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Structural dynamic responses analysis applying differential quadrature method

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Abstract: Unconditionally stable higher-order accurate time step integration algorithms based on the differential quadrature method (DQM) for second-order initial value problems were applied and the quadrature rules of DQM, computing of the weighting coefficients and choices of sampling grid points were discussed. Some numerical examples dealing with the heat transfer problem, the second-order differential equation of imposed vibration of linear single-degree-of-freedom systems and double-degree-of-freedom systems, the nonlinear move differential equation and a beam forced by a changing load were computed, respectively. The results indicated that the algorithm can produce highly accurate solutions with minimal time consumption, and that the system total energy can remain conservative in the numerical computation.

Key words: Differential quadrature method (DQM), Dynamic response analysis, Conservation of energy
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INTRODUCTION

The differential quadrature method (DQM) was proposed by Bellman and Casti as an analogous extension of the quadrature method for integrals (Bellman and Casti, 1971; Bellman *et al*., 1972). It can be essentially expressed as the values of the derivatives at each grid point as weighted linear sums approximately of the function values at all grid points within the domain under consideration. As a distinct numerical solution technique to obtain the initial and/or boundary value problems of engineering and physical sciences, the advantage of producing highly accurate solutions with minimal DQM computational effort is gradually emerging (Bert and Malik, 1996; Malik and Civan, 1995) compared with the conventional numerical solution techniques such as the finite difference and finite element method. Currently, the research areas where DQM is applied include fluid mechanics, static and dynamic structural mechanics, heat transfer, biosciences, transport processes, aeroelasticity, lubrication mechanics and petrochemical engineering. Bert *et al*.(1988; 1989) first applied DQM to solve structural mechanics problems in 1987. The algorithm was successfully applied to static and dynamic structural analysis of the structural components soon afterwards (Bert *et al*., 1993) and highly accurate solutions can be achieved for free vibrational analysis of beam and plate (Wang *et al*., 2004; Malekzadeh, 2005; Karami *et al*., 2006). DQM can be used for both spatial and temporal discretization. In general, the boundary conditions are expressed by applying differential quadrature analog equations at the sampling grid points on or near the boundaries, these analog equations are then used to replace the differential quadrature analog equations of the governing differential equations at these points in order to solve the boundary value problems. However, this procedure will become very tedious when higher order derivations or multi-boundary conditions are involved along the boundaries. A δ-technique was proposed (Bert *et al*., 1988; Jang *et al*., 1989) to impose the first derivative boundary conditions, although this technique may also create problems of ill-conditioned weighting coefficient matrices and unexpected oscillation behaviour of the solution (Bert and Malik, 1996). Several methods were proposed (Wang *et al*., 1993; Malik and Bert, 1996) to incorporate the boundary conditions in the weighting coefficient matrices. Shu and Du (1997) pointed out that these techniques have some major limitations and cannot be used to tackle general boundary conditions. Some methods (Chen *et al*., 1997; Wang and Gu, 1997) have also been proposed for the higher-order derivatives boundary conditions imposed exactly by modifying the trial functions to incorporate the degrees of freedom of the higher-order derivatives at the boundary or by using the differential quadrature element method directly. However, the general formulas for the explicit weighting coefficients are not yet available. In order to solving second- and higher-order initial value problems based on DQM, a scheme on how to impose the given initial conditions has to be considered. Fung (2001a; 2001b; 2003a; 2003b) proposed unconditionally stable higher-order accurate time step integration algorithms, whose numerical solutions are found to be equivalent to the generalized Pade approximations, for second- and higher-order initial value problems based on DQM. In this paper, some numerical examples dealing with the heat transfer problem, the second-order differential equation of imposed vibration of linear single-degree-of-freedom systems and double-degree-of-freedom systems, the nonlinear move differential equation and a beam forced by a changing load (Pu, 2004) were computed, respectively. Highly precise computational results can be obtained and the system total energy can remain conservative in the numerical computation (Kuhl and Crisfield, 1999).

DIFFERENTIAL QUADRATURE METHOD

A differential form can be written approximately as Eq.(1) by virtue of the coefficient matrix A_{ij} determined in various fashions

$$f'(x_i) \approx \sum_{j=1}^{N} A_{ij} f(x_j), \quad i = 1,\ 2,\ ...,\ N. \tag{1}$$

This procedure is called "differential quadrature", and in essence is expressed as the values of the derivatives at each grid point regarded as weighted linear sums approximately of the function values at all grid points within the domain under consideration. In the method, all sampling grid points are used to analogize the each-order derivation of the function at each point, so a highly accurate numerical solution can be obtained by using a few sampling grid points. The numerical method for solving initial or boundary value problem is to seek a transformation through a differential or an integral formulation so that the governing differential or integro-differential equations are changed into a set of first-order or algebraic analogous equations in terms of the discrete values of the field variable at some prespecified discrete points of the solution domain. In DQM, this is accomplished by expressing at each grid point, the calculus operator value of a function with respect to a coordinate direction at any discrete point as the weighted linear sum of the function values at all the discrete points chosen in that direction. For a function $\psi=\psi(x,y)$ in the given domain, the values of the rth-order x-partial derivations of the function at a discrete point $x=x_i$ along any line $y=y_j$ parallel to the x-axis can be expressed approximately as the weighted linear sums of the function values

$$\left.\frac{\partial^r \psi}{\partial x^r}\right|_{x=x_i} = \sum_{k=0}^{N_x-1} A_{ik}^{(r)} \psi_{kj},\ i = 1,\ 2,\ 3,\ ...,\ N_x, \tag{2}$$

and sth-order y-partial derivations at a discrete point $y=y_j$ along any line $x=x_i$ parallel to the y-axis

$$\left.\frac{\partial^s \psi}{\partial y^s}\right|_{y=y_j} = \sum_{l=1}^{N_y} B_{jl}^{(s)} \psi_{il},\ j = 1,\ 2,\ 3,\ ...,\ N_y, \tag{3}$$

where $\psi_{ij}=\psi(x_i,y_j)$ with $A_{ik}^{(r)}$ and $B_{jl}^{(s)}$ standing for the respective weighting coefficients. According to the quadrature rules, Eq.(2) can be written in matrix form as

$$\boldsymbol{\psi}_{x,j}^{(r)} = \boldsymbol{A}^{(r)} \boldsymbol{\psi}_j, \tag{4}$$

where, $\boldsymbol{\psi}_j$ and $\boldsymbol{\psi}_{x,j}^{(r)}$ are the column vectors of the N_x values each of the function and its rth-order x-partial derivations, respectively, at the sampling points on a line $y=y_j$. $\boldsymbol{A}^{(r)}$ is the $N_x \times N_x$ matrix of weighting coefficients of the rth-order derivations. According to the definition of the differential operators, the quadrature rule from Eq.(2) can be expressed as

$$\left.\frac{\partial^r \psi}{\partial x^r}\right|_{x=x_i} = \sum_{k=1}^{N_x} A_{ik}^{(1)} \sum_{m=1}^{N_x} A_{km}^{(r-1)} \psi_{mj} = \sum_{k=1}^{N_k} A_{ik}^{(r-1)} \sum_{m=1}^{N_x} A_{km}^{(1)} \psi_{mj}. \tag{5}$$

In Eqs.(4) and (5) there are given with respect to the x-coordinate only, the formulae with respect to the y-coordinate would follow in an identical manner. Following Eqs.(4) and (5), a recurrence relationship for the weighting coefficients can be obtained as

$$\boldsymbol{A}^{(r)} = \boldsymbol{A}^{(1)} \boldsymbol{A}^{(r-1)} = \boldsymbol{A}^{(r-1)} \boldsymbol{A}^{(1)}. \tag{6}$$

It is shown that the matrix $\boldsymbol{A}^{(1)}$ of first-order derivative weighting coefficients can obtain more weighting coefficients of higher-order derivatives by successive multiplications of the $\boldsymbol{A}^{(1)}$ matrix by itself. The weighting coefficient matrix in Eq.(2) is the Vandermonde rank expressions, so the unique value of the weighting coefficient $A_{ik}^{(r)}$ can be obtained in the process of solving it.

CHOOSING WEIGHT COEFFICIENT AND SAMPLE GRID POINTS

Noticing the recurrence relationship of the weighting coefficients in Eq.(6), a first-order weighting coefficient can be expressed as

$$A_{ij} = \prod_{k=1, k\neq i,j}^{N} (x_i - x_k) \Big/ \prod_{k=1, k\neq j}^{N} (x_j - x_k), \quad i \neq j, \tag{7}$$

$$A_{ii} = \sum_{k=1, k\neq i}^{N} \frac{1}{(x_i - x_k)}, \tag{8}$$

By substituting Eqs.(7) and (8) into Eq.(6), the weighting coefficient of higher-order derivative can be obtained by successive multiplications. An explicit formulae proposed by Shu and Richards for the off-diagonal and diagonal terms of the weighting coefficient matrix of the first- and higher-order derivative are given detailedly in (Bert and Malik, 1996), a simple and convenient choice for simplifying grid points is that of equally spaced points, although more accurate results will be obtained with unequally spaced points such as in the Chebyshev-Gauss-Lobatto scheme in (Bert and Malik, 1996). In (Fung, 2001), an explicit formula was proposed for the weighting coefficient matrices of the first-order derivative, the sample grid points $\tau_1, \ldots, \tau_n$ can be regarded as the roots of the nth order polynomial given by

$$\tau^n - W_1 - W_2\tau - W_3\tau^2 - W_n\tau^{n-1} = 0, \tag{9}$$

where

$$W_k = \frac{(-1)^{n-k} n!n!(n+k-2)!}{(k-1)!(k-1)!(n+1-k)!(2n)!} \frac{2(n+\mu(k-1))}{1+\mu}, \quad 0 \le \mu \le 1. \tag{10}$$

DQM FOR INITIAL PROBLEM GOVERNED BY HIGHER-ORDER EQUATIONS

For N-degree-of-freedom systems governed by higher-order equations, a linear mth-order ordinary differential equation can be expressed as Eq.(11) using the DQM

$$\boldsymbol{K}_0 \frac{\mathrm{d}^m \boldsymbol{q}}{\mathrm{d}t^m} + \boldsymbol{K}_1 \frac{\mathrm{d}^{m-1} \boldsymbol{q}}{\mathrm{d}t^{m-1}} + \cdots + \boldsymbol{K}_m \boldsymbol{q} = \boldsymbol{f}(t), \ m \ge 1, \tag{11}$$

where, $\boldsymbol{f}(t)=[f_1(t),\ldots,f_N(t)]^{\mathrm{T}}$ is the excitation vector; $\boldsymbol{q}=[q_1,\ldots,q_N]^{\mathrm{T}}$ includes the N generalized coordinates; $\boldsymbol{K}_0, \ldots, \boldsymbol{K}_m$ are $N\times N$ square matrices. Each order variable at the end of the time interval can be obtained by external interpolation according to Eq.(12).

$$\boldsymbol{q}^{(r)}(1) = L_0^n(1)\boldsymbol{q}_0^{(r)} + [L_1^n(1) \ \ldots \ L_n^n(1)] \otimes \boldsymbol{I}\boldsymbol{Q}^{(r)}, \tag{12}$$

where, $L_k^n(1)$ is nth order Lagrange polynomial, $\otimes$ is the Kronecker product for two matrices.

$$\boldsymbol{Q}^{(r)} = \{\boldsymbol{q}_1^{(r)} \ \ldots \ \boldsymbol{q}_n^{(r)}\}^{-1}, \ r = 0, 1, 2, \ldots, m, \tag{13}$$

where

$$\begin{Bmatrix} \boldsymbol{q}_1^{(1)} \\ \vdots \\ \boldsymbol{q}_n^{(1)} \end{Bmatrix} = \begin{bmatrix} G_{11}\boldsymbol{I} & \cdots & G_{1n}\boldsymbol{I} \\ \vdots & \ddots & \vdots \\ G_{n1}\boldsymbol{I} & \cdots & G_{nn}\boldsymbol{I} \end{bmatrix} \begin{Bmatrix} \boldsymbol{q}_1^{(0)} \\ \vdots \\ \boldsymbol{q}_n^{(0)} \end{Bmatrix} + \begin{bmatrix} G_{10}\boldsymbol{I} \\ \vdots \\ G_{n0}\boldsymbol{I} \end{bmatrix} \boldsymbol{q}_0^{(0)}. \tag{14}$$

The weighted coefficient matrix can be expressed as follows:

$$\boldsymbol{G}=\begin{bmatrix} A_{11}^{(1)} & A_{12}^{(1)} & \cdots & A_{1n}^{(1)} \\ A_{21}^{(1)} & A_{22}^{(2)} & \cdots & A_{2n}^{(1)} \\ \vdots & \vdots & \vdots & \vdots \\ A_{n1}^{(1)} & A_{n2}^{(1)} & \cdots & A_{nn}^{(1)} \end{bmatrix}, \quad \boldsymbol{G}_0=\begin{Bmatrix} A_{10}^{(1)} \\ A_{20}^{(1)} \\ \vdots \\ A_{n0}^{(1)} \end{Bmatrix}. \quad (15)$$

The higher-order derivatives can be expressed as

$$\boldsymbol{Q}^{(r)}=\boldsymbol{G}^r \otimes \boldsymbol{IQ}+\sum_{k=0}^{r-1}\boldsymbol{G}_k \otimes \boldsymbol{q}_0^{(r-k-1)}, \ r=1,\ 2,\ ...,\ m, \quad (16)$$

where

$$\boldsymbol{G}_k=\boldsymbol{G}^k\boldsymbol{G}_0, \quad k=1,\ 2,\ ...,\ N. \quad (17)$$

NUMERICAL EXAMPLES

Example 1

Consider a one-dimensional heat transmission case, the equation governing the temperature may be written in dimensional form as

$$\xi\frac{\mathrm{d}^2\Theta}{\mathrm{d}\xi^2}+\frac{\mathrm{d}\Theta}{\mathrm{d}\xi}=m^2\Theta,\ 0\le\xi\le1, \quad (18)$$

where Θ is the nondimensional temperature and ξ is the nondimensional axial coordinate, m is a dimensionless parameter and taken as 1. The boundary conditions for Eq.(18) are: $\mathrm{d}\Theta/\mathrm{d}\xi=0$ at $\xi=0$ and $\Theta=1$ at $\xi=1$. According to the rules of DQM, the quadrature analog of the governing differential equation and the boundary condition at $\xi=0$ can be written as follows, respectively:

$$\sum_{j=1}^{N-1}[\xi_i A_{ij}^{(2)}+A_{ij}^{(1)}]\Theta_j-m^2\Theta_i=-\xi_i A_{iN}^{(2)}-A_{iN}^{(1)},$$

$$i=2,\ 3,\ ...,\ N-1, \quad (19)$$

$$\sum_{j=1}^{N-1}A_{ij}^{(1)}\Theta_j=-A_{iN}^{(1)}, \quad i=1. \quad (20)$$

The boundary conditions at $\xi=1$ have been built into Eqs.(19) and (20), a set of ($N-1$) linear equations yielding temperature values at the sampling points ξ_i (i=1, 2, 3, …, $N-1$). The quadrature solutions of the problem are obtained by solving Eqs.(19) and (20) using uniformly sampling grid points, Chebyshev-Gauss-Lobatto sampling grid points and the sampling grid points according to Eq.(9). The computing results are shown in Table 1 at ξ=0.1 intervals.

Table 1 Solution of the DQ with different methods

ξ	Θ			
	Exact	Uniformly	Chebyshev	n=9, μ=1
0.0	0.438676	0.480540	0.451936	0.443566
0.1	0.483653	0.502338	0.486722	0.484453
0.2	0.530897	0.542570	0.532956	0.530785
0.3	0.580485	0.588618	0.581550	0.579804
0.4	0.632494	0.638352	0.633721	0.633201
0.5	0.687003	0.691231	0.687756	0.687074
0.6	0.744096	0.747070	0.744454	0.743595
0.7	0.803855	0.805815	0.804334	0.804085
0.8	0.866367	0.867476	0.866595	0.866470
0.9	0.931718	0.932079	0.931809	0.931585

Table 1 shows that the precision using uniformly sampling points is lower than that using Chebyshev-Gauss-Lobatto sampling grid points. Using sampling grid points for n=9 and μ=1 according to Eq.(9), the numerical computing precision is higher than using that Chebyshev-Gauss-Lobatto grid points. The results are shown in Fig.1.

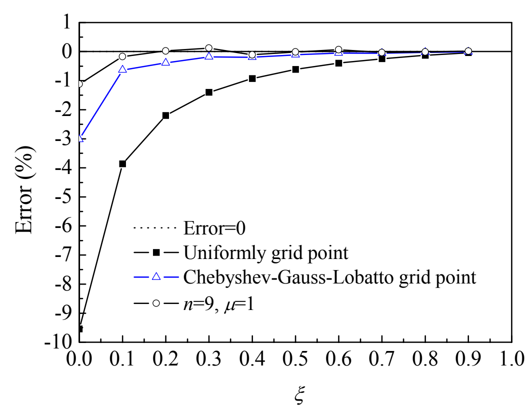

Fig.1 Error comparison with different methods

Example 2

Consider a linear single-degree-of-freedom system, with governing differential equation given by

$$\frac{\mathrm{d}^2u}{\mathrm{d}t^2}+2\xi\omega\frac{\mathrm{d}u}{\mathrm{d}t}+\omega^2u=f(t). \quad (21)$$

In Eq.(21) the initial conditions and the coefficients are: $f(t)=0$, $u_0=\sqrt{2}$, $v_0=0$ at $\xi=0$ and $\omega=1$. The exact solution of the differential equation is $u(t)=\sqrt{2}\cos(t)$, the system total energy is equal to one and should be conservative. The displacement and energy were computed numerically by various methods, using time step $\Delta t=0.2\pi$. Taking $n=3$ and $\mu=1$, the energy of the numerical solution is 1.0003 at the end of 1000 time steps, the error of the numerical solution is only $\varepsilon=(1-1.0003)\times100\%=-0.03\%$. Fig.2 shows that the precision and stability of the algorithm are better than those of the conventional numerical algorithm.

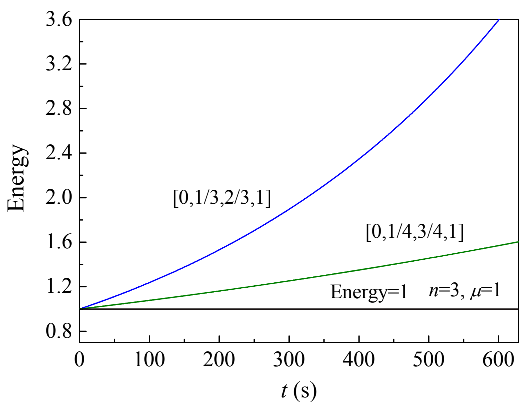

Fig.2 Energy with various methods (Time step number: 1000; $\Delta t=0.2\pi$)

In Eq.(21) taking each item as: $\xi=0.3$, $\omega=2$, $f(t)=h\sin(\theta t)$, $\theta=1$, $u_0=1$ and $v_0=1$, the time step is $\Delta t=6.28$, the coefficients are $h=0$ and $h=3$, respectively. The curves of displacements for free and forced vibration with various methods are shown in Fig.3.

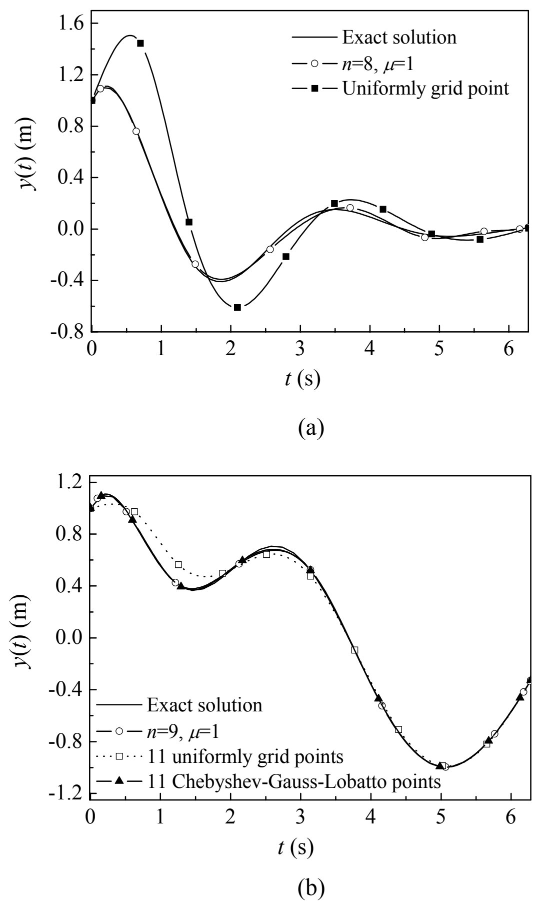

Fig.3 Free (a) and forced (b) vibration cures with various methods ($\Delta t=6.28$)

Example 3

Considering a simple pendulum consisting of a mass attached to a hinged weightless rod, the nonlinear motion equation can be expressed as

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Phi+\omega^2\sin\Phi=0, \qquad (22)$$

where $\Phi(t)$ is the angle between the rod and a vertical line at time t, $\omega=\sqrt{g/L}$, g is the gravitational acceleration, L is the rod length.

The DQM can be used to solve this nonlinear problem. First Eq.(22) is rewritten in a nondimensional form; the quadrature rule equation can be expressed as

$$(\boldsymbol{G}^2+\omega^2\Delta t^2\cos\boldsymbol{\Phi}_i)\Delta\boldsymbol{\Phi}_i =-\boldsymbol{G}^2\boldsymbol{\Phi}_i-\boldsymbol{G}_0\Omega_0\Delta t-\boldsymbol{G}\boldsymbol{G}_0\Phi_0-\omega^2\Delta t^2\sin\boldsymbol{\Phi}_i, \quad i=1,\ 2,\ \ldots,\ n. \qquad (23)$$

where Φ_0 and Ω_0 are initial angle and initial velocity, respectively. Using Newton-Raphson iterative method, $\boldsymbol{\Phi}_i$ for $i=1, 2, \ldots, n$ for each time step Δt can be obtained according to Eq.(24)

$$\boldsymbol{\Phi}_{k+1}=\boldsymbol{\Phi}_k+\Delta\boldsymbol{\Phi}. \qquad (24)$$

Taking coefficient $\omega=1$ and $\Phi_0=0$, the nonlinear equation is solved by using different numerical methods; the computing results is shown in Table 2. The change curves of amplitude Φ and angular velocity Ω are shown in Fig.4.

Table 2 Numerical solution with various methods

	Method/grid points	$T_f/\Delta t$	$\Phi(T_f)$	Error in Φ (%)
$\Omega_0=1$, $T_f=1.6858$	Exact	−	1.04720	−
	$n=2$, $\mu=1$	1	1.04485	−0.2240
	$n=2$, $\mu=1$	2	1.04713	−0.0067
	$n=2$, $\mu=1$	3	1.04718	0.0019
	[0,1/2,1]	1	0.87809	−16.1487
	[0,1/2,1]	5	1.04412	−0.2941
	[0,1/2,1]	10	1.04664	−0.0534
	$n=3$, $\mu=1$	1	1.04721	0.0008
	$n=3$, $\mu=1$	2	1.04719	−0.0006
	[0,1/4,3/4,1]	1	1.04051	0.6388
	[0,1/4,3/4,1]	3	1.04751	−0.0296
	[0,1/4,3/4,1]	5	1.04728	−0.0076
	[0,1/3,2/3,1]	1	1.03787	0.8909
	[0,1/3,2/3,1]	3	1.04812	−0.0878
	[0,1/3,2/3,1]	5	1.04742	−0.0210
$\Omega_0=1.84776$, $T_f=2.4001$	Exact	−	2.35619	−
	$n=2$, $\mu=1$	1	2.20934	−6.2327
	$n=2$, $\mu=1$	3	2.35707	−0.0373
	$n=2$, $\mu=1$	5	2.35623	0.0014
	[0,1/2,1]	1	1.56487	−33.5847
	[0,1/2,1]	5	2.30487	−2.1781
	[0,1/2,1]	10	2.34464	−0.4902
	[0,1/2,1]	20	2.35351	−0.1137
	$n=3$, $\mu=1$	1	2.37920	0.9764
	$n=3$, $\mu=1$	2	2.35571	−0.0204
	$n=3$, $\mu=1$	3	2.35609	0.0042
	[0,1/4,3/4,1]	1	2.02710	13.9670
	[0,1/4,3/4,1]	3	2.35610	0.0038
	[0,1/4,3/4,1]	5	2.35653	−0.0144
	[0,1/4,3/4,1]	10	2.35627	−0.0034
	[0,1/4,3/4,1]	20	2.35621	−0.0008
	[0,1/3,2/3,1]	1	1.78101	24.4114
	[0,1/3,2/3,1]	3	2.35613	0.0025
	[0,1/3,2/3,1]	5	2.35721	−0.0433
	[0,1/3,2/3,1]	10	2.35640	−0.0089
	[0,1/3,2/3,1]	20	2.35623	−0.0017

T_f is the oscillation period, $\Phi(T_f)$ is the oscillation angle

Example 4

Consider double-degree-of-freedom system differential equations in the form

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 3\cos 4t \\ 0 \end{Bmatrix}, \quad (25)$$

with initial condition $\{u_i\} = \{\dot{u}_i\} = 0$ for i=1, 2. The

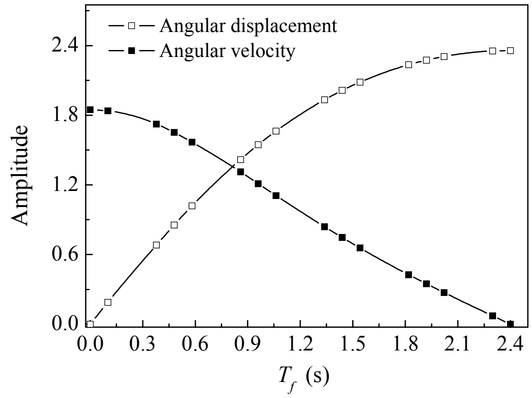

Fig.4 Changing curves of pendulum angle Φ and angular velocity Ω with time

system dynamic response equations were computed for different time steps by using DQM with the numerical results (μ=1) compared with Newmark method (NM) and precise integral method (PIM) (Table 3).

Table 3's computational results show that values are changed minimally with time step Δt and changed remarkably when DQM is used, and that the solving process is stable so that the solving efficiency is quite remarkable.

Example 5

A simply supported beam was forced by a sine load at the middle of the beam with length of L=6 m, section area A=0.12 m^2, inertia moment I=0.0036 m^4, elastic modulus E=2.0×10^5 MPa, density ρ=7850 kg/m^3, load P=320sin(200t) kN. The beam was divided into 100 beam elements, at each end of which are two freedom degrees such as a vertical displacement and angular displacement.

The computing results for the beam were obtained by using NM, PIM and DQM (n=3, μ=1), respectively (Fig.5). The numerical solution accuracy will be lost at time step Δt=0.0025 with NM, so a smaller time step ought to be selected to obtain accurate solution, but at the cost of erectly increased time consumption, the computational results being not precise enough due to cumulating error. Use of DQM yields relatively stable numerical results at time step Δt=0.005, when the precision is the same as that by using the precise time integral method at Δt=0.0025, but the computational efficiency is obviously enhanced.

Table 3 Computational results of the displacement and velocity (t=5.6)

Methods	Δt	u_1	u_2	$\dot{u}_1$	$\dot{u}_2$
Analytical solution	−	0.4093294	−0.1133778	−0.4854754	−0.0788401
DQM n=5	1.4	0.4096396	−0.1134443	−0.4852643	−0.0790288
DQM n=5	0.7	0.4093266	−0.1133765	−0.4854741	−0.0788393
DQM n=5	0.28	0.4093256	−0.1133760	−0.4854734	−0.0788387
DQM n=9	2.8	0.4114815	−0.1159241	−0.4842253	−0.0770746
DQM n=9	1.4	0.4115752	−0.1134285	−0.4879472	−0.0794684
DQM n=9	0.7	0.4096207	−0.1132396	−0.4868947	−0.0787022
PIM	2.8	1.2323203	−0.1209424	−0.5132277	−0.5509784
PIM	1.4	0.3169916	−0.9783549	−0.5018512	−0.0396914
PIM	0.7	0.4105805	−0.1137822	−0.4850493	−0.0792688
PIM	0.28	0.4093320	−0.1133786	−0.4854747	−0.0788410
NM	0.28	0.2953956	−0.0685144	−0.2893168	−0.1206785
NM	0.028	0.4081649	−0.1129032	−0.4829121	−0.0795419
NM	0.0028	0.4093177	−0.1133731	−0.4854498	−0.0788471

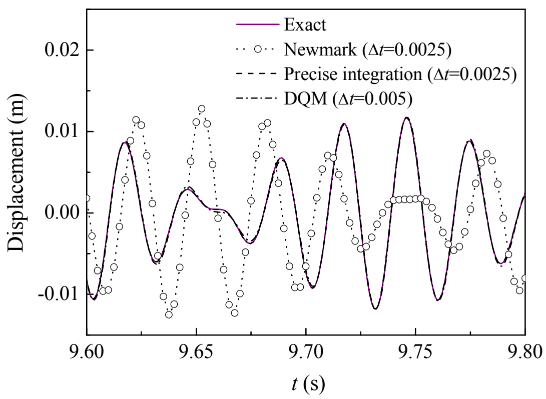

Fig.5 Vertical displacement in the middle of a beam by various methods at 9.6~9.8 s

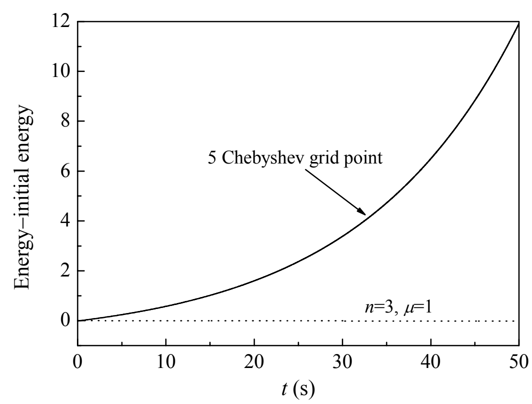

Fig.6 Energy computation for a beam (Time step number: 10000; Δt=0.005)

When there are some initial displacements with unit force P=1 in the middle of the beam, the total energy is then defined as the sum of the kinetic energy and the potential energy. Taking the time step as Δt=0.005, the energy computations are shown in Fig.6 within 10000 time steps. It is evident that the total energy can remain conservation with coefficient n=3, μ=1, taking five Chebyshev grid points; the energy is rapidly dispersed and the effectiveness of the algorithm will be lost.

CONCLUSION

Using unconditionally stable higher-order accurate time step integration algorithms for second-order initial value problems, the dynamic response was computed numerically for double-degree-of-freedom systems and a beam forced by a changing load based on the DQM. Numerical results changing is not distinct for the algorithm with changing time step Δt; the solving process is stable. The time step can be taken to be much larger than that of other algorithms and the solving efficiency is remarkable.

The results showed that the algorithm can yield highly accurate solutions with minimal time consumption compared with the conventional numerical method, and that the energy is perfectly conserved for dynamic response analysis.

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