{aligned}$$

Individual i is a free individual, so $\|\boldsymbol{x}^i-\boldsymbol{x}^p\|>\delta$.

As $\exp(-\|\boldsymbol{x}^i-\boldsymbol{x}^p\|^2/c_t)\|\boldsymbol{x}^i-\boldsymbol{x}^p\|$ is a decreasing function of the distance $\|\boldsymbol{x}^i-\boldsymbol{x}^p\|$ with the maximum occurring at $\|\boldsymbol{x}^i-\boldsymbol{x}^p\|=\delta$, we have

$$\begin{aligned}\dot{V}_i&\le-2a_t\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|^2\\&\quad+\sum_{p=1}^{2}b_t\exp(-\delta^2/c_t)\delta\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|\\&=-2a_t\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|^2\\&\quad+2b_t\delta\exp(-\delta^2/c_t)\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|\\&=-2[a_t\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|-b_t\delta\exp(-\delta^2/c_t)]\\&\quad\cdot\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|.\end{aligned}$$

We need $\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|\ge\dfrac{b_t\delta}{a_t}\exp(-\delta^2/c_t)$ to make the term in square brackets be positive, for $\dfrac{b_t}{a_t}\exp(-\delta^2/c_t)=1$. Therefore, when $\|\boldsymbol{e}^i\|=\|\boldsymbol{x}^i-(\boldsymbol{x}^1+\boldsymbol{x}^2)/2\|\ge\delta$, we have $\dot{V}_i<0$, which proves the assertion.

Remark Intuitively, once individual j gets to the vicinity of individual i, the repulsive force will be in effect. When the repulsive force of individual j is in the same direction as the attractive force imposed by $\boldsymbol{x}_{\mathrm{goal}}$, the stability of individual i is the same as the stability of free individuals with the attractive constant a increased. When the repulsive and attractive force are in opposite direction as shown in Fig.5, there exists a positive constant λ to describe the balance distance $\|\boldsymbol{x}^1-\boldsymbol{x}^2\|=\delta-\lambda$. Furthermore, it is in direct ratio with the distance $\|\boldsymbol{x}^1-\boldsymbol{x}_{\mathrm{goal}}\|$.

When the repulsion force is not pointing to the same or opposite direction as the attractive force

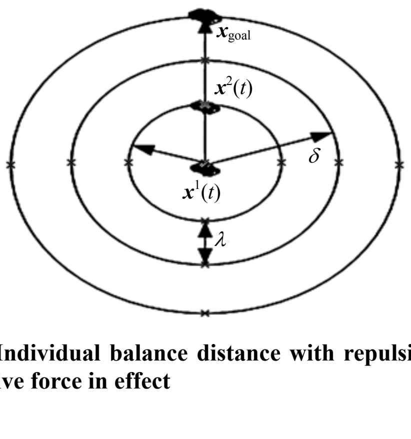

Fig.5 Individual balance distance with repulsive and attractive force in effect

imposed by object position, the component of repulsion force along $e^i(t)$ will be in effect in association with the attractive force. Individual i will move along the combined direction of attractive and repulsive force except that individual j is replaced by other nearest neighbor (individual k) or disappear in the repulsive range of individual i. The latter individual will be a free individual and classification of the former individual k is similar to that of individual j.

SIMULATION

Simulation examples are provided in this section to illustrate the previous results. The results as shown in Figs.6~8 are for the contrast between the two-layer model proposed in this paper and that in (Gazi and Passino, 2004). We use the region 20 cm×40 cm in the space, global object position is at (5 cm, 38 cm). As parameters we use r=1 cm, ρ=0.5 cm, a_t=0.05, b_t=10, c_t=0.05, ε=10 cm. When the initial distribution of the swarm is the same as that shown in Fig.6, the swarm remains cohesive at all times as shown in Fig.7 with the model and motion equation proposed in this paper, while with the model and motion equation proposed in (Gazi and Passino, 2004), the swarm splits into two sub-swarms as shown in Fig.8.

The reason is as follows: the model in this paper is based on two-layer topology which can assure cohesiveness both in swarm layer and neighbor layer, so the swarm will remain cohesive at all times as shown in Fig.7. Nevertheless, the model in (Gazi and Passino, 2004) just considered a single neighbor layer and cannot assure the continuation of the mutual observability between the individuals when the neighbor layer of every individual is different, which means not all individuals know the position of each other [the assumption in (Gazi and Passino, 2004) cannot be satisfied and so original track in Fig.8 is not all convergent but split into two sub-swarms]. There is eventual convergence of the two sub-swarms in the latter part of the track shown in Fig.8 because every individual in the two sub-swarms knows the exact position of all other individuals in the whole swarm when the two sub-swarms separately approach the vicinity of the goal position. As we know, when every individual can observe the exact position of all other individuals in the swarm, the member in the swarm layer and the neighbor layer will be the same.

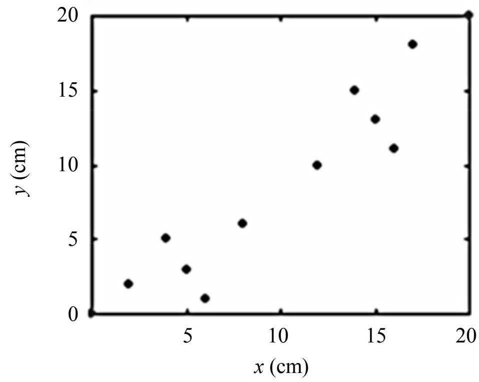

Fig.6 Initial distribution of the swarm

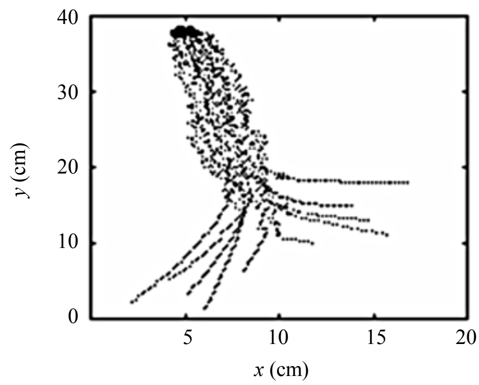

Fig.7 Track of the two-layer swarm

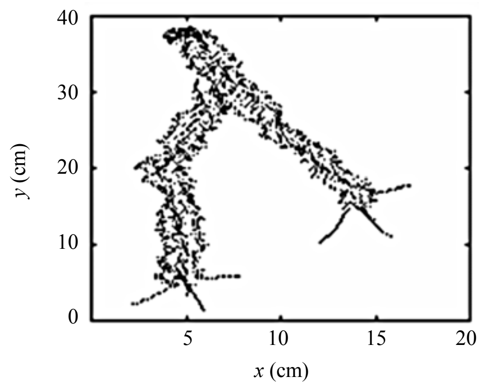

Fig.8 Track of the single-layer swarm

CONCLUSION

Growing employment of intelligent vehicles system has highlighted the power of scalable swarm systems for surveillance, reconnaissance and rescue tasks in both military and civil applications. In this paper we propose a two-layer emergent model for these systems and study the stability properties of the emergent behavior of the swarm based on Lyapunov stability theory. From the analysis and simulation we can conclude that the scale of the swarm will not be limited because only local individual information is used for modelling in the two-layer topology.

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