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On the fracture resistance of adhesively jointing structures*

ZHOU Zhi-qiang[1], YANG Qing-da[2], CHEN Wei-qiu[‡1]
([1]*Department of Civil Engineering, Zhejiang University, Hangzhou 310027, China*)
([2]*Rockwell Scientific Co., LLC, 1049 Camino Dos Rios, Thousand Oaks, CA 91360, USA*)
E-mail: zjuzzq@hotmail.com; qyang@rwsc.com; chenwq@zju.edu.cn

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Abstract: The interface toughness of adhesively bonded structural members is one of the critical parameters for adhesive joint design. It is often assumed that the joint toughness is a material constant so that its value can be obtained from fracture tests of simple geometries such as DCB for Mode-I, ENF for Mode-II, using linear elastic fracture mechanics (LEFM). However, the LEFM assumption of point-wise crack-tip fracture process is overly simplistic and may cause significant error in interpreting fracture test data. In this paper, the accuracy and applicability of various traditional beam-bending-theory based methods for fracture toughness evaluation, such as simple beam theory (SBT), corrected beam theory (CBT) and experimental compliance method (ECM), were assessed using the cohesive zone modelling (CZM) approach. It was demonstrated that the fracture process zone (FPZ) size has profound influence on toughness calculation and unfortunately, all the classic beam-bending theories based methods fail to include this important element and are erroneous especially when the ratio of crack length to FPZ size is relatively small (<5.0). It has also been demonstrated that after the FPZ size is incorporated into simple beam formulations, they provide much improved evaluation for fracture toughness. Formulation of first order estimate of FPZ size is also given in this paper.

Key words: Fracture mechanics, Adhesive, Cohesive zone model (CZM)
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INTRODUCTION

Adhesive bonding technology has been widely applied in modern structures in industries such as automotive, aerospace and in microelectronic device. The core idea of this technology is to bond two similar or dissimilar structure members with a thin continuous interface layer which can provide far better stress transfer across the interface than those traditional point-wise joining technologies such as spot welding, riveting and bolting. Furthermore, the damage tolerance properties of many adhesive materials provide extra gains in structure design (Kinloch *et al*., 1994; Yang *et al*., 1999; 2000; Yang and Thouless, 2001).

The interface toughness of an adhesive joint is one of the most critical parameters for joint design purposes. It is often assumed that the joint toughness is a material constant so that its value can be obtained from fracture tests of simple beam-like geometries such as double cantilever beam (DCB) test for Mode-I and end-loaded split (ELS) test for Mode-II (Fig.1). Linear elastic fracture mechanics (LEFM) coupled with simple beam-bending theory is widely used for deducing toughness value from fracture test data, typically in the form of load vs load-point displacement. However, the LEFM assumption of point-wise crack-tip fracture process is overly simplistic and may cause significant error in interpreting fracture test data (Yang *et al*., 1999).

According to LEFM, the fracture of adhesively bonded structure occurs when the fracture energy release rate (ERR), G, reaches a critical value, G_C, usually considered as inherent material property although it may vary with fracture mode ratio (Hutchinson and Suo, 1992). Modes I and II are the most

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‡ Corresponding author

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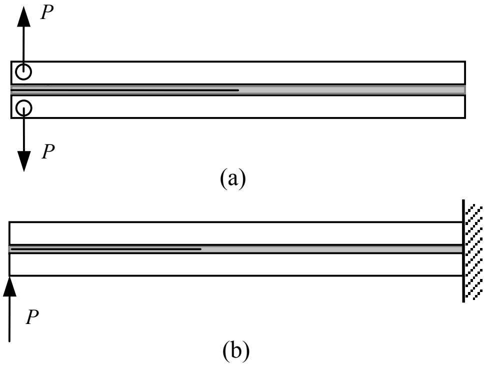
significant to engineering applications because they inevitably occur when a structure is subjected to in-plane tension or out-of-plane bending. Different test configurations have been developed to experimentally determine the single-mode fracture ERR. For Mode I, DCB geometries have been widely used, while ELS test is generally used for Mode II. The configurations of the two test specimens are depicted in Fig.1.



(a)

(b)

Fig.1 Modes I and II test specimens. (a) Double cantilever beam for Mode I; (b) End-loaded split for Mode II

Based on simple beam (bending) theory (SBT), the critical ERR for DCB (Mode-I) and ELS (Mode-II) are:

$$G_{\mathrm{IC}}=\frac{4P^2}{E_{\mathrm{s}}B^2}\left(\frac{3a^2}{h^3}+\frac{1}{h}\right), \tag{1}$$

and

$$G_{\mathrm{IIC}}=\frac{9P^2a^2}{4E_{\mathrm{s}}B^2h^3}. \tag{2}$$

where a is the crack length, E_{s}, h, and B are the Young's modulus, height, and width of the substrates, respectively.

A key assumption in the above equations is the fixed boundary condition in the beams bonded directly above or beneath the crack-tip (Hutchinson and Suo, 1992), which essentially assumes that the adhesive deformation is negligibly small ahead of a crack-tip. This assumption is not valid in most adhesive joints wherein the adhesive layers experience large deformation before they eventually fail in leading to crack extension. Therefore using simple beam theory for calculating fracture toughness may be erroneous especially when initial crack or notch length is small compared to the characteristic fracture zone size. Various amendment methods have been proposed in the literature to improve the accuracy of beam theory in fracture analysis. For example, Blackman *et al.*(2003; 2005) proposed the corrected beam theory (CBT) to calculated fracture ERR using the following modified beam-theory equations

$$G_{\mathrm{IC}}=\frac{12P^2(a+|\Delta_{\mathrm{I}}|)^2}{E_{\mathrm{s}}B^2h^3}\cdot F_{\mathrm{I}}=\frac{3P\delta}{2B(a+|\Delta_{\mathrm{I}}|)}\cdot F_{\mathrm{I}}, \tag{3}$$

$$G_{\mathrm{IIC}}=\frac{9P^2(a+|\Delta_{\mathrm{II}}|)^2}{4B^2h^3E_{\mathrm{s}}}\cdot F_{\mathrm{II}}, \tag{4}$$

where δ is the measured load-line displacement, and F_{I} and F_{II} are correction factors accounting for large displacement (Blackman *et al.*, 2003; 2005). Δ_{I} in Eq.(3) and Δ_{II} in Eq.(4) are the correction lengths due to beam root rotation; Δ_{I} is found from the negative intercept of a plot of $C^{1/3}$ vs a, with C being the compliance of the beam, while $\Delta_{\mathrm{II}}=0.42\Delta_{\mathrm{I}}$ as recommended by Blackman *et al.*(2005).

The crack length corrections (Δ_{I} in Eq.(3) and Δ_{II} in Eq.(4) for that matter) are derived from Kanninen's seminal analysis (Kanninen, 1973) of DCB under transverse shear loading (Fig.1a). They were derived in (Kanninen, 1973) to account for the shear deformation zone ahead of a crack-tip under such loading, and were then used by Williams and Kinloch to account for the possible crack-tip rotation (root rotation) in peel tests (Williams, 1989; Kinloch *et al.*, 1994). In theory it works only for the condition that the adhesive layer is elastically deformed before fracture. Nevertheless, it has been used to amend elasto-plastic peel tests by Kinloch *et al.*(1994). The justification of such modification on nonlinearly deformed adhesives remains an open research topic that has not been completely resolved, and this will be one of the focusing points of this paper.

Another alternative is use of the experimental compliance method (ECM), which gives the following expression for computing joint toughnesses

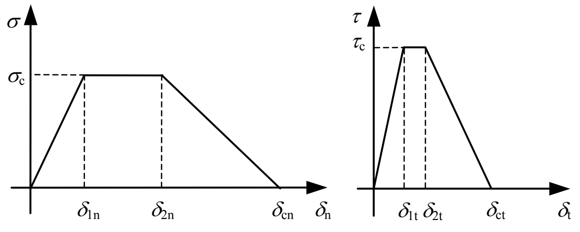
$$G_{\mathrm{IC}}=\frac{nP\delta}{2Ba}\cdot F_{\mathrm{I}} \tag{5}$$

for Mode I, and

$$G_{\mathrm{IIC}}=\frac{3P^2a^2m}{2B}\cdot F_{\mathrm{II}} \tag{6}$$

for Mode II. n and m are experimentally determined constants. Again these equations are based on LEFM and simple beam theory. The only difference is that the two constants are deduced from experimental data directly. Also it should be noted that in deducing n and m, certain functionalities of the experimental load-displacement curves have to be assumed in advance and this could bring extra error into the analysis.

Typically, the above different modification methods will give different toughness values when applied to a fracture test. It is of great importance to check the validity and applicability of these different methods using an independent method. In this paper, the cohesive zone model (CZM) scheme (Yang *et al.*, 1999; Yang and Thouless, 2001) is employed to evaluate these different beam-theory based methods. The core of a cohesive zone model is the traction-separation law (also called cohesive law) that describes the evolution of interface stresses as functions of interface separations as depicted in Fig.2. The cohesive law can be obtained by comparing numerical simulation results using trial cohesive parameters with experimental load-displacement curves (Yang *et al.*, 1999; Yang and Thouless, 2001; Yang and Cox, 2005). Furthermore, since in a cohesive model the fracture toughness is an input model parameter, it serves as a perfect reference to judge the accuracy of various beam-theory methods. Section 2 of this paper will give a brief description of the cohesive zone modelling approach. In Section 3, the CZM will be applied to analyze a Mode-II test reported in the literature to demonstrate its excellent capability of describing non-steady state, nonlinear fracture processes. The calibrated (Mode-II) toughness will be used as a yard-stick for measuring the accuracy of beam-bending theory based facture models (ECM, SBT, CBT, etc.). The root cause of the inaccuracy of these simple beam theories, especially when crack length is small, will be demonstrated and discussed in Section 4. Finally, Section 5 will conclude the paper.



(a) (b)

Fig.2 Traction-separation law in cohesive zone model (CZM). (a) $G_{\rm IC}$; (b) $G_{\rm IIC}$

COHESIVE ZONE MODEL (CZM) SCHEME

A cohesive zone model involves representing the adhesive bonded interface by a layer of special elements whose constitutive properties describe the traction (or cohesive stress) evolution as the interface is being opened (Needleman, 1987; 1997). Typically the cohesive stress increases initially with the opening displacement, reaches a peak and then drops continuously to zero again at a certain critical displacement, as shown in Fig.2. The area encompassed by the cohesive law and the horizontal displacement axis ($G_{\rm IC}$ and $G_{\rm IIC}$) is the fracture toughness in the classical LEFM sense and the peak cohesive stresses (σ_c and τ_c) represent the maximum load bearing capability of the adhesive. These are the two most important parameters for a cohesive law—the other two parameters δ_1/δ_c and δ_2/δ_c (Fig.1) are only of secondary importance (Tvergaard and Hutchinson, 1992; 1993). The CZM model has been successfully used to explore a variety of nonlinear fracture problems—a recent review of this can be found in (Yang and Cox, 2005).

In this paper, the mode-dependent CZM of (Yang and Thouless, 2001) will be used. This cohesive model features with independent descriptions for Mode-I and Mode-II fractures and an energy-based failure criterion for mixed mode fracture.

The energy absorbed per unit area during the facture process, i.e., energy release rate, can be calculated as

$$G_{\rm I}=\int_0^{\delta_{\rm cn}}\sigma(\delta_{\rm n})\mathrm{d}\delta_{\rm n}, \tag{7}$$

for Mode I, and

$$G_{\rm II}=\int_0^{\delta_{\rm ct}}\tau(\delta_{\rm t})\mathrm{d}\delta_{\rm t}, \tag{8}$$

for Mode II. For pure modes, the Mode-I or Mode-II fracture toughness ($G_{\rm IC}$ or $G_{\rm IIC}$) is the value when the critical opening or shear displacement ($\delta_{\rm cn}$ and $\delta_{\rm ct}$) is reached. For mixed-mode fracture, an energy-based failure criterion in agreement with many experimental data is defined as

$$G_{\rm I}/G_{\rm IC}+G_{\rm II}/G_{\rm IIC}=1. \tag{9}$$

It can be shown that with such a mixed-mode failure criterion the mode mixedness of any fracture problem will become a natural outcome of a CZM analysis (Yang and Thouless, 2001), which is of great value because for many engineering problems there is no way of knowing the mode-mixedness a priori.

It should be emphasized here that, a CZ analysis is inherently a multiscale analysis which involves at least two length scales. At microscale level, the length scale is associated with the critical displacement when failure occurs; in the meanwhile, at structural level, a CZ law is to introduce a damage zone (cohesive zone) wherein the structural materials experience certain degree of damages but have not lost load bearing capability completely yet. The cohesive length scale for Mode-I and Mode-II fractures can be estimated as follows (Yang and Cox, 2005; Yang *et al.*, 2006):

$$l_{\mathrm{c}}^{\mathrm{I}}=\left(\frac{G_{\mathrm{IC}}E_{\mathrm{s}}}{\sigma_{\mathrm{c}}^{2}}\right)^{1/4}h^{3/4},\quad \text{(Mode-I)} \tag{10}$$

$$l_{\mathrm{c}}^{\mathrm{II}}=\left(\frac{G_{\mathrm{IIC}}E_{\mathrm{s}}}{\tau_{\mathrm{c}}^{2}}\right)^{1/2}h^{1/2}.\quad \text{(Mode-II)} \tag{11}$$

To achieve reasonably numerical accuracy, the characteristic elemental size of those adherend element joined by the cohesive element should be smaller than the cohesive zone sizes—1/5~1/10 smaller is recommended.

Furthermore, due to the existence of such a finite sized fracture zone, it is obvious that simple beam theory based solutions need modification, because there is beam deformation that is not necessarily limited to the crack wake, but also ahead of the crack-tip in the cohesive zone too. We believe this is the one of the major root causes leading to the insufficiency of beam-theory based results but this point has not been well appreciated in the adhesion and fracture research community. In the following section, we will demonstrate this point through a detailed analysis on experimental ELS results reported by Blackman *et al.*(2005).

CZM analysis of an ELS test

To demonstrate how such a cohesive zone model can capture the key features of nonlinear fracture problems, the ELS test of (Blackman *et al.*, 2005) (a Mode-II geometry) was analyzed using CZM. Exact specimen geometry of the test specimen was followed and a layer of cohesive zone element was placed along the center line of the ELS beam. The substrate is a unidirectional CFRP composite and the adhesive is epoxy; the length of substrate is 120 mm, and the height is 2 mm; the initial crack length is 70 mm; and the thickness of the adhesive layer is 2 mm. The parameters of CZM for adhesive test were calibrated using the load-displacement curve reported in (Blackman *et al.*, 2005), as shown in Fig.3. The peak stress and the critical displacement of the adhesive joint were found to be 28 MPa and 0.46 mm, by comparing simulation results of different choices of τ_{c} and δ_{ct} and finding the best fit of the experimental data. It follows from Eq.(8) that the critical fracture energy release rate is 6.96 N/mm. It should be noted that the initial elastic slope of the experimental data indicates that the modulus of the substrate should be 93 GPa, rather than the value of 126 GPa reported in (Blackman *et al.*, 2005).

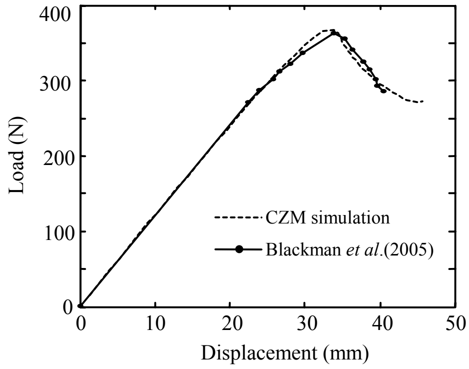


Fig.3 A typical load-displacement curve from (Blackman *et al.*, 2005) and the numerical simulation with cohesive zone model (CZM)

The numerically calculated load-displacement curve and current crack length were then subjected to various beam theory analyses to calculate fracture toughness. The results are shown in Fig.4, where the CBT refers to the corrected beam theory in which the correction of crack length is $\Delta_{\mathrm{II}}=0.42\Delta_{\mathrm{I}}$ with Δ_{I} being found from the negative intercept of a plot of $C^{1/3}$ vs a for Mode I test specimen. In the MBT, the correction length (41 mm) is directly calculated from Eq.(10). It can be seen from Fig.4 that almost all the beam theory calculated toughnesses are off from the input value of 6.96 N/mm. ECM, SBT, and CBT all considerably under predict the toughness and the MBT slightly over predicts it. The rising of calculated toughness when crack length is closed or larger than 100 mm is purely due to end effects—crack propagation becomes increasingly difficult when it approaches the clamped end. It is obvious that in terms of accuracy, MBT gives much improved toughness evaluation than the other methods. But it still misses quite a lot, especially near the initiation stage.

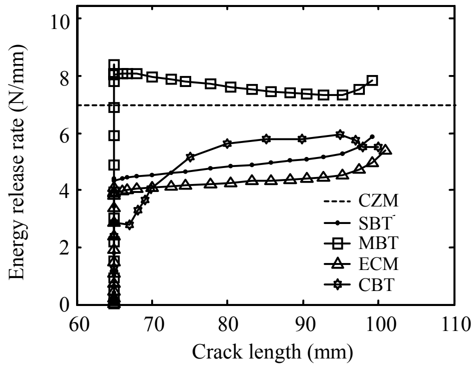


Fig.4 Comparison of several methods

One of the major contributions to the insufficiency of beam theory based LEFM formulations for fracture toughness evaluation is the assumption that crack length is much larger than the cohesive zone size ahead of a crack tip. For many adhesive joints this condition may not be valid. Fig.5 shows the shear deformation zone of the above ELS simulation. The cohesive zone is indicated in Fig.5 by the shear stresses across the bondline. A fully-developed cohesive zone is indicated in Fig.5 by the solid-black curve, which shows that the zone length is about 30 mm. The zone length of 30 mm is about half of the initial crack length. That why even the MBT, which has the correct crack length modification, overestimated the toughness significantly at the early stage of crack propagation—the results improve quite a lot as the crack becomes longer. For all the other solutions, they consistently underestimate the toughness substantially because none of them explicitly take into account the large cohesive zone. To further demonstrate the point, we proceed to the next section wherein a CZM study on both Mode-I and Mode-II geometries will be detailed.

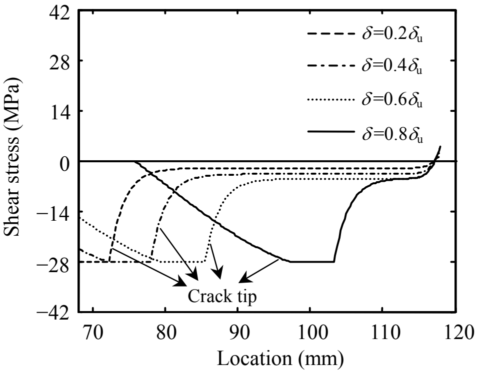
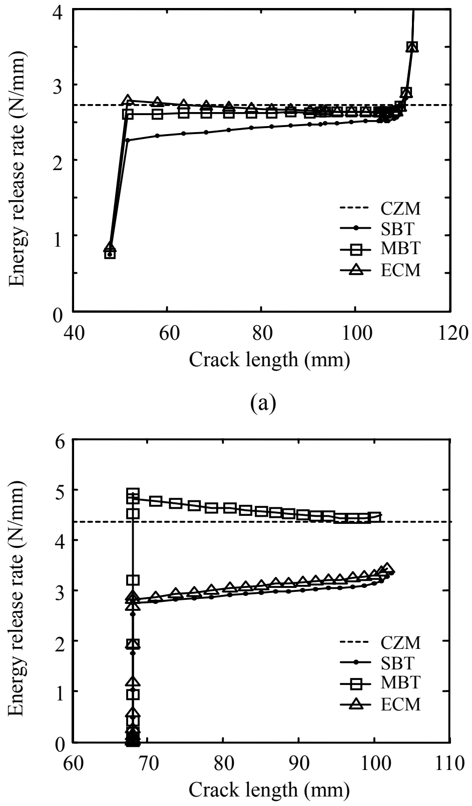


Fig.5 Shear stress distributions ahead of the crack-tip predicted by cohesive zone model (CZM), δ_u being the ultimate displacement of load point

Comparison of beam theories

Having obtained the above primary understanding for a Mode-II specimen, we will compare the three beam theories with the CZM scheme to check their validity either for Mode I or for Mode II. We will focus again on the effect of cohesive zone size on the accuracy of LEFM results. Numerical simulation for Mode-I test was conducted with the value of σ_c=50 MPa, and G_{IC}=2.725 N/mm and for Mode II τ_c=40 MPa, and G_{IIC}=4.360 N/mm. The substrate's height and elastic modulus in the numerical analysis are both taken as 4 mm and 93 MPa, respectively. The initial crack length is 52 mm for Mode I, and 68 mm for Mode II. The cohesive zone length is 9.0 mm for Mode-I and 31.8 mm for Mode-II test, respectively.

Fig.6 gives the Mode-I and Mode-II toughnesses vs the crack length predicted by the three beam theories, with the horizontal line being the CZM input toughness as a benchmark. Note that for Mode-I the estimated cohesive zone size is about 9.0 mm and the initial crack length is 52 mm, i.e., the cohesive zone is at most 1/5 that of the crack length. Under such conditions all the beam-theory based solutions do a reasonably good job in calculating the fracture toughness, especially when the crack length is large. However, this is not true for the Mode-II case, for which the estimated cohesive zone size is 31.8 mm, almost half of the initial crack length (68 mm). Therefore, initially none of the solutions are sufficiently close to the input toughness. As the crack length increases, the ratio between the cohesive zone length and the crack length decreases. The result of MBT, which has cohesive zone length correction, approaches to the input value steadily. All the other solutions fail to capture the true toughness even at very large cracks. This indicates that their crack length modifying scheme is problematic. For example, the CBT by Blackman *et al.*



(a)

(b)

Fig.6 Comparison of different methods for (a) Mode I and (b) Model II

(2005) uses $\Delta_{\rm II}$=0.42$\Delta_{\rm I}$ for the correction of crack length in Mode-II test. However, as mentioned earlier, the cohesive zone length ahead of the crack-tip for Mode II is generally larger than that for Mode I. Thus, the correction of crack length in the CBT for Mode II test may be inappropriate. After a series of numerical experiments, we concluded that the results of SBT, CBT, ECM all can be considered to be reliable if the initial crack length were at least 10 times the fracture process zone length. In the case of brittle adhesive, the fracture process length will be small, and hence SBT, CBT and ECM all could perform well. On the other hand, the results predicted by the three beam theories should be examined when the adhesive is so ductile and tough that the cohesive zone size is not too small compared to the crack length. The CZM scheme, with parameters carefully calibrated from the experimental curve of load vs displacement, can play the role of judge.

CONCLUSION

In this paper, the accuracy and applicability of various traditional beam-bending theory based methods for fracture toughness evaluation, such as simple beam theory (SBT), corrected beam theory (CBT) and experimental compliance method (ECM), were assessed using the cohesive zone modelling (CZM) approach. It has been demonstrated that the fracture process zone (FPZ) size has profound influence on toughness calculation using such simple beam theory and LEFM. However, this important feature is not included in the classic beam-bending theory based methods. As a result, such beam theory results are erroneous especially when the ratio of crack length to FPZ size is relatively small (<5.0). It has also been demonstrated that after the FPZ size is incorporated into simple beam formulations, much improved evaluation for fracture toughness is provided. Formulation of first order estimate of FPZ size is also given in this paper.

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