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Design of H_∞ robust fault detection filter for nonlinear time-delay systems*

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Abstract: In this paper, the robust fault detection filter (RFDF) design problems are studied for nonlinear time-delay systems with unknown inputs. First, a reference residual model is introduced to formulate the RFDF design problem as an H_∞ model-matching problem. Then appropriate input/output selection matrices are introduced to extend a performance index to the time-delay systems in time domain. The reference residual model designed according to the performance index is an optimal residual generator, which takes into account the robustness against disturbances and sensitivity to faults simultaneously. Applying robust H_∞ optimization control technique, the existence conditions of the RFDF for nonlinear time-delay systems with unknown inputs are presented in terms of linear matrix inequality (LMI) formulation, independently of time delay. An illustrative design example is used to demonstrate the validity and applicability of the proposed approach.

Key words: Nonlinear time-delay systems, Robust fault detection filter (RFDF), H_∞ optimization, Linear matrix inequality (LMI)
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INTRODUCTION

Due to increasing demand for higher performance, as well as for higher safety and reliability standards, the model-based approaches to fault detection and isolation (FDI) for dynamic systems have received more and more attention during the last two decades (Chen and Patton, 1999; Chen and Saif, 2006; Frank and Ding, 1997; Gao and Wang, 2006; Jiang and Zhou, 2005; Zhou *et al*., 1998). Among these model-based approaches, the most common way is the observer-based approach (Chen and Patton, 1999), i.e., using state observers or filters to generate residuals and using these residuals to set a threshold to detect the fault. Recently, with the rapid development of robust control theory and H_∞ optimization control technique, more and more methods have been presented to solve robust fault detection and isolation (RFDI) problems (Chen and Patton, 1999; Ding *et al*., 2000; Frank and Ding, 1997; Zhong *et al*., 2003). Different from robust control, the goal of robust fault detection is to distinguish between the faults effects and the effects of uncertain signals and perturbations. Therefore the performance of RFDI systems should be measured by a suitable trade-off between robustness and sensitivity.

As is well known, time delays are inherent in many real physical systems, such as chemical processes, long transmission lines in pneumatic systems, power and water distribution networks, air pollution systems, econometric systems, etc. Since the delayed state very often causes instability and poor performance of systems (Richard, 2003), increasing attention has recently been devoted to the robust fault detection filter (RFDF) design problems of the linear state delayed systems. Ding *et al*.(2001) studied the RFDF problems for time-delay LTI systems via introducing an idealized reference residual model. However, it only considered the sensitivity of the residual to the

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faults when the reference residual model was designed. In addition, the existing results achieved in delay-free systems are not suitable for time-delay systems because of existence of the state delay. To our knowledge, for the FDI case, relevant literature is relatively few for nonlinear time-delay systems, and so far, little attention has been paid to the RFDF design problems in the simultaneous presence of time delay, nonlinear parts and exogenous disturbance input. This motivates the present research on designing H_∞ RFDF for nonlinear time-delay systems with exogenous disturbance inputs.

This paper deals with the problem of H_∞ RFDF design for a class of state-delay nonlinear systems with exogenous disturbance inputs. This problem aims at designing the RFDF such that, for exogenous disturbances, the RFDF system is stable and has a prescribed H_∞ performance, independently of the time delay. The class of systems under consideration is described by a linear delayed state space model with the addition of known nonlinearities, which depend on state as well as delayed state and satisfy the global Lipschitz conditions. In this paper, a reference residual model is used to reduce the RFDF design problem to a standard H_∞ model-matching problem. The performance index used to design the reference residual model considers both the robustness against disturbances and the sensitivity to faults. In addition, applying the H_∞ optimization technique, an delay-independent LMI approach to design the RFDF is proposed.

PROBLEM STATEMENT

Consider the following class of fault nonlinear time-delay systems

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{A}_{\mathrm{d}}\boldsymbol{x}(t-h)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{G}g(\boldsymbol{x}(t),\boldsymbol{x}(t-h))+\boldsymbol{B}_{\mathrm{f}}\boldsymbol{f}(t)+\boldsymbol{B}_{\mathrm{d}}\boldsymbol{d}(t), \tag{1}$$

$$\boldsymbol{y}(t)=\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{D}\boldsymbol{u}(t)+\boldsymbol{D}_{\mathrm{f}}\boldsymbol{f}(t)+\boldsymbol{D}_{\mathrm{d}}\boldsymbol{d}(t), \tag{2}$$

$$\boldsymbol{x}(t)=0\quad \forall t\in[-h,0],$$

where $\boldsymbol{x}(t)\in\mathbb{R}^n$ is the state vector, $\boldsymbol{u}(t)\in\mathbb{R}^p$ is the control input vector, $\boldsymbol{y}(t)\in\mathbb{R}^q$ is the measurement output vector, $\boldsymbol{d}(t)\in\mathbb{R}^m$ is the disturbance input that belongs to $L_2^m[0,+\infty)$, $\boldsymbol{f}(t)\in\mathbb{R}^l$ is the fault to be detected, $h\geq 0$ is an unknown but constant delay, $g(\cdot,\cdot)$: $\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^{n_g}$ is a known nonlinear function. $\boldsymbol{A}$, $\boldsymbol{A}_{\mathrm{d}}$, $\boldsymbol{B}$, $\boldsymbol{B}_{\mathrm{f}}$, $\boldsymbol{B}_{\mathrm{d}}$, $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{D}_{\mathrm{f}}$, $\boldsymbol{D}_{\mathrm{d}}$ and $\boldsymbol{G}$ are known matrices with appropriate dimensions.

Throughout this paper, we make the following assumptions:

Assumption 1 $(\boldsymbol{C}, \boldsymbol{A})$ is detectable.

Assumption 2 (Lipschitz condition)

(1) $g(\mathbf{0},\mathbf{0})=\mathbf{0}$;

(2) $\|g(\boldsymbol{x}_1,\boldsymbol{x}_2)-g(\boldsymbol{y}_1,\boldsymbol{y}_2)\|\leq\|\rho_1(\boldsymbol{x}_1-\boldsymbol{y}_1)\|+\|\rho_2(\boldsymbol{x}_2-\boldsymbol{y}_2)\|$ for all $\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{y}_1,\boldsymbol{y}_2\in\mathbb{R}^n$, where ρ_1 and ρ_2 are known real constant matrices.

We are interested in designing the so-called fault detection filter

$$\begin{aligned}\dot{\hat{\boldsymbol{x}}}(t)&=\boldsymbol{A}\hat{\boldsymbol{x}}(t)+\boldsymbol{A}_{\mathrm{d}}\hat{\boldsymbol{x}}(t-h)+\boldsymbol{B}\boldsymbol{u}(t)\\&\quad+\boldsymbol{G}g\left(\hat{\boldsymbol{x}}(t),\hat{\boldsymbol{x}}(t-h)\right)+\boldsymbol{H}\left(\boldsymbol{y}(t)-\hat{\boldsymbol{y}}(t)\right),\end{aligned} \tag{3}$$

$$\hat{\boldsymbol{y}}(t)=\boldsymbol{C}\hat{\boldsymbol{x}}(t)+\boldsymbol{D}\boldsymbol{u}(t),\quad \boldsymbol{r}(t)=\boldsymbol{V}\left(\boldsymbol{y}(t)-\hat{\boldsymbol{y}}(t)\right), \tag{4}$$

where $\hat{\boldsymbol{x}}(t)\in\mathbb{R}^n$ and $\hat{\boldsymbol{y}}(t)\in\mathbb{R}^q$ represent the state and output estimation vectors, respectively; $\boldsymbol{r}(t)$ is the so-called generated residual signal. The design parameters of an RFDF are the observer gain matrix $\boldsymbol{H}$ and the residual weighting matrix $\boldsymbol{V}$. Define the error state $\boldsymbol{e}(t)=\boldsymbol{x}(t)-\hat{\boldsymbol{x}}(t)$. Then it follows from Eqs.(1)~(4) that

$$\begin{aligned}\dot{\boldsymbol{e}}(t)&=(\boldsymbol{A}-\boldsymbol{H}\boldsymbol{C})\boldsymbol{e}(t)+(\boldsymbol{B}_{\mathrm{f}}-\boldsymbol{H}\boldsymbol{D}_{\mathrm{f}})\boldsymbol{f}(t)+\boldsymbol{G}\boldsymbol{\Psi}\\&\quad+\boldsymbol{A}_{\mathrm{d}}\boldsymbol{e}(t-h)+(\boldsymbol{B}_{\mathrm{d}}-\boldsymbol{H}\boldsymbol{D}_{\mathrm{d}})\boldsymbol{d}(t),\end{aligned} \tag{5}$$

$$\boldsymbol{r}(t)=\boldsymbol{V}\boldsymbol{C}\boldsymbol{e}(t)+\boldsymbol{V}\boldsymbol{D}_{\mathrm{f}}\boldsymbol{f}(t)+\boldsymbol{V}\boldsymbol{D}_{\mathrm{d}}\boldsymbol{d}(t), \tag{6}$$

where $\boldsymbol{\Psi}=g\left(\boldsymbol{x}(t),\boldsymbol{x}(t-h)\right)-g\left(\hat{\boldsymbol{x}}(t),\hat{\boldsymbol{x}}(t-h)\right)$.

Note that the dynamics of the residual signal depends not only on $\boldsymbol{f}(t)$ and $\boldsymbol{d}(t)$, but also on the nonlinear part $\boldsymbol{\Psi}$. The methods mentioned in (Ding *et al*., 2000; Garcia and Frank, 1997; Zhong *et al*., 2003) for delay-free case are not suitable for solving RFDF design problem, because there are time delays in $\boldsymbol{e}(t-h)$ and $\boldsymbol{\Psi}$. Here, we propose to use a reference residual model describing the desired behavior of the residual vector $\boldsymbol{r}(t)$, to formulate the RFDF design problem as an H_∞ model-matching problem, that is to find an idealized reference residual and minimize the worst case distance between the generated residual and the idealized reference residual. In the idealized case, the observed states $\hat{x}(t)$ and $\hat{x}(t-h)$ should be equal to $x(t)$ and $x(t-h)$ (i.e. $\boldsymbol{\Psi}=\mathbf{0}$). Therefore, according to Eqs.(5) and (6) and assuming $\boldsymbol{\Psi}=\mathbf{0}$, the corresponding reference residual error model is given by

$$\begin{aligned}\dot{\boldsymbol{e}}_{\mathrm{f}}(t) &= (\boldsymbol{A}-\bar{\boldsymbol{H}}\boldsymbol{C})\boldsymbol{e}_{\mathrm{f}}(t)+\boldsymbol{A}_{\mathrm{d}}\boldsymbol{e}_{\mathrm{f}}(t-h)\\ &\quad+(\boldsymbol{B}_{\mathrm{f}}-\bar{\boldsymbol{H}}\boldsymbol{D}_{\mathrm{f}})\boldsymbol{f}(t)+(\boldsymbol{B}_{\mathrm{d}}-\bar{\boldsymbol{H}}\boldsymbol{D}_{\mathrm{d}})\boldsymbol{d}(t),\end{aligned} \tag{7}$$

$$\boldsymbol{r}_{\mathrm{f}}(t)=\bar{\boldsymbol{V}}\boldsymbol{C}\boldsymbol{e}_{\mathrm{f}}(t)+\bar{\boldsymbol{V}}\boldsymbol{D}_{\mathrm{f}}\boldsymbol{f}(t)+\bar{\boldsymbol{V}}\boldsymbol{D}_{\mathrm{d}}\boldsymbol{d}(t), \tag{8}$$

$$\boldsymbol{e}_{\mathrm{f}}(t)=\mathbf{0}\quad(t\le 0),$$

where $\boldsymbol{e}_{\mathrm{f}}(t)\in\mathbb{R}^n$ is the reference model error state vector, $\boldsymbol{r}_{\mathrm{f}}(t)$ is the reference model residual signal, $\bar{\boldsymbol{H}}$ and $\bar{\boldsymbol{V}}$ are the parameters of the reference residual model to be designed.

Thus the overall system can be described by

$$\dot{\boldsymbol{\eta}}(t)=\bar{\boldsymbol{A}}\boldsymbol{\eta}(t)+\bar{\boldsymbol{A}}_{\mathrm{d}}\boldsymbol{\eta}(t-h)+\bar{\boldsymbol{B}}\boldsymbol{w}(t)+\bar{\boldsymbol{G}}\boldsymbol{\Psi}, \tag{9}$$

$$\boldsymbol{r}_{\mathrm{e}}(t):=\boldsymbol{r}(t)-\boldsymbol{r}_{\mathrm{f}}(t)=\bar{\boldsymbol{C}}\boldsymbol{\eta}(t)+\bar{\boldsymbol{D}}\boldsymbol{w}(t), \tag{10}$$

where

$$\boldsymbol{\eta}=\begin{bmatrix}\boldsymbol{e}\\ \boldsymbol{e}_{\mathrm{f}}\end{bmatrix},\quad \boldsymbol{w}=\begin{bmatrix}\boldsymbol{f}\\ \boldsymbol{d}\end{bmatrix},\quad \bar{\boldsymbol{A}}=\begin{bmatrix}\boldsymbol{A}-\boldsymbol{H}\boldsymbol{C} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{A}-\bar{\boldsymbol{H}}\boldsymbol{C}\end{bmatrix},$$

$$\bar{\boldsymbol{A}}_{\mathrm{d}}=\begin{bmatrix}\boldsymbol{A}_{\mathrm{d}} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{A}_{\mathrm{d}}\end{bmatrix},\quad \bar{\boldsymbol{G}}=\begin{bmatrix}\boldsymbol{G}\\ \mathbf{0}\end{bmatrix},\quad \bar{\boldsymbol{C}}=[\boldsymbol{V}\boldsymbol{C}\quad -\bar{\boldsymbol{V}}\boldsymbol{C}],$$

$$\bar{\boldsymbol{B}}=\begin{bmatrix}\boldsymbol{B}_{\mathrm{f}}-\boldsymbol{H}\boldsymbol{D}_{\mathrm{f}} & \boldsymbol{B}_{\mathrm{d}}-\boldsymbol{H}\boldsymbol{D}_{\mathrm{d}}\\ \boldsymbol{B}_{\mathrm{f}}-\bar{\boldsymbol{H}}\boldsymbol{D}_{\mathrm{f}} & \boldsymbol{B}_{\mathrm{d}}-\bar{\boldsymbol{H}}\boldsymbol{D}_{\mathrm{d}}\end{bmatrix},\quad \bar{\boldsymbol{D}}=\begin{bmatrix}(\boldsymbol{V}\boldsymbol{D}_{\mathrm{f}}-\bar{\boldsymbol{V}}\boldsymbol{D}_{\mathrm{f}})^{\mathrm{T}}\\ (\boldsymbol{V}\boldsymbol{D}_{\mathrm{d}}-\bar{\boldsymbol{V}}\boldsymbol{D}_{\mathrm{d}})^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}.$$

Here we can see that in the design of RFDF, one main objective of this work is formulated as an H_∞ model-matching design problem, i.e. applying robust H_∞ optimization control technique, for all exogenous disturbance inputs and nonlinear parts, the generated residual $\boldsymbol{r}(t)$ will be designed as closely as possible to the reference model residual $\boldsymbol{r}_{\mathrm{f}}(t)$, independently of the unknown time-delay h. Thus, the problem of designing an observer-based RFDF can be described as designing observer gain matrix $\boldsymbol{H}$ and residual weighting matrix $\boldsymbol{V}$ such that

(1) System of Eqs.(9) and (10) is robustly asymptotically stable;

(2) Under zero initial condition, for given constant $\gamma>0$ and any non-zero $\boldsymbol{w}(t)\in L_2[0,\infty)$, system of Eqs.(9) and (10) satisfies the following inequality

$$\|\boldsymbol{r}_{\mathrm{e}}(t)\|_2<\gamma\|\boldsymbol{w}(t)\|_2. \tag{11}$$

After designing RFDF, the remaining important task of this paper is the evaluation of the generated residual. One of the widely adopted approaches is to choose a so-called threshold $J_{\mathrm{th}}>0$ and, based on this, use the following logical relationship for fault detection

$$\left.\begin{aligned}&\|\boldsymbol{r}(t)\|_{2,\tau}>J_{\mathrm{th}}\Rightarrow \text{a fault has occurred}\Rightarrow\text{a alarm},\\ &\|\boldsymbol{r}(t)\|_{2,\tau}\le J_{\mathrm{th}}\Rightarrow\text{no fault has occurred},\end{aligned}\right\} \tag{12}$$

where $\|\boldsymbol{r}(t)\|_{2,\tau}=\left[\int_{t_1}^{t_2}\boldsymbol{r}^{\mathrm{T}}(t)\boldsymbol{r}(t)\mathrm{d}t\right]^{1/2}$, $\tau=t_1-t_2$, $t\in[t_1, t_2]$.

Note that the length of the time window is finite (i.e. τ instead of ∞). Since in practice it is desired that the faults will be detected as early as possible, an evaluation of residual signal over the whole time range makes little sense (Ding *et al*., 2001; Zhong *et al*., 2003).

MAIN RESULTS AND PROOFS

As mentioned earlier, the design of RFDF for nonlinear time-delay systems can be formulated as an H_∞ model-matching problem. We should design the reference residual model first; the RFDF design problems for nonlinear time-delay systems can then be solved.

The following lemmas will be useful in designing the RFDF for nonlinear time-delay systems.

Lemma 1 (Wang *et al*., 2002) Given constant matrices χ_1, χ_2, χ_3, where $\chi_1=\chi_1^{\mathrm{T}}$ and $\mathbf{0}<\chi_2=\chi_2^{\mathrm{T}}$, then $\chi_1+\chi_3^{\mathrm{T}}\chi_2^{-1}\chi_3<\mathbf{0}$ if and only if

$$\begin{bmatrix}\chi_1 & \chi_3^{\mathrm{T}}\\ \chi_3 & -\chi_2\end{bmatrix}<\mathbf{0}\ \text{ or equivalently }\ \begin{bmatrix}-\chi_2 & \chi_3\\ \chi_3^{\mathrm{T}} & \chi_1\end{bmatrix}<\mathbf{0}.$$

Lemma 2 (Lien, 2004) Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be real matrices of appropriate dimensions. For any scalar $\varepsilon>0$ and vectors $\boldsymbol{x},\boldsymbol{y}\in\mathbb{R}^n$, then

$$2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{B}\boldsymbol{y}\le\varepsilon^{-1}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}+\varepsilon\boldsymbol{y}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{y}.$$

Choice of reference residual model

The selection of a suitable reference residual model is one of the key steps for designing an RFDF for nonlinear time-delay systems. If the reference residual model is not selected suitably, miss alarms or false alarms may occur. In order to select a suitable reference residual model, we first consider the following performance index

$$J_{\mathrm{f}}=\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{d}}(s)\right\|_{\infty}-\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{f}}(s)\right\|_{\infty}, \tag{13}$$

where $\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{f}}$ and $\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{d}}$ are the transfer functions from $\boldsymbol{f}$ and $\boldsymbol{d}$ to reference model residual $\boldsymbol{r}_{\mathrm{f}}$, respectively.

It is interesting to notice that setting $J_{\mathrm{f}}\rightarrow\min$ yields $\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{f}}\right\|_{\infty}\rightarrow\max$ and $\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{d}}\right\|_{\infty}\rightarrow\min$.

Therefore the reference residual model can be designed according to the performance index Eq.(13), which takes into account the robustness of the reference model residual against disturbance and sensitivity to faults simultaneously.

For the sake of simplicity, we assume that $l=m$. In fact, if $l>m$ (or $l<m$), by extending $\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{d}}$ (or $\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{f}}$) and $\boldsymbol{d}$ (or $\boldsymbol{f}$) to $\tilde{\boldsymbol{T}}_{r_{\mathrm{f}}\boldsymbol{d}}=\begin{bmatrix}\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{d}} & \boldsymbol{\varphi}_{l-m}\end{bmatrix}$ (or $\tilde{\boldsymbol{T}}_{r_{\mathrm{f}}\boldsymbol{f}}=\begin{bmatrix}\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{f}} & \boldsymbol{\varphi}_{m-l}\end{bmatrix}$) and $\tilde{\boldsymbol{d}}=\begin{bmatrix}\boldsymbol{d}\\ \boldsymbol{\varphi}_{l-m}\end{bmatrix}$ (or $\tilde{\boldsymbol{f}}=\begin{bmatrix}\boldsymbol{f}\\ \boldsymbol{\varphi}_{m-l}\end{bmatrix}$), we can have the same results, where $\boldsymbol{\varphi}$ denotes null matrix with appropriate dimensions.

Consider the following transfer function

$$\boldsymbol{T}\triangleq\boldsymbol{M}\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{\omega}}\boldsymbol{N}=\boldsymbol{M}\begin{bmatrix}\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{f}} & \boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{d}}\end{bmatrix}\boldsymbol{N}, \tag{14}$$

where the matrices $\boldsymbol{M}\in\mathbb{R}^{q\times q}$ and $\boldsymbol{N}\in\mathbb{R}^{2l\times l}$ select the appropriate input/output channels or channels combinations.

$\boldsymbol{T}$ admits the following realization

$$\boldsymbol{T}:\begin{cases}\dot{\boldsymbol{e}}_{\mathrm{f}}(t)=(\boldsymbol{A}-\bar{\boldsymbol{H}}\boldsymbol{C})\boldsymbol{e}_{\mathrm{f}}(t)+\boldsymbol{A}_{\mathrm{d}}\boldsymbol{e}_{\mathrm{f}}(t-h)+\boldsymbol{B}_1\boldsymbol{N}\boldsymbol{\omega}(t),\\ \boldsymbol{r}_{\mathrm{f}}(t)=\boldsymbol{M}\bar{\boldsymbol{V}}\boldsymbol{C}\boldsymbol{e}_{\mathrm{f}}(t)+\boldsymbol{M}\boldsymbol{D}_1\boldsymbol{N}\boldsymbol{\omega}(t),\end{cases} \tag{15}$$

where

$\boldsymbol{B}_1=[\boldsymbol{B}_{\mathrm{f}}-\bar{\boldsymbol{H}}\boldsymbol{D}_{\mathrm{f}}\quad \boldsymbol{B}_{\mathrm{d}}-\bar{\boldsymbol{H}}\boldsymbol{D}_{\mathrm{d}}]$, $\boldsymbol{D}_1=[\bar{\boldsymbol{V}}\boldsymbol{D}_{\mathrm{f}}\quad \bar{\boldsymbol{V}}\boldsymbol{D}_{\mathrm{d}}]$ and $\boldsymbol{\omega}=\boldsymbol{d}-\boldsymbol{f}$.

Choosing $\boldsymbol{M}=\boldsymbol{I}_{q\times q}$, $\boldsymbol{N}=[-\boldsymbol{I}_{l\times l}\ \ \boldsymbol{I}_{l\times l}]^{\mathrm{T}}$ and giving $\alpha>0$, we have

$$\|\boldsymbol{T}\|_{\infty}=\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{d}}-\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{f}}\right\|_{\infty}>\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{d}}\right\|_{\infty}-\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{f}}\right\|_{\infty},$$
$$\|\boldsymbol{T}\|_{\infty}<\alpha\Rightarrow\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{d}}\right\|_{\infty}-\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{f}}\right\|_{\infty}<\alpha.$$

Then the reference residual model can be designed by the following optimization problem:

$$\min_{\bar{\boldsymbol{H}},\bar{\boldsymbol{V}}}\alpha\quad \text{s. t. Eq.(13) and Eq.(15).} \tag{16}$$

The following theorem provides us a sufficient condition ensuring that for a given $\alpha>0$, the reference RFDF satisfies Eq.(16).

Theorem 1 Given $\alpha>0$ and reference residual model of Eqs.(7) and (8), if there exist symmetric positive-definite matrices $\boldsymbol{P}>\boldsymbol{0}$, $\boldsymbol{Q}>\boldsymbol{0}$ and $\boldsymbol{Z}>\boldsymbol{0}$ as well as matrix $\boldsymbol{Y}$ such that

$$\boldsymbol{\Xi}:=\begin{bmatrix}\boldsymbol{\Xi}_1 & \boldsymbol{\Xi}_2 & \boldsymbol{P}\boldsymbol{A}_{\mathrm{d}}\\ * & \boldsymbol{\Xi}_3 & \boldsymbol{0}\\ * & * & -\boldsymbol{Q}\end{bmatrix}<\boldsymbol{0} \tag{17}$$

holds, then system of Eqs.(7) and (8) is stable and satisfies $\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{d}}(s)\right\|_{\infty}-\left\|\boldsymbol{T}_{r_{\mathrm{f}}\boldsymbol{f}}(s)\right\|_{\infty}<\alpha$, moreover $\bar{\boldsymbol{H}}=\boldsymbol{P}^{-1}\boldsymbol{Y}$, $\bar{\boldsymbol{V}}=\boldsymbol{Z}^{1/2}$. Here

$$\boldsymbol{\Xi}_1=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}-\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Y}^{\mathrm{T}}-\boldsymbol{Y}\boldsymbol{C}+\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{C}+\boldsymbol{Q},$$
$$\boldsymbol{\Xi}_2=\boldsymbol{P}(\boldsymbol{B}_{\mathrm{d}}-\boldsymbol{B}_{\mathrm{f}})+\boldsymbol{Y}(\boldsymbol{D}_{\mathrm{f}}-\boldsymbol{D}_{\mathrm{d}})+\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Z}(\boldsymbol{D}_{\mathrm{d}}-\boldsymbol{D}_{\mathrm{f}}),$$
$$\boldsymbol{\Xi}_3=(\boldsymbol{D}_{\mathrm{d}}-\boldsymbol{D}_{\mathrm{f}})^{\mathrm{T}}\boldsymbol{Z}(\boldsymbol{D}_{\mathrm{d}}-\boldsymbol{D}_{\mathrm{f}})-\alpha^2\boldsymbol{I},$$

$\boldsymbol{Z}^{1/2}$ denotes the square root factorization of the matrix $\boldsymbol{Z}$.

Proof Define Lyapunov-Krasovskii function candidate

$$V_1(\boldsymbol{e}_{\mathrm{f}}(t),t)=\boldsymbol{e}_{\mathrm{f}}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_{\mathrm{f}}(t)+\int_{t-h}^{t}\boldsymbol{e}_{\mathrm{f}}^{\mathrm{T}}(\tau)\boldsymbol{Q}\boldsymbol{e}_{\mathrm{f}}(\tau)\mathrm{d}\tau, \tag{18}$$

where $\boldsymbol{P}>\boldsymbol{0}$ and $\boldsymbol{Q}>\boldsymbol{0}$.

Consider the following index

$$\begin{aligned}J_1&:=\int_0^{\infty}\boldsymbol{r}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{r}_{\mathrm{f}}\mathrm{d}t-\alpha^2\int_0^{\infty}\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega}\mathrm{d}t\\ &=\int_0^{\infty}\left[\boldsymbol{r}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{r}_{\mathrm{f}}-\alpha^2\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega}+\dot{V}_1(\boldsymbol{e}_{\mathrm{f}},t)\right]\mathrm{d}t\\ &\quad-V_1(\boldsymbol{e}_{\mathrm{f}},t)\big|_t+V_1(\boldsymbol{e}_{\mathrm{f}},t)\big|_{t=0}.\end{aligned} \tag{19}$$

Denote $\boldsymbol{Y}=\boldsymbol{P}\bar{\boldsymbol{H}}$, $\boldsymbol{Z}=\bar{\boldsymbol{V}}^{\mathrm{T}}\bar{\boldsymbol{V}}$, set $\boldsymbol{M}=\boldsymbol{I}_{q\times q}$, $\boldsymbol{N}=[-\boldsymbol{I}_{l\times l}\ \boldsymbol{I}_{l\times l}]^{\mathrm{T}}$. Because $V_1(\boldsymbol{e}_\mathrm{f}, t)|_{t=0}=0$ under zero initial condition and $V_1(\boldsymbol{e}_\mathrm{f}, t)>0$ unless $\boldsymbol{e}_\mathrm{f}(t)=\boldsymbol{0}$, substituting the time derivative of $V_1(\boldsymbol{e}_\mathrm{f}, t)$ along Eq.(15) into J_1 leads to

$$J_1<\int_0^\infty[\boldsymbol{r}_\mathrm{f}^{\mathrm{T}}\boldsymbol{r}_\mathrm{f}-\alpha^2\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega}+\dot{V}_1(\boldsymbol{e}_\mathrm{f},t)]\mathrm{d}t$$
$$=\int_0^\infty\begin{bmatrix}\boldsymbol{e}_\mathrm{f}(t)\\ \boldsymbol{\omega}(t)\\ \boldsymbol{e}_\mathrm{f}(t-h)\end{bmatrix}^{\mathrm{T}}\boldsymbol{\Xi}\begin{bmatrix}\boldsymbol{e}_\mathrm{f}(t)\\ \boldsymbol{\omega}(t)\\ \boldsymbol{e}_\mathrm{f}(t-h)\end{bmatrix}\mathrm{d}t.$$

$\boldsymbol{\Xi}<\boldsymbol{0}$ implies that $J_1<0$. According to Eq.(17), $\|\boldsymbol{T}_{r_\mathrm{f}d}(s)-\boldsymbol{T}_{r_\mathrm{f}f}(s)\|_\infty<\alpha$ holds. Because $\|\boldsymbol{T}_{r_\mathrm{f}d}(s)\|_\infty-\|\boldsymbol{T}_{r_\mathrm{f}f}(s)\|_\infty<\|\boldsymbol{T}_{r_\mathrm{f}d}(s)-\boldsymbol{T}_{r_\mathrm{f}f}(s)\|_\infty$, so $\|\boldsymbol{T}_{r_\mathrm{f}d}(s)\|_\infty-\|\boldsymbol{T}_{r_\mathrm{f}f}(s)\|_\infty<\alpha$ holds.

Consider the following inequality

$$\begin{bmatrix}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}-\boldsymbol{Y}\boldsymbol{C}-\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Y}^{\mathrm{T}}+\boldsymbol{Q} & \boldsymbol{P}\boldsymbol{A}_\mathrm{d}\\ \boldsymbol{A}_\mathrm{d}^{\mathrm{T}}\boldsymbol{P} & -\boldsymbol{Q}\end{bmatrix}<\boldsymbol{0}, \quad (20)$$

which guarantees $\dot{V}_1(\boldsymbol{e}_\mathrm{f},t)<0$ in case $\boldsymbol{\omega}=\boldsymbol{0}$. If the LMI Eq.(17) is feasible, then the LMI Eq.(20) is also feasible. Thus the system of Eqs.(7) and (8) is stable. This completes the proof.

Remark In Theorem 1, the reference residual model has two parameters (i.e. $\bar{\boldsymbol{H}}$ and $\bar{\boldsymbol{V}}$) to be designed. But in (Ding *et al.*, 2001), the reference residual model is only determined by the matrix $\bar{\boldsymbol{H}}$. Ding *et al.*(2001) did not consider the matrix $\bar{\boldsymbol{V}}$, which is only a special case. In fact, if the residual weighting matrix $\bar{\boldsymbol{V}}$ is made to identity matrix $\boldsymbol{I}$, the form in the present reference residual model is equal to the one in (Ding *et al.*, 2001). In addition, the method presented in (Ding *et al.*, 2001) requires that the coefficient matrix of fault $\boldsymbol{D}_\mathrm{f}$ has to be of full column rank; in fact, when considering the faults appearing in state and output equations, the matrix $\boldsymbol{D}_\mathrm{f}$ cannot possibly have full column rank. Therefore, the applications of the approach proposed in (Ding *et al.*, 2001) to fault diagnosis are very limited. Yet in this paper, the matrix $\boldsymbol{D}_\mathrm{f}$ that is of full column rank is not required, which will be seen later in Section 4.

Design of RFDF

The next thing to be solved is to design the RFDF for nonlinear time-delay systems, which is formulated as an H_∞ model-matching problem and solved via an LMI formulation. The following theorem presents a sufficient condition to guarantee that the RFDF system is stable and has a prescribed H_∞ performance, independently of time delay.

Theorem 2 Consider the nonlinear time-delay system

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{A}_\mathrm{d}\boldsymbol{x}(t-h)+\boldsymbol{G}g\big(\boldsymbol{x}(t),\boldsymbol{x}(t-h)\big)+\boldsymbol{B}\boldsymbol{w}(t), \quad (21)$$
$$\boldsymbol{z}=\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{D}\boldsymbol{w}(t), \quad (22)$$
$$\boldsymbol{x}(t)=\boldsymbol{0},\ \forall t\in[-h,\ 0].$$

For a given positive constant $\gamma>0$, if there exist scalars $\varepsilon>0$ ($2\varepsilon\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{\rho}_2<\boldsymbol{Q}$) such that LMI

$$\begin{bmatrix}\boldsymbol{\Gamma} & \boldsymbol{P}\boldsymbol{A}_\mathrm{d} & \boldsymbol{P}\boldsymbol{B} & \boldsymbol{C}^{\mathrm{T}} & \boldsymbol{P}\boldsymbol{G}\\ * & 2\varepsilon\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{\rho}_2-\boldsymbol{Q} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & -\gamma^2\boldsymbol{I} & \boldsymbol{D}^{\mathrm{T}} & \boldsymbol{0}\\ * & * & * & -\boldsymbol{I} & \boldsymbol{0}\\ * & * & * & * & -\varepsilon\boldsymbol{I}\end{bmatrix}<\boldsymbol{0} \quad (23)$$

has symmetric positive-definite matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$, then the system is robustly asymptotically stable and satisfies $\|\boldsymbol{z}\|_2<\gamma\|\boldsymbol{w}\|_2$, where

$$\boldsymbol{\Gamma}=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}+2\varepsilon\boldsymbol{\rho}_1^{\mathrm{T}}\boldsymbol{\rho}_1+\boldsymbol{Q}.$$

Proof Define the following Lyapunov-Krasovskii function candidate

$$V_2\big(\boldsymbol{x}(t),t\big)=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t)+\int_{t-h}^{t}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{Q}\boldsymbol{x}(\tau)\mathrm{d}\tau. \quad (24)$$

The time-derivative of $V_2(\boldsymbol{x}(t), t)$ along the solution of Eq.(21) is then given by

$$\begin{aligned}\dot{V}_2(\boldsymbol{x}(t),t)=&2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}[\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{A}_\mathrm{d}\boldsymbol{x}(t-h)\\ &+\boldsymbol{G}g(\boldsymbol{x}(t),\boldsymbol{x}(t-h))+\boldsymbol{B}\boldsymbol{w}(t)]\\ &+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-h)\boldsymbol{Q}\boldsymbol{x}(t-h).\end{aligned} \quad (25)$$

Using Assumption 2, we have

$$\left.\begin{aligned}&\|g(\boldsymbol{x}(t),\boldsymbol{x}(t-h))\|\le\|\boldsymbol{\rho}_1\boldsymbol{x}(t)\|+\|\boldsymbol{\rho}_2\boldsymbol{x}(t-h)\|,\\ &\|g(\boldsymbol{x}(t),\boldsymbol{x}(t-h))\|^2\le 2\|\boldsymbol{\rho}_1\boldsymbol{x}(t)\|^2+2\|\boldsymbol{\rho}_2\boldsymbol{x}(t-h)\|^2,\end{aligned}\right\} \quad (26)$$

Considering this and using Lemma 2, we can deduce that for any scalar $\varepsilon>0$

$$\begin{aligned}&2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PG}g(\boldsymbol{x}(t),\boldsymbol{x}(t-h))\\&\quad\le\varepsilon g^{\mathrm{T}}(\boldsymbol{x}(t),\boldsymbol{x}(t-h))g(\boldsymbol{x}(t),\boldsymbol{x}(t-h))\\&\qquad+\varepsilon^{-1}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PGG}^{\mathrm{T}}\boldsymbol{Px}(t)\\&\quad\le2\varepsilon\left[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{\rho}_1^{\mathrm{T}}\boldsymbol{\rho}_1\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t-h)\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{\rho}_2\boldsymbol{x}(t-h)\right]\\&\qquad+\varepsilon^{-1}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PGG}^{\mathrm{T}}\boldsymbol{Px}(t).\end{aligned}$$

Thus

$$\begin{aligned}\dot V_2\left(\boldsymbol{x}(t),t\right)&\le2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}[\boldsymbol{Ax}(t)+\boldsymbol{A}_{\mathrm{d}}\boldsymbol{x}(t-h)+\boldsymbol{Bw}(t)]\\&\quad+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Qx}(t)-\boldsymbol{x}^{\mathrm{T}}(t-h)\boldsymbol{Qx}(t-h)\\&\quad+\varepsilon^{-1}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PGG}^{\mathrm{T}}\boldsymbol{Px}(t)\\&\quad+2\varepsilon\left[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{\rho}_1^{\mathrm{T}}\boldsymbol{\rho}_1\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t-h)\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{\rho}_2\boldsymbol{x}(t-h)\right].\end{aligned}$$

Consider the following index

$$J_2=\int_0^\infty\left[\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)\right]\mathrm{d}t, \tag{27}$$

because $V_2(\boldsymbol{x}(t), t)|_{t=0}=0$ under zero initial condition and $V_2(\boldsymbol{x}(t), t)>0$ unless $\boldsymbol{x}(t)=\boldsymbol{0}$, substitute the time derivative of $V_2(\boldsymbol{e}_{\mathrm{f}}, t)$ into J_2, then we have

$$\begin{aligned}J_2&\le\int_0^\infty\left[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}(t)-\gamma^2\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)+\dot V_2(\boldsymbol{x}(t),t)\right]\mathrm{d}t\\&=\int_0^\infty\boldsymbol{X}^{\mathrm{T}}\boldsymbol{\Omega X}\mathrm{d}t,\end{aligned}$$

where

$$\boldsymbol{X}=\begin{bmatrix}\boldsymbol{x}(t)\\\boldsymbol{x}(t-h)\\\boldsymbol{w}(t)\end{bmatrix},\ \boldsymbol{\Omega}=\begin{bmatrix}\boldsymbol{U}&\boldsymbol{PA}_{\mathrm{d}}&\boldsymbol{PB}+\boldsymbol{C}^{\mathrm{T}}\boldsymbol{D}\\\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{P}&2\varepsilon\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{\rho}_2-\boldsymbol{Q}&\boldsymbol{0}\\\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{D}^{\mathrm{T}}\boldsymbol{C}&\boldsymbol{0}&\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}-\gamma^2\boldsymbol{I}\end{bmatrix},$$

$$\boldsymbol{U}=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{PA}+\varepsilon^{-1}\boldsymbol{PGG}^{\mathrm{T}}\boldsymbol{P}+2\varepsilon\boldsymbol{\rho}_1^{\mathrm{T}}\boldsymbol{\rho}_1+\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}+\boldsymbol{Q}.$$

$\boldsymbol{\Omega}<\boldsymbol{0}$ implies that $J_2<0$. After some manipulation using Lemma 1, the inequality $\boldsymbol{\Omega}<\boldsymbol{0}$ is equivalently changed to the condition of Eq.(23), thus $\|\boldsymbol{z}\|_2<\gamma\|\boldsymbol{w}\|_2$ holds.

Consider the following inequality

$$\begin{bmatrix}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{PA}+2\varepsilon\boldsymbol{\rho}_1^{\mathrm{T}}\boldsymbol{\rho}_1+\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}+\boldsymbol{Q}&\boldsymbol{PA}_{\mathrm{d}}&\boldsymbol{PG}\\ *&2\varepsilon\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{\rho}_2-\boldsymbol{Q}&\boldsymbol{0}\\ *&*&-\varepsilon\boldsymbol{I}\end{bmatrix}<\boldsymbol{0} \tag{28}$$

which guarantees $\dot V_2(\boldsymbol{x}(t),t)<0$ in case $\boldsymbol{w}(t)=\boldsymbol{0}$. If the LMI (23) is feasible, then the LMI Eq.(28) is also feasible. Thus the system of Eqs.(21) and (22) is stable. This completes the proof.

Using Theorem 2, the RFDF design problem formulated earlier can be easily solved.

Theorem 3 For a given positive constant $\gamma>0$, if there exist scalar $\varepsilon>0$ ($2\varepsilon\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{\rho}_2<\boldsymbol{Q}_1$) and symmetric positive-define matrices $\boldsymbol{P}_1>\boldsymbol{0}$, $\boldsymbol{P}_2>\boldsymbol{0}$, $\boldsymbol{Q}_1>\boldsymbol{0}$, $\boldsymbol{Q}_2>\boldsymbol{0}$, matrices $\boldsymbol{Y}$ and $\boldsymbol{V}$ such that LMI (29) (see below) holds, then the system of Eqs.(9) and (10) is robustly asymptotically stable and satisfies $\|\boldsymbol{r}_{\mathrm{e}}\|_2<\gamma\|\boldsymbol{w}\|_2$. Furthermore, the observer gain matrix is given by

$$\boldsymbol{H}=\boldsymbol{P}_1^{-1}\boldsymbol{Y}, \tag{30}$$

where

$$\boldsymbol{\Omega}_1=\boldsymbol{P}_1\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_1-\boldsymbol{YC}-\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Y}^{\mathrm{T}}+2\varepsilon\boldsymbol{\rho}_1^{\mathrm{T}}\boldsymbol{\rho}_1+\boldsymbol{Q}_1,$$
$$\boldsymbol{\Omega}_2=\boldsymbol{P}_2\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_2-\boldsymbol{P}_2\bar{\boldsymbol{H}}\boldsymbol{C}-\boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{H}}^{\mathrm{T}}\boldsymbol{P}_2+\boldsymbol{Q}_2.$$

Proof The proof can be obtained directly by using Theorem 2.

The condition of Eq.(29) is an LMI condition. Therefore for fixed $\gamma>0$, the observer gain matrix $\boldsymbol{H}$

$$\begin{bmatrix}\boldsymbol{\Omega}_1&\boldsymbol{0}&\boldsymbol{P}_1\boldsymbol{A}_{\mathrm{d}}&\boldsymbol{0}&\boldsymbol{P}_1\boldsymbol{B}_{\mathrm{f}}-\boldsymbol{YD}_{\mathrm{f}}&\boldsymbol{P}_1\boldsymbol{B}_{\mathrm{d}}-\boldsymbol{YD}_{\mathrm{d}}&\boldsymbol{C}^{\mathrm{T}}\boldsymbol{V}^{\mathrm{T}}&\boldsymbol{P}_1\boldsymbol{G}\\ *&\boldsymbol{\Omega}_2&\boldsymbol{0}&\boldsymbol{P}_2\boldsymbol{A}_{\mathrm{d}}&\boldsymbol{P}_2(\boldsymbol{B}_{\mathrm{f}}-\bar{\boldsymbol{H}}\boldsymbol{D}_{\mathrm{f}})&\boldsymbol{P}_2(\boldsymbol{B}_{\mathrm{d}}-\bar{\boldsymbol{H}}\boldsymbol{D}_{\mathrm{d}})&-\boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{V}}^{\mathrm{T}}&\boldsymbol{0}\\ *&*&2\varepsilon\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{\rho}_2-\boldsymbol{Q}_1&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ *&*&*&-\boldsymbol{Q}_2&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ *&*&*&*&-\gamma^2\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{D}_{\mathrm{f}}^{\mathrm{T}}[\boldsymbol{V}^{\mathrm{T}}-\bar{\boldsymbol{V}}^{\mathrm{T}}]&\boldsymbol{0}\\ *&*&*&*&*&-\gamma^2\boldsymbol{I}&\boldsymbol{D}_{\mathrm{d}}^{\mathrm{T}}[\boldsymbol{V}^{\mathrm{T}}-\bar{\boldsymbol{V}}^{\mathrm{T}}]&\boldsymbol{0}\\ *&*&*&*&*&*&-\boldsymbol{I}&\boldsymbol{0}\\ *&*&*&*&*&*&*&-\varepsilon\boldsymbol{I}\end{bmatrix}<\boldsymbol{0} \tag{29}$$

and the residual weighting matrix $\boldsymbol{V}$ can be easily obtained by using some software such as Matlab LMI toolbox. In order to achieve an RFDF with minimal γ, the Matlab LMI Mincx solver can be used.

The last step of fault detection is to evaluate the residual. Under the assumption of $\boldsymbol{d}\in L_2$, we can further have $\|\boldsymbol{d}(t)\|_2=S$ (S>0). By using Theorem 2, we can obtain $\gamma_d=\sup_{d\in L_2}(\|\boldsymbol{r}\|_2/\|\boldsymbol{d}\|_2)$. In the fault-free case, the generated residual $\boldsymbol{r}(t)$ is only affected by the disturbance input $\boldsymbol{d}(t)$. Therefore, the threshold can be determined by

$$J_{\mathrm{th}}=\gamma_{\mathrm{d}}\|\boldsymbol{d}(t)\|_2=\gamma_{\mathrm{d}}S. \tag{31}$$

According to the logical relationship (12), we can detect the fault.

NUMERICAL EXAMPLES AND SIMULATIONS

Consider the nonlinear time-delay system of Eqs.(1) and (2) with parameters given by

$$\boldsymbol{A}=\begin{bmatrix}-1.80 & 0.50 & -0.50\\ 0.50 & -3.00 & 0.90\\ -0.30 & 0.70 & -2.40\end{bmatrix},\ \boldsymbol{B}=\begin{bmatrix}0.00\\ 0.20\\ -0.40\end{bmatrix},$$

$$\boldsymbol{A}_{\mathrm{d}}=\begin{bmatrix}0.40 & -0.10 & -0.01\\ 0.10 & -0.30 & 0.20\\ 0.10 & -0.10 & 0.50\end{bmatrix},\ \boldsymbol{B}_{\mathrm{f}}=\begin{bmatrix}0.70 & 0\\ -0.50 & 0\\ 0.80 & 0\end{bmatrix},$$

$$\boldsymbol{B}_{\mathrm{d}}=\begin{bmatrix}0 & 1.00\\ 0 & 0.20\\ 0 & -0.30\end{bmatrix},\ \boldsymbol{G}=\begin{bmatrix}0.1 & 0.2 & 0.3\\ 0.2 & 0.5 & 0.1\\ 0.4 & 0.1 & 0.5\end{bmatrix}, \boldsymbol{D}=\boldsymbol{0},$$

$$\boldsymbol{C}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\ \boldsymbol{D}_{\mathrm{f}}=\begin{bmatrix}0 & 0.40\\ 0 & 0.80\\ 0 & -1.20\end{bmatrix},\ \boldsymbol{D}_{\mathrm{d}}=\begin{bmatrix}1.50 & 0\\ 0.20 & 0\\ 1.00 & 0\end{bmatrix}.$$

It is evident that the method presented in (Ding *et al*., 2001) cannot be suitable for diagnosing the fault because $\boldsymbol{D}_{\mathrm{f}}$ does not have full column rank. Here we use a new approach to design observer-based RFDF.

Using Theorem 1, we have the reference residual model in the form of Eqs.(7) and (8) with $\bar{\boldsymbol{H}}$, $\bar{\boldsymbol{V}}$ and the minimal $\alpha_{\min}$ respectively, as follows

$$\bar{\boldsymbol{H}}=\begin{bmatrix}-0.7094 & -0.2687 & 0.4178\\ 0.6860 & -1.0665 & -0.3157\\ -0.6911 & 0.8273 & 0.0712\end{bmatrix},$$

$$\bar{\boldsymbol{V}}=\begin{bmatrix}0.4443 & -0.4007 & -0.2321\\ 0 & 0.5846 & 0.1717\\ 0 & 0 & 0.3552\end{bmatrix},$$

$$\alpha_{\min}=0.5847.$$

Subject to the constraint $2\varepsilon\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{\rho}_2<\boldsymbol{Q}_1$, we choose

$$\varepsilon=0.1,\ \boldsymbol{\rho}_1=\begin{bmatrix}0.2 & 0.1 & 0.3\\ 0.1 & 0.4 & 0.5\\ 0.2 & 0.3 & 0.6\end{bmatrix},\ \boldsymbol{\rho}_2=\begin{bmatrix}0.1 & 0.2 & 0.4\\ 0.3 & 0.2 & 0.5\\ 0.1 & 0.4 & 0.1\end{bmatrix}.$$

By using Theorem 3, we obtain the solutions to LMI (29) with $\boldsymbol{H}$, $\boldsymbol{V}$ and minimal $\gamma_{\min}$ respectively, as follows

$$\boldsymbol{H}=\begin{bmatrix}0.3998 & 1.0138 & -0.0020\\ -1.3579 & 2.8764 & 1.3933\\ -0.7825 & 1.9165 & 1.1755\end{bmatrix},$$

$$\boldsymbol{V}=\begin{bmatrix}0.5495 & -0.0796 & -0.3104\\ 0.2197 & 0.1486 & -0.0807\\ 0.1170 & -0.1711 & 0.2893\end{bmatrix},$$

$$\gamma_{\min}=0.4445.$$

To demonstrate the effectiveness of the design, the nonlinear part is assumed to be $g(\boldsymbol{x}(t), \boldsymbol{x}(t-h))=(\sin 0.2)\boldsymbol{x}(t)+(\sin 0.1)\boldsymbol{x}(t-h)$, the time delay h is assumed to be 0.5 s, an unknown input $\boldsymbol{d}(t)$ is assumed to be band-limited white noise with power 0.001 (zero-order holds with sampling time 0.01 s) and the input $\boldsymbol{u}(t)$ is taken as unit step signal. The fault signal $\boldsymbol{f}(t)$ is simulated as a square ware of unit amplitude that occurred from 5 s to 10 s.

The fault signal $\boldsymbol{f}(t)$, the output signal $\boldsymbol{y}(t)$ and the generated residual signals $\boldsymbol{r}(t)$ (including $r_1(t)$ and $r_2(t)$) are shown in Fig.1, Fig.2 and Fig.3, respectively. Fig.4 shows the evolution of residual evaluation function $\|\boldsymbol{r}(t)\|_{2,\tau}$, from which we can calculate when the fault can be detected. By using Theorem 2, we have minimal γ_{d}=0.7148. Suppose S=1, then the threshold is $J_{\mathrm{th}}=\gamma_{\mathrm{d}}S\approx0.7148$. In Fig.4, we can see that $\|\boldsymbol{r}(t)\|_{2,6}\approx0.73>0.7148$ for t_1=0 s and t_2=6 s. This means that the fault $\boldsymbol{f}(t)$ can be detected 1 s after its occurrence.

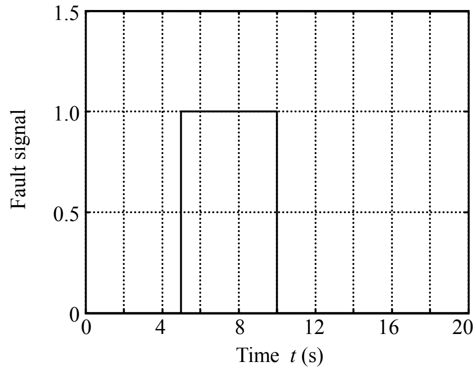

Fig.1 Fault signal *f*(*t*)

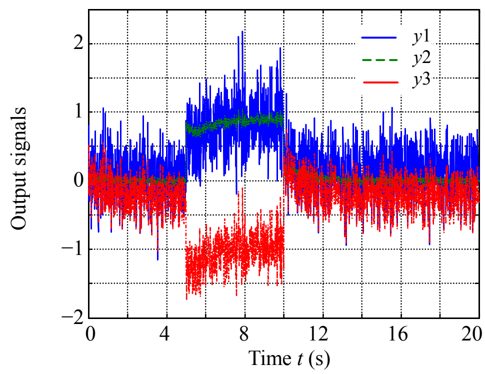

Fig.2 Output signal *y*(*t*)

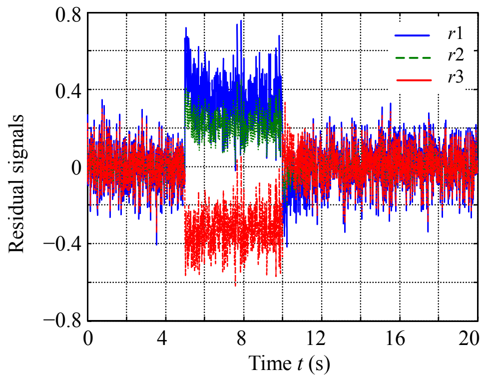

Fig.3 Generated residual *r*(*t*)

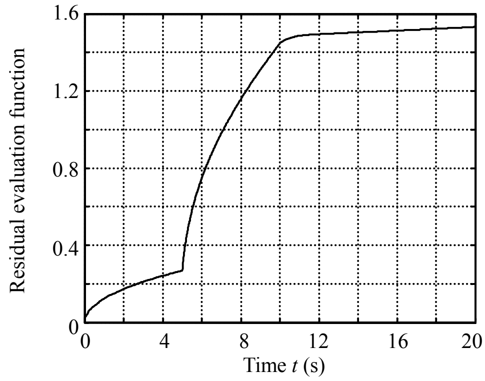

Fig.4 Evolution of residual evaluation function

CONCLUSION

In this paper, the observer-based RFDF design problem is studied for nonlinear time-delay systems with unknown inputs. The main contribution of our study is the introduction of an optimal reference residual model (considering both observer gain matrix and residual weighting matrix) to formulate the RFDF design problem as an H_∞ model-matching problem. Moreover, we introduce the appropriate input/output selection matrices to extend a performance index to the time-delay systems in time domain. The reference residual model, which can be used to describe the robustness against disturbances and sensitivity to faults simultaneously, is an optimal solution of the RFDF. The existence of the solution is presented in terms of LMI formulation, which can be obtained conveniently by using Matlab LMI toolbox. An illustrative example has demonstrated the validity and applicability of the proposed approach.

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