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Studies on vibration characteristics of a pear using finite element method*

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Abstract: The variation of the vibration characteristics of a Huanghua pear was investigated using finite element simulations. A new image processing technique was used to obtain the unsymmetrical and un-spherical geometrical model of a pear. The vibration characteristics of this type of pear with the correlation of its behavior with geometrical configurations and material characteristics were investigated using numerical modal analysis. The results showed that the eigenfrequency increased with the increasing pear Young's modulus, while decreased with increasing pear density, and decreased with increasing pear volume. The results of this study provided foundation for further investigations of the physical characteristics of fruits and vegetables by using finite element simulations.

Key words: Pear, Vibration characteristics, Unsymmetrical and un-spherical model, Finite element analysis
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INTRODUCTION

The quality of fruits can be determined by its external and internal characteristics. The most important external characteristics are the size, shape, smell, appearance and product presentation, and the most important internal characteristics are the taste and texture (Abbott *et al*., 1968; Abbott and Lu, 1996; Petrell *et al*., 1980; Chen and de Baerdemaeker, 1993a; Wang and Sheng, 2005). The flesh firmness is a texture attribute and one of the major fruit quality indicators. The demands for high quality fruits make it necessary for growers and distributors to set up an integrated quality control system for monitoring the quality of the fruits during picking, storage, and distribution. The traditional destructive technique for measuring the firmness is the Magness-Taylor firmness test with penetrometer. This method which is defined in terms of resistance to penetration is, however, destructive and cannot be used for on-line control of fruit quality.

Currently, there is a growing interest in non-destructive methods for on-line sorting. Many researchers proposed some non-destructive methods based on the dynamic principles, such as the acoustical signal and resonance frequency produced by the machinery knock and shock power, which utilize eigenfrequency of fruit vibration or shock power to measure its firmness (Abbott *et al*., 1992; Schotte *et al*., 1999; de Belie *et al*., 2000; Wang, 2003; Wang *et al*., 2004a; 2004b). Although the measurements and technologies may be different, their purposes in all studies are to evaluate the Young's modulus of fruit firmness and obtain their relationship.

Many reports are available in the scientific literature on finite element (FE) modal analysis and resonant frequencies of various kinds of near-spherical agricultural objects such as apples (Chen and de Baerdemaeker, 1993b; Lu and Abbott, 1997), peaches (Verstreken and de Baerdemaeker, 1994), melons (Chen *et al*., 1996) and tomatoes (Lan-

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genakens *et al.*, 1997). These researches showed that, finite element (FE) model analysis is a suitable method for calculating the resonance properties of near-spherical solid materials. However, for the unsymmetrical and un-spherical geometrical model of complex-shaped fruits such as pears, further research is required. For example, Dewulf *et al.*(1999) studied the vibration patterns of a 'conference' pear by FE modal analysis in which the correlation between the material properties and the dynamic characteristics was analyzed. A computer vision-based modelling system was used to establish geometrical models. Jancsok *et al.*(2001) studied the effect of the shape of 'conference' pears on their resonant frequencies using FE modal analysis, with the FE models being validated by resonance experiment measurements.

A new image processing technique was used in this paper to obtain the unsymmetrical and un-spherical geometrical model of a pear by the ANSYS7.0 FE program (ANSYS Inc.). The variation of the dynamic characteristics of a Huanghua pear was investigated using finite element simulations. The vibration characteristics of this type of pear and the correlation of its behavior with geometrical configurations and material characteristics were investigated by using numerical modal analysis. Where results provided foundation for further investigation of the physical characteristic of fruits and vegetable by using finite element simulations.

MATERIALS AND METHODS

Geometrical model for the pear

In this work, a new image process technique was used to obtain the unsymmetrical and un-spherical geometrical model of a pear (Fig.1). The image processing system used consisted of a camera and a capture card. The object was placed on a table and pictures (24-bit Bitmap) were taken from one of several views. Digital analysis software was used to load, the images into the memory for display in the screen. A threshold value was used to separate the points of the object from the background. After filtering the noise, the program determines the edges of the object and extracts the boundary points; then Visual Basic software (lab made) was used to scan and obtain the coordinate of this outline; and then a series of round cross section outlines were obtained on the basis of the axle section outline by the ANSYS7.0 program. Finally, the geometry modelling was completed by linking the external outline in the surface of the pear and filling it.

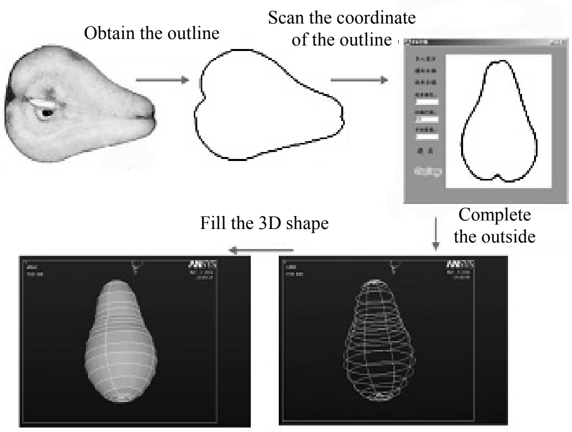

Fig.1 Geometry modelling of the pear

Finite element modal analysis

Finite element modal analysis is a numerical procedure in which the natural frequencies and the mode shapes (the pattern of the deformations during the vibration) of objects are calculated and in the analysis ANSYS7.0 FE program was used. The principle is as follows:

The displacement at any point of any element e $\boldsymbol{u}(\xi,t)$ can be approximately expressed in terms of the nodal displacements $\boldsymbol{q}^e(t)$ as:

$$\boldsymbol{u}(\xi,t)=\boldsymbol{N}^e(\xi)\cdot\boldsymbol{q}^e(t), \tag{1}$$

where $\boldsymbol{N}^e(\xi)$ is the shape function matrix.

After applying the prescribed time-varying loads $\boldsymbol{F}(t)$, and the equilibrium, compatibility and stress-strain conditions to all the degrees of freedom (DOFs), the following well-known matrix equation describing the dynamic motion of the pear can be obtained:

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t)+\boldsymbol{C}\dot{\boldsymbol{q}}(t)+\boldsymbol{K}\boldsymbol{q}(t)=\boldsymbol{F}(t). \tag{2}$$

It is assumed that the system is not damped. The undamped vibration of the nodes can be described by Eq.(3).

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t)+\boldsymbol{K}\boldsymbol{q}(t)=\boldsymbol{F}(t), \tag{3}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}$ is the damped matrix, $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{q}(t)$, $\dot{\boldsymbol{q}}(t)$ and $\ddot{\boldsymbol{q}}(t)$ are the displacement, velocity and acceleration vectors, and t is the time. In this FE modal analysis free vibration is assumed, so no force is applied and correspondingly $\boldsymbol{F}(t)=\boldsymbol{0}$. For a linear system this vibration will be harmonic of the form:

$$\boldsymbol{q}_i(t)=\phi_i\cdot\sin\omega_i t, \tag{4}$$

where ϕ_i is the eigenvector representing the ith natural frequency, ω_i is the ith natural angular frequency and t is the time. By substituting Eq.(4) into Eq.(3) it follows, that

$$(\boldsymbol{K}-\omega_i^2\boldsymbol{M})\phi_i=0. \tag{5}$$

For a non-trivial solution this corresponds to the following equation:

$$\det(\boldsymbol{K}-\omega_i^2\boldsymbol{M})=0. \tag{6}$$

This Eq.(6) represents an eigenvalue problem that can be solved for n values of ω^2 (eigenvalues) and n eigenvectors, where n is the number of all the degrees of freedom (DOFs). This eigenvalue problem was solved by using ANSYS7.0 FE program using the subspace iteration method.

The first 50 mode shapes of the pear were calculated. The mode shapes were identified by examining the deformation plot and by the animated mode shape display.

Vibration characteristics analysis of FE modal

The pear is considered an elastic homogeneous object and the influence of pericarp and the core can be neglected and assumed to be the same as listed in (Dewulf *et al*., 1999; Jancsok *et al*., 2001). The bottom boundary constraint has been applied.

The geometrical model of the pear is meshed by using 3D structural 8 nodes 45 solid elements (Fig.2). From the results of the validation experiments, the following material properties are used: density of 1000 kg/m^3, Young's modulus of 4.0 MPa and Poisson's ratio was assumed to be 0.3 (Teng, 2002; Wang, 2003; Wang *et al*., 2004a; 2004b). Three different volume pear samples including pear A (volume 325 cm^3), pear B (volume 250 cm^3) and pear C (volume 200 cm^3), were used and their vibration frequencies were calculated separately.

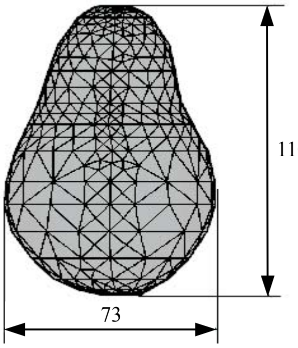

Fig.2 Finite element meshing of the pear B

In order to investigate the dynamic behaviour of this type of pear and the correlation of its behaviour with geometrical configurations and material characteristics, the following material properties were used: (1) Young's modulus of 2.0~4.0 MPa, density of 860 kg/m^3, Poisson's ratio assumed to be 0.3 and volume of 325 cm^3 were to analyze the correlation of Young's modulus and frequency; (2) Young's modulus of 2.0 MPa, density of 850~1000 kg/m^3, Poisson's ratio assumed to be 0.3 and volume of 325 cm^3 were used to analyze the correlation of density and frequency; (3) Young's modulus of 2.0 MPa, density of 860 kg/m^3, Poisson's ratio assumed to be 0.3 and volume of 200~350 cm^3 were used to analyze the correlation of volume and frequency.

RESULTS AND DISCUSSION

Fifty modes were obtained by the finite element analysis on the selected pears. There were six rigid body modes which could be observed at a frequency below 100 Hz. The first vibration mode (No. 7) was a mode where the stem of the pear is deformed. This mode is called a bending mode. Mode No. 8 was the same as mode No. 7 but the deformation was perpendicular to the one in mode No. 7 (Fig.3a). Mode No. 9 was a torsion mode (Fig.3b). Mode No. 10 was a mode where the whole pear was deformed in longitudinal direction (Fig.3c). In this mode as the pear was compressed and elongated, it became shorter and longer. This mode is called compression or longitudinal mode. Modes No. 11 and No. 12 were also bending modes, but there were two nodal regions where the deformations were high. The deformations on the two modes were perpendicular to each other. Mode No. 13 was also a torsion mode. The difference between modes No. 9 and No. 13 was that in No. 9 there was only one nodal region without deformation while at No. 13 there were two. The deformations in the modes No. 14 and No. 15 were also mutually perpendicular, and they were more complex bending modes. The higher modes were complex modes with vibration pattern not related to this study.

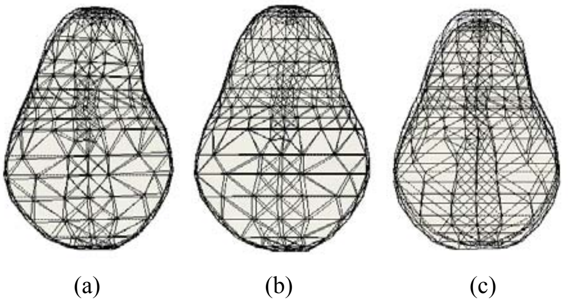

Fig.3 Mode shapes of the finite element model. (a) Bending mode (No. 8); (b) Torsion mode (No. 9); (c) Compression mode (No. 10)

Fig.4 shows the one-way distributed stress of the bended vibration model of the mode (No. 8). In the illustration, the dark color of the calyx end of the pear is the positive largest stress area and the dark color of the pear stem is negative largest stress area.

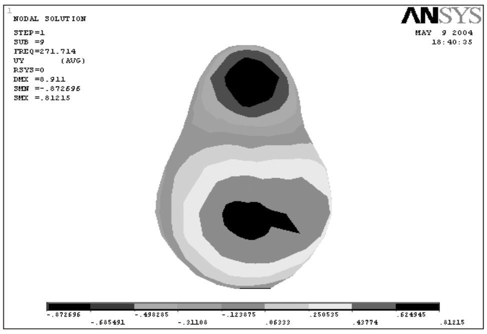

Fig.4 Stress distribution of bending model (No. 8)

Table 1 shows the result of the FE simulation compared with the result of the experimental modal analysis (Teng, 2002; Wang, 2003; Wang *et al*., 2004a; 2004b). The calculated data agree wholely with the experimental data (absolute error<5%). The discrepancy between the calculated and measured results may be due to: (1) the pear is considered to be a linear elastic object and homogeneous, with the influence of pericarp and the core neglected; (2) the difference between experimental knock point and numerical model knock point; (3) the difference between material characteristic parameter, the form of the geometry of the real pear and numerical model; (4) the measured frequency was based on the real pear with pericarp and core, and the density of which are higher than flesh. So the measured frequency is generally higher than finite element method calculated frequency.

As shown in Fig.5, the lines are corresponding to 1~50 modal frequency arrays sequentially from bottom to top.

The eigenfrequency increased with increasing pear Young's modulus, but decreased with increasing pear density and volume. In which the Young's modulus *E*, the volume *V* (or quality) more obviously affect frequency *f*. This is completely consistent with the experimental conclusion (Teng, 2002; Wang, 2003; Wang *et al*., 2004a; 2004b).

Table 1 Calculated and measured resonant frequencies

	Pear A (volume 300 cm^3)	Pear B (volume 250 cm^3)	Pear C (volume 200 cm^3)	Mode
FEM calculated frequency (Hz)	287.42	344.01	430.12	No. 7: Bend
	288.03	345.16	431.20	No. 8: Bend
Measured frequency (Hz)*	305.30*	358.70*	450.50*	Bend
FEM calculated frequency (Hz)	432.26	518.61	648.15	No. 11: Second order bending
	433.74	519.81	650.15	No. 12: Second order bending
Measured frequency (Hz)*	451.20*	535.60*	670.40*	Second order bending

* Quoted from the result of experiment (Teng, 2002; Wang, 2003; Wang *et al*., 2004a; 2004b); FEM: Finite element method

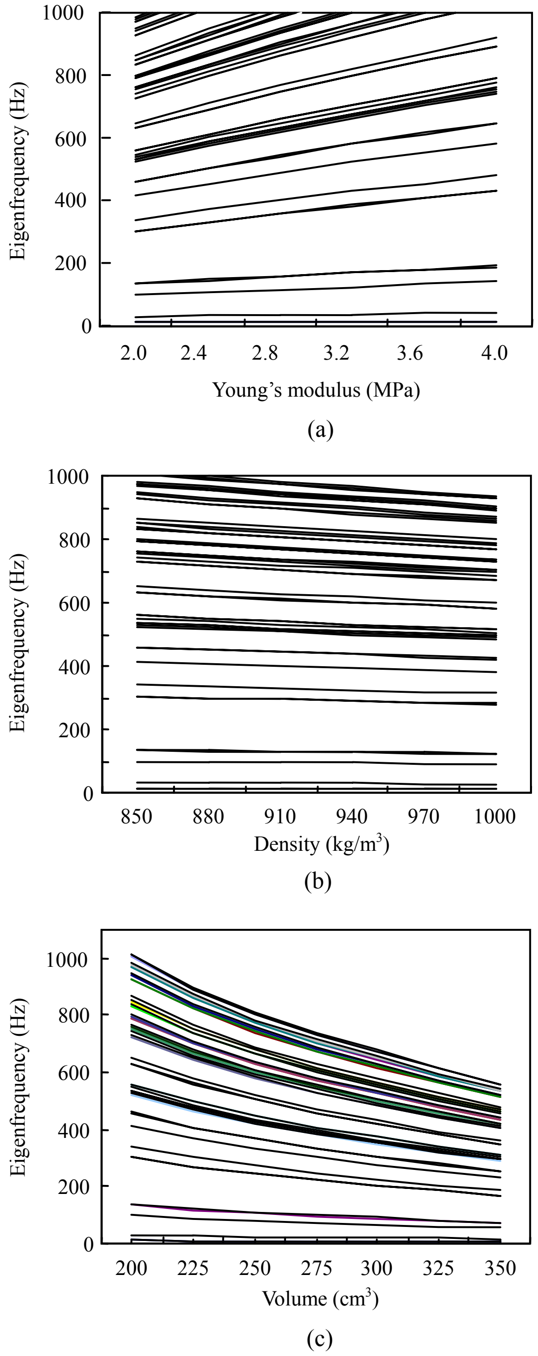

Fig.5 The correlation between the frequencies and the Young's modulus, density, volume (1~50 modal frequency arrays sequentially from bottom to top). (a) Frequency vs Young's modulus; (b) Frequency vs density; (c) Frequency vs volume

CONCLUSIONS

1. The new image processing technique has been used to obtain the unsymmetrical and un-spherical geometrical model of a pear using ANSYS7.0 FE program, the variations of the resonant frequencies and modes of a pear were investigated by using finite element simulations.

2. Fifty modes were obtained by the finite element analysis. Six rigid body modes (Nos. 1~6) could be observed at a frequency below 100 Hz. Mode 16 to higher modes are complex modes with their vibration pattern not related to this study.

3. The vibration characteristics of this type of pear with the correlation of its behavior with geometrical configurations and material characteristics was investigated using numerical modal analysis. The results showed that the eigenfrequency increased with increasing pear Young's modulus, decreased with increasing pear density and volume. In which the Young's modulus E, the volume V (or quality) is more obviously affect frequency f.

References

Abbott, J.A., Lu, R., 1996. Anisotropic mechanical properties of apples. *Transactions of the ASAE*, **39**(4):1451-1459.

Abbott, J.A., Bachman, G.S., Childers, N.F., Fiztgerald, J.V., Matusik, F.J., 1968. Sonic techniques for measuring texture of fruit and vegetables. *Food Technology*, **22**:635-645.

Abbott, A.J., Affeldt, A.H., Liljedahl, A.L., 1992. Firmness measurement of stored 'Delicious' apples by sensory methods, Magness-Taylor, and sonic transmission. *Journal of the American Society for Horticultural Science*, **117**:590-595.

Chen, H., de Baerdemaeker, J., 1993a. Madal analysis of the dynamic behavior of pineapples and its relation to fruit firmness. *Transactions of the ASAE*, **36**(5):1439-1444.

Chen, H., de Baerdemaeker, J., 1993b. Finite-element-based modal analysis of fruit firmness. *Transactions of the ASAE*, **36**(6):1827-1833.

Chen, H., de Baerdemaeker, J., Bellon, V., 1996. Finite element study of the melon for nondestructive sensing of firmness. *Transactions of the ASAE*, **39**(3):1057-1065.

de Belie, N., Schotte, S., Coucke, P., de Baerdemaerker, J., 2000. Development of an automated monitoring device to quantify changes in firmness of apple during storage. *Postharvest Biology and Technology*, **18**(1):1-8. [doi:10.1016/S0925-5214(99)00063-0]

Dewulf, W., Jancsók, P., Nicolaï, B.M., de Roeck, G., Briassoulis, D., 1999. Determining the firmness of a pear using finite element modal analysis. *Journal of Agricultural Engineering Research*, **74**(3):217-224. [doi:10.1006/jaer.1999.0451]

Jancsok, P.T., Clijmans, L., Nicolai, B.M., de Baerdemaeker, J., 2001. Investigation of the effect of shape on the acoustic response of ‘conference’ pears by finite element modeling. *Postharvest Biology and Technology*, **23**(1): 1-12.

Langenakens, J., Vandewalle, X., de Baerdemaeker, J., 1997. Influence of global shape and internal structure of tomatoes on the resonant frequency. *Journal of Agricultural Engineering Research*, **66**(1):41-49. [doi:10.1006/jaer.1996.0117]

Lu, R., Abbott, J.A., 1997. Finite element modeling of transient responses of apples to impulse excitation. *Transactions of the ASAE*, **40**(2):1395-1406.

Petrell, R.J., Mohsenin, N.N., Wallner, S., 1980. Dynamic mechanical properties of the apple sortex in relation to sample location and ripening. *Journal of Texture Studies*, **10**(3):217-229.

Schotte, S., de Belie, N., de Baerdemaerker, J., 1999. Acoustic impulse-response technique for evaluation and modeling of firmness of tomato fruit. *Postharvest Biology and Technology*, **17**(2):105-115. [doi:10.1016/S0925-5214(99)00041-1]

Teng, B., 2002. Studies on the Characteristics of Frequency of Pear and Peach. Master Thesis Dissertation, Zhejiang University, China (in Chinese).

Verstreken, E., de Baerdemaeker, J., 1994. Evolution of maturity of peaches: non-destructive firmness measurement from the acoustic impulse response. *International Agrophysics*, **8**:469-473.

Wang, J., 2003. Anisotropic relaxation properties of pear. *Biosystems Engineering*, **85**(1):59-65. [doi:10.1016/S1537-5110(03)00017-5]

Wang, J., Sheng, K.C., 2005. Variations in firmness and sugar content in ‘Huanghua’ pear (Pyrus pyrifolia ‘Nakai’). *Journal of Horticultural Science and Biotechnology*, **80**(3):307-312.

Wang, J., Teng, B., Yu, Y., 2004a. Pear dynamic characteristics and firmness detection. *European Food Research and Technology*, **218**(3):289-294. [doi:10.1007/s00217-003-0850-9]

Wang, J., Teng, B., Zhou, M., 2004b. Dynamic response of impact forces on pear. *Transactions of the Chinese Society for Agricultural Machinery*, **35**(2):65-68.