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Research on bursting pressure formula of mild steel pressure vessel

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Abstract: Of several formulas for calculating bursting pressure of mild steel vessel, the Faupel formula is the most famous one. In fact, Faupel formula is conservative in calculating mild steel pressure. Based on hundreds of bursting experiments on mild steel pressure vessels such as Q235(Gr.D), 20R(1020) and, after statistically analyzing data on bursting pressure, it was found that the Faupel formula had some errors in calculation. The authors derived a more approximate modified formula from the data, which proved more general after examining the data on other mild steel pressure vessels with different diameters and shell thickness.

Key words: Bursting pressure, Pressure vessel, Mild steel, Formula
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INTRODUCTION

In industrial production, mild steel is chosen as the structural material of the shell of many pressure vessels, such as tower reactors and exchangers. Mild steel has high plasticity, toughness and good performance in welding due to the presence of some microelements in it such as silicon, manganese and so on. Being inexpensive, they become main production materials of pressure vessels or chemical equipments. Pressure vessels hold a large amount of energy at working pressure, so it will be disastrous if the pressure vessels burst. So, the problem of bursting pressure vessel has been under constant and extensive research. In recent years, design criterion analysis of important equipments need the bursting value of pressure vessel. Accurate calculation of bursting pressure can decrease the safety factors and improve design precision, and can also decrease the quantity of materials used and lighten the weight on pressure vessels. The Faupel formula obtained after many bursting experiments on pressure vessels of mild steel, low alloy steel and high alloy steel has been applied widely to estimate their bursting pressure. There are additionally other formulas based on plastic theory. Though they have been widely used for a long time, they have big error for mild steel pressure vessels. After many bursting experiments on mild steel pressure vessels, the authors found that the error caused in estimating the bursting pressure of mild steel pressure vessels by the Faupel formula exceeds 10%. If the Faupel formula is modified in some way, the error of bursting pressure of mild steel pressure vessels will be reduced. It is proved that the modified formula can be applied to other different mild steel pressure vessels.

PRESENT FORMULAS OF BURSTING PRESSURE

Typical bursting process of mild steel

The usual destruction of mild steel is excess intensity destruction of pressure vessels. They are destroyed after great plastic deformation under increasing operating pressure or decreasing of shells thickness. Take the material of Q235(Gr.D) for an example, it has four states before bursting. They are the state of elastic deformation, the state of yield, the state of plastic deformation and the state of fracture. When plastic deformation occurs, the increasing value of loading capacity caused by the intensifying strain of materials becomes smaller than the decreasing value of loading capacity caused by reduction of the shell thickness due to plastic deformation, so fracture will occur.

For pressure vessels, the plastic deformation of materials will occur when the vessels pressure exceeds the yield limit. However, the strain hardening of materials can increase the loading capacity at the same time, so even if the whole body of a pressure vessel yields, the pressure vessel cannot be considered as useless, as it can still load pressure for the action of strain hardening. The obvious strain hardening of materials can prevent plastic materials from fracturing. Only when the pressure increases to certain value, does the great deformation lead to the decreasing of the shells thickness and loading capacity, will the pressure vessels burst. This is the theoretical basis of the principle of bursting invalidation design criterion. There are many methods for estimation bursting pressure, but the Faupel formula and the formula based on experimental data on torsion and bending are widely accepted by many countries because of their correctness and long history.

Faupel formula

The Faupel formula holds that the bursting pressure of pressure vessels is between the pressure of fully plastic yields and pressure of ultimate intensity of the material. It means that the bursting pressure is between the yield limit and the ultimate intensity limit. When the material is fully plastic without strain hardening, the bursting pressure is the lowest value (Ding, 1994; Giglio, 1997; Zhu and Manesh, 2004).

$$P_{\mathrm{bmin}} = \frac{2}{\sqrt{3}}\sigma_{\mathrm{s}} \ln k.$$

The yield limit will rise when strain hardening occurs. The limiting condition is $\sigma_s=\sigma_b$, the bursting pressure is the highest value at this time. That is:

$$P_{\mathrm{bmax}} = \frac{2}{\sqrt{3}}\sigma_{\mathrm{b}} \ln k.$$

The actual bursting pressure of pressure vessels is linearly related with the value of σ_s/σ_b. The bursting pressure is

$$P_{\mathrm{b}} = \frac{2}{\sqrt{3}}\sigma_{\mathrm{s}}\left(2-\frac{\sigma_{\mathrm{s}}}{\sigma_{\mathrm{b}}}\right)\ln k, \tag{1}$$

where P_b is bursting pressure; P_{bmax} the highest bursting pressure; P_{bmin} the lowest bursting pressure; σ_s the yield stress of materials; σ_b the ultimate tensile strength of materials; k the value of the ratio of R_o to R_i; R_o outer radius of pressure vessel; R_i inner radius of pressure vessel.

Eq.(1) is a half-empirical formula obtained through experiments, and is simple for engineering application. But the application of the Faupel formula has revealed some unsatisfying problems on strength calculations of some high pressure vessels of petroleum and chemical engineering equipments in recent years. The calculation error of bursting pressure is bigger than ±15%. After hundreds of bursting experiments on the same size pressure vessels of mild steel, the authors found that the average calculation error of bursting pressure comes to about 20% and is too conservative (Huang, 1992; Lin, 1996). So the Faupel formula needs to be modified for mild steel pressure vessels.

Formula based on the experimental data on torsion and bending

The formula based on the experimental data on torsion and bending is an analytical solution, deduced and obtained through the method of plastic mechanics and the theory of great deformation. The hardening effect of materials is considered after great plastic deformation on the bursting pressure. The formula is (Liu, 1991):

$$\begin{aligned} P_{\mathrm{b}} = {} & A\left[\gamma^{1/2}\left(2-\frac{\gamma}{3}\right)+\left(\frac{\gamma^{2.5}}{30}-\frac{\gamma^{4.5}}{3240}\right)\right]\Bigg|_{\gamma_{\mathrm{o}}}^{\gamma_{\mathrm{i}}} \\ & + B\left[\gamma^{1/4}\left(4-\frac{\gamma}{2.5}\right)+\left(\frac{\gamma^{2.25}}{27}-\frac{\gamma^{4.25}}{3060}\right)\right]\Bigg|_{\gamma_{\mathrm{o}}}^{\gamma_{\mathrm{i}}} \\ & + C\left[\gamma^{1/8}\left(8-\frac{\gamma}{2.25}\right)+\left(\frac{\gamma^{2.125}}{25.5}-\frac{\gamma^{4.125}}{2970}\right)\right]\Bigg|_{\gamma_{\mathrm{o}}}^{\gamma_{\mathrm{i}}}, \end{aligned} \tag{2}$$

where A, B, C are the constants obtained by letting the formula of $\tau=A\gamma^{1/2}+B\gamma^{1/4}+C\gamma^{1/8}$ fit the curve of shearing strength τ and shearing strain γ.

Based on extensive experimental data, Eq.(2) shows that the error between theoretical value and actual bursting pressure is less than ±5%. However, the formula is complicated, and γ_i, γ_o must be solved by iteration technique:

$$e^{\gamma_i}-1=K(e^{\gamma_o}-1),$$
$$\frac{A\gamma_i^{1/2}+B\gamma_i^{1/4}+C\gamma_i^{1/8}}{A\gamma_o^{1/2}+B\gamma_o^{1/4}+C\gamma_o^{1/8}}=e^{\gamma_i-\gamma_o}.$$

Furthermore, the mechanical curve chart of material should be obtained before calculation, and is inconvenient for engineering design and usually it cannot be obtained. To solve this problem, the authors found a new formula that can accurately calculate burst pressure value of mild steel pressure vessels. The formula is deduced by modifying the Faupel formula and closely simulating through real experimental data. The modified formula has been proven to have good accuracy in engineering application.

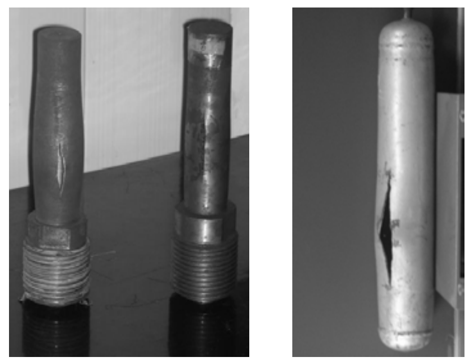
BURSTING EXPERIMENT

Experimental device

Hydraulic pressure burst is applied for bursting experiment. The pressure vessel is put into a safeguarded bursting hole. Two kinds of vessel shapes are shown in Fig.1. The small pressure vessel (Fig.1a) has inner and outer diameter listed in Table 1. Their lengths are 250 mm (excluding the screw thread part). Their material is Q235(Gr.D). Their mechanical properties are: σ_s=235 MPa, σ_b=375 MPa. In order to get more precise result, 20 pressure vessels were manufactured from the same block of material. The big pressure vessels (Fig.1b) has inner and outer diameter listed in Table 2 (Nos. 5~7). Their lengths are 500 mm (excluding the screw thread part). Their material is 20R(1020). Their mechanical properties are: σ_s=285 MPa, σ_b=484 MPa. For the same reason, 3 pressure vessels are manufactured from the same block of material.

Choosing experimental sample

In order to get more accurate value, 20 pressure vessels of the same batch of material were chosen for the experiment. After checking them up, 5 vessels whose size error exceeds the limit were deleted and 15



(a) (b)

Fig.1 Small (a) and big (b) pressure vessels

Table 1 Calculation of bursting pressure

Sequence number	D_o (mm)	D_i (mm)	k	P_b' (MPa)	Modified bursting formula		Theoretical bursting formula	
					Pressure (MPa)	Error (%)	P_b (MPa)	Error (%)
1	39.78	36.00	1.105	49.2	47.80	−2.85	37.50	−23.78
2	38.94	34.89	1.116	55.2	52.54	−4.82	40.90	−25.90
3	39.23	35.12	1.117	52.4	52.97	1.08	41.23	−21.31
4	38.89	34.65	1.122	50.0	55.11	10.20	42.90	−14.20
5	40.00	35.50	1.127	60.5	57.24	−5.39	44.55	−26.36
6	39.80	35.00	1.134	67.5	60.20	−10.80	46.86	−30.57
7	40.10	35.20	1.139	66.8	62.31	−6.72	48.50	−27.39
8	39.87	34.94	1.141	63.2	63.15	−0.10	49.12	−19.67
9	39.89	34.94	1.142	61.6	63.57	3.20	49.48	−19.67
10	40.01	35.04	1.142	64.0	63.57	−0.67	49.48	−22.68
11	39.95	34.87	1.146	60.8	65.24	7.30	50.79	−16.46
12	39.95	34.80	1.148	62.0	66.08	6.58	51.44	−17.03
13	39.80	34.66	1.150	66.0	66.91	1.38	52.08	−20.10
14	40.00	34.70	1.153	66.4	68.10	2.56	53.05	−21.09
15	40.11	34.74	1.155	64.0	68.98	7.78	53.70	−16.09

D_o: Outside diameter; D_i: Inner diameter; $k=D_o/D_i$; P_b': Actual bursting pressure; P_b: Theoretical bursting pressure

Table 2 Error calculation of bursting pressure

Sequence number	Fitting formula		Faupel formula	
	Pressure (MPa)	Error (%)	Pressure (MPa)	Error (%)
1	46.67	−5.159	37.50	−23.78
2	51.67	−6.409	40.90	−25.90
3	52.16	−0.458	41.23	−21.31
4	54.64	9.282	42.90	−14.20
5	57.11	−5.601	44.55	−23.36
6	60.55	−10.294	46.86	−30.57
7	63.00	−5.695	48.50	−27.39
8	63.97	1.219	49.48	−19.67
9	64.46	4.638	49.48	−19.67
10	64.46	0.714	49.48	−22.68
11	66.40	9.209	50.79	−16.46
12	67.37	8.658	51.44	−17.03
13	69.78	5.094	53.05	−20.10
14	68.34	3.538	52.08	−21.09
15	70.75	10.540	53.70	−16.09

vessels were available. All 15 vessels burst and all the breaking mouths lie in the middle of the vessels (Fig.1a). The fracture has the character of plastic material bursting after great deformation. Fifteen samples were available. Their data are listed in Table 1.

Treatment of experimental data

Curves of actual bursting pressure and bursting pressure calculated in Faupel formula are shown in Fig.2. The chart is for different thickness of shell or the value of k. The curve of the actual bursting pressure can be fitted by least quadratic multiplication. The formula of the fitting curve is:

$$P_b' \approx 555.56(\ln k - 0.0168).$$

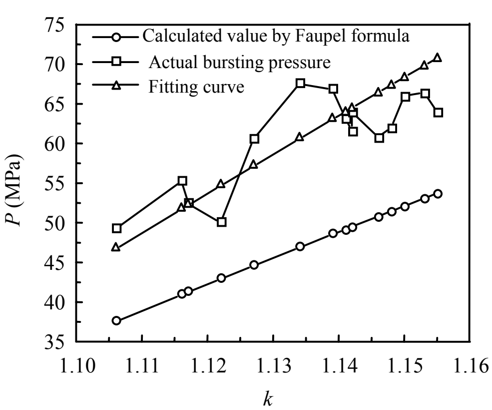


Fig.2 Curve of bursting pressure

Analyzing the error between the fitted formula and the Faupel formula, we show error and list it in Table 2 and Fig.3.

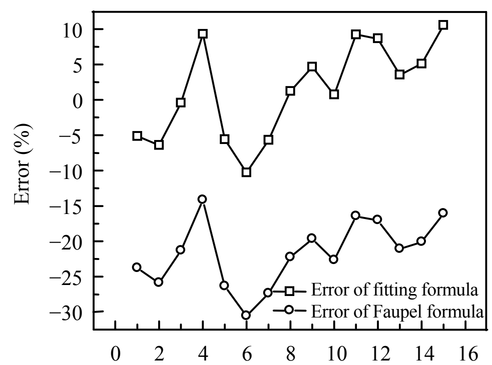


Fig.3 Curve of error of bursting pressure

Examining accuracy of another type of pressure vessel

From Fig.2 and Fig.3 we see that:

(1) The values of the actual bursting pressure vary unsteadily. The fitted formula curve fluctuates at the actual bursting pressure. Their average error is under 10%.

(2) The error of bursting pressure calculated by the Faupel formula is greater. The greatest error even exceeds 30%. The Faupel formula is conservative.

(3) The error still remains at about 15% with increasing value of k. However, the fluctuation tends to become steady.

From the Faupel formula, we can see that the bursting pressure varies linearly with the yield stress of material σ_s and the value of σ_s/σ_b. Thus, the fitting formula can be improved as follows:

$$P_b' \approx 13.21\sigma_s(\sigma_s/\sigma_b)^4 \ln k. \quad (3)$$

Eq.(3) is named "modified formula", because it was obtained from experiments on pressure vessel, and simulated from experimental data, its universality needs to be examined by more pressure vessels. So, another group of pressure vessels with bigger diameter were burst. The vessel's shape is shown in Fig.1b. Its length is 500 mm. They are No. 5 to No. 7 listed in Table 3. The others are obtained from other references (Liu, 1991; Cheng, 1992; Dixon, 2002; Fernando and Claudio, 2004; Rajan, 2002). All the experimental data on bursting pressure are shown in Table 3.

Table 3 Bursting pressure data on pressure vessel with different diameter

Sequence number	Material	k	P_b' (MPa)	Modified formula		Faupel formula	
				Pressure (MPa)	Error (%)	Pressure (MPa)	Error (%)
1	Q235(Gr.D)	1.014	6.28	6.65	5.89	5.15	−17.90
2	Q235(Gr.D)	1.013	5.83	6.18	6.00	4.82	−17.30
3	Q235(Gr.D)	1.012	5.32	5.71	7.33	4.45	−16.30
4	Q235(Gr.D)	1.011	5.12	5.23	2.15	4.10	−19.90
5	20R(1020)	1.102	47.80	43.97	−7.77	41.29	−13.30
6	20R(1020)	1.102	47.60	43.97	−7.62	41.29	−13.20
7	20R(1020)	1.102	45.10	43.97	−2.51	41.29	−8.44
8	20R(1020)	1.192	76.03	79.49	4.60	81.56	7.27
9	20R(1020)	1.300	119.68	118.75	−1.07	121.83	1.80
10	20R(1020)	1.330	128.32	129.08	0.64	132.43	3.20
11	20R(1020)	1.422	167.26	159.35	−4.70	163.48	2.26
12	20R(1020)	1.600	212.39	210.39	−0.95	218.25	2.76
13	20R(1020)	2.000	311.85	310.27	0.51	321.87	3.21
14	20R(1020)	2.400	381.48	391.88	2.66	406.53	6.57
15	20R(1020)	2.800	456.90	460.89	0.87	478.11	4.65
17	20R(1020)	3.200	526.62	520.66	−1.15	540.12	2.56
18	20R(1020)	3.600	574.69	573.38	−0.23	594.88	3.51

$k=D_o/D_i$; P_b': Actual bursting pressure

It is obvious that the bursting pressure calculated by the modified formula is close to the actual bursting pressure, and that the error is not more than ±8%. This is fit for calculating bursting pressure of mild steel pressure vessels. The modified formula is suitable only for mild steel. It may have greater errors for other types of steel vessels (Huang and Friedrich, 1994).

CONCLUSION

After many bursting experiments on mild steel pressure vessels, such as Q235(Gr.D) and 20R(1020), and statistically analyzing the bursting pressure of mild steel pressure vessels, the conclusion can be drawn that the error of the Faupel formula is big. Then the modified formula is presented which closely accords with the actual value. Different diameter pressure vessels made of 20R(1020) were used to prove the validity of the formula. The experimental data in Table 3 indicate that the modified formula can be more adaptable for mild steel pressure vessels. The formula has the characteristics of simplicity, wide range of application, high precision in calculation, small error, etc. However, the adaptability of the formula to pressure vessels made of other materials needs to be further proven.

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