



A vague-set-based fuzzy multi-objective decision making model for bidding purchase^{*}

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Abstract: A vague-set-based fuzzy multi-objective decision making model is developed for evaluating bidding plans in a bidding purchase process. A group of decision-makers (DMs) first independently assess bidding plans according to their experience and preferences, and these assessments may be expressed as linguistic terms, which are then converted to fuzzy numbers. The resulting decision matrices are then transformed to objective membership grade matrices. The lower bound of satisfaction and upper bound of dissatisfaction are used to determine each bidding plan's supporting, opposing, and neutral objective sets, which together determine the vague value of a bidding plan. Finally, a score function is employed to rank all bidding plans. A new score function based on vague sets is introduced in the model and a novel method is presented for calculating the lower bound of satisfaction and upper bound of dissatisfaction. In a vague-set-based fuzzy multi-objective decision making model, different valuations for upper and lower bounds of satisfaction usually lead to distinct ranking results. Therefore, it is crucial to effectively contain DMs' arbitrariness and subjectivity when these values are determined.

Key words: Fuzzy multi-objective decision making model, Vague set, Score function, Lower bound of satisfaction, Upper bound of dissatisfaction

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INTRODUCTION

The rapid growth of Chinese economies over the past three decades has significantly changed the landscape of supply and demand in the market: China has evolved from a seller's market in virtually all economic fields to an extremely competitive buyer's market in many business segments. Customers in China nowadays enjoy an unprecedented wide variety of high-quality products and services and, hence, possess a higher negotiation power in reaching a deal. Generally speaking, there exist several different purchasing modes such as public bidding, inviting bidding, competitive negotiation, and single source negotiation. To take advantage of this buyer's market,

bidding purchase has been increasingly employed to reduce the cost without sacrificing the quality of products and services. Bidding purchase refers to a process that a buyer selects a suitable bidding plan by competitive bidding. Usually, the buyer convenes a panel consisting of several experts referred to as judges and this panel evaluates all bidding plans and generates a report. Based on the panel's evaluation report, the buyer chooses a successful bidding plan (Wu S.R., 2004). At present, several methods in bidding purchase are widely used in practice, including the TOPSIS comprehensive assessment (Chen, 2000; Yan *et al.*, 2002), value engineering assessment (Liu *et al.*, 2005), AHP (Zhu *et al.*, 2001; Leung and Cao, 2000), evaluation method (Paek *et al.*, 1992), and D-Evidence theory (Wu H.Q., 2004), to name a few. These approaches generally cannot handle problems that contain fuzzy or uncertain information.

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To evaluate a bidding plan, several aspects have to be considered, for instance, the bidding plan's creditability and offered price. Among these different aspects, some information is inherently fuzzy or vague. In addition, when the judge panel presents its report to the buyer, individual judges' preferences have to be aggregated to form a collective preference over bidding plans. Therefore, bidding decision-making is essentially a multi-objective group decision-making problem (Szmidski and Kacprzyk, 2002; Bozdag *et al.*, 2003).

Vague sets were proposed by Gau and Buehrer (1993). Similar to rough sets (Skowron, 2005; Chen *et al.*, 2006), this theory constitutes a valuable extension to Zadeh's fuzzy sets theory (Zadeh, 1965; Hong and Choi, 2000). A key characteristic of vague sets theory is that it simultaneously considers both membership and non-membership information (Lu and Ng, 2005). Therefore, vague-set based decision-making tends to be more exact, effective, and flexible in comparison with traditional fuzzy-sets based methods (Lo *et al.*, 2006; Polkowski, 2006). Vague sets have been used to solve fuzzy multi-objective decision-making problems by Chen and Tan (1994), in which score functions and weighted score functions are employed to provide decision aid. By analyzing this approach, Hong and Kim (1999) and Li F. *et al.* (2001) provided various improved versions.

In this paper, bidding purchase is investigated within a multi-objective decision making context. DMs' evaluations of bidding plans against the objectives are converted to weighted relative membership grade in this model. The lower bound of satisfaction and upper bound of dissatisfaction are used to determine each bidding plan's supporting, opposing, and neutral objective sets, which together determine the vague value of a bidding plan. Finally, a score function is employed to rank all bidding plans. In a vague-set-based fuzzy multi-objective decision making model, different valuations for upper and lower bounds of satisfaction, implying max-max and max-min approaches, usually lead to distinct ranking results. Therefore, it is crucial to effectively contain DMs arbitrariness and subjectivity when these values are determined. A new method is provided for calculating the lower/upper bound of satisfaction/dissatisfaction. A novel score function is designed to calculate the vague value in bidding purchase. This

score method takes into account supporting sets, opposing sets, and neutral sets. The most satisfying bidding plan is chosen by using a Borda function along with the weights of the DMs (reflecting the importance level of the judges in the panel) and the vague value determined by the score function.

The rest of the paper is structured as follows. A multi-objective decision making model is established next. Then the decision-making method and the algorithm for the model are presented, followed by an illustrative example. The paper concludes with some comments.

A MULTI-OBJECTIVE DECISION MAKING MODEL FOR BIDDING PURCHASE

Our model for bidding purchase is a decision-making model in which the judge panel can select the best supplier on the basis of a range of evaluating indicators. This model is designed to solve group decision-making problems with fuzzy judgmental information in bidding purchase process. It is assumed that DM group $P = \{P_1, P_2, \dots, P_l\}$ consists of l judge P_k ($k=1, 2, \dots, l$), and bidding plan set $X = \{x_1, x_2, \dots, x_n\}$ is comprised of n elements, and there are m evaluating indicators f_i ($i=1, 2, \dots, m$). For each $x \in X$, DM P_k 's evaluation with respect to the m indicators can be expressed as a vector $f_{P_k}(x) = [f_1(x), f_2(x), \dots, f_m(x)]^T$. Then the aim of this fuzzy multi-objective group decision-making model for bidding purchase is to select some $x^* \in X$ such that

$$\max_{x \in X} \{f(x) = [f_{p_1}(x), f_{p_2}(x), \dots, f_{p_l}(x)]^T\}. \quad (1)$$

In a multiple-participant multi-objective decision making setting, DMs' objectives can be different. But in a general way, the DMs tend to have the same overall goal in a bidding purchase model. We assume that:

- (1) All DMs share the same set of evaluating indicators;
- (2) Different DMs may have distinct opinions on the importance of various indicators, reflected as varied weights;
- (3) The weights of DMs vary;
- (4) Generally speaking, bidding purchase group decision is an unstructured complex problem;

(5) The quality of bidding purchase group decision is affected by the decision rules applied.

A FUZZY MULTI-OBJECTIVE DECISION MAKING METHOD BASED ON VAGUE SETS

Weighted relative membership grade determination

For the bidding purchase model in this paper, as both m and n are finite, DM P_k 's evaluation results for all bidding plans with respect to the indicators can be expressed intuitively by a decision matrix:

$$F^k = (f_{ij}^k)_{m \times n}, \quad (2)$$

$$i=1, 2, \dots, m; j=1, 2, \dots, n; k=1, 2, \dots, l$$

where f_{ij}^k is the i th evaluating indicator value made by P_k on bidding plan $x_j \in X$. This value can be a fuzzy linguistic variable such as "good", "very good", and can also be a specific crisp value. The values of integers i, j, k are taken from $1 \sim m, 1 \sim n$ and $1 \sim l$ respectively throughout the remainder of the paper.

According to the fuzzy set theory, linguistic variables can be expressed by fuzzy sets defined in an interval of real numbers and triangular membership functions are probably the most commonly used ones to describe such linguistic variables (Li K.W. et al., 2001; Li, 2002). In this paper, triangular fuzzy numbers are therefore adopted to characterize the linguistic variables as shown in Table 1.

Table 1 Triangular fuzzy numbers for linguistic variables

Linguistic variable	Triangular fuzzy number
Worst	(0, 0, 0.2)
Worse	(0, 0.1, 0.3)
Bad	(0, 0.2, 0.4)
Moderate	(0.3, 0.5, 0.7)
Good	(0.6, 0.7, 0.8)
Better	(0.7, 0.8, 0.9)
Very good	(0.8, 0.9, 1.0)
Extremely good	(0.9, 1.0, 1.0)

We then convert triangular fuzzy numbers into relative membership grades by using the average area measuring method for ranking fuzzy numbers. The results are shown in Table 2.

Table 2 Corresponding relationship between linguistic variables and relative membership grade

Linguistic variable	Relative membership grade
Worst	0.050
Worse	0.125
Bad	0.200
Moderate	0.500
Good	0.700
Better	0.800
Very good	0.900
Extremely good	0.975

For evaluating indicators with specific crisp values, it is hard, if not possible, to attain the supremum and the infimum of f_i^k in the process of resolving practical problems. Therefore, we take the relative membership grade instead of the absolute membership grade. The following formula (Liu, 2004) can be applied to determine the relative membership grade because all evaluating indicators in bidding purchase are profit-related indexes.

$$\mu_{ij}^k = (f_{ij}^k / \mu_{i \max}^k)^\theta, \quad (3)$$

where θ is a parameter given by the DM, usually below 1. Then, the decision matrix can be transformed into a relative membership grade matrix as per Eq.(3).

$$U^k = (\mu_{ij}^k)_{m \times n}, \quad (4)$$

DM P_k 's relative importance evaluation for all evaluation indicators is given as a weight vector:

$$\bar{\omega}^k = [\bar{\omega}_1^k, \bar{\omega}_2^k, \dots, \bar{\omega}_m^k]^T, \quad (5)$$

The weight factor in Eq.(5) is then standardized according to Eq.(6):

$$\omega_i^k = \bar{\omega}_i^k / \sum_{i=1}^m \bar{\omega}_i^k. \quad (6)$$

Vague estimates and ranking of bidding plans

Definition 1 $f_{P_k}(x) = [f_1(x), f_2(x), \dots, f_m(x)]^T$ is DM P_k 's evaluation vector in respect to the m indicators, where $x \in X$, and

- (1) If $\mu_{ij}^k \geq \xi_k^U$, where ξ_k^U is the lower bound of

satisfaction that the DMs can accept, then the i th objective is considered supporting the j th plan;

(2) If $\mu_{ij}^k \leq \xi_k^L$, where ξ_k^L is the upper bound of dissatisfaction that the DMs cannot accept, then the i th objective is considered opposing the j th plan;

(3) If $\xi_k^L < \mu_{ij}^k < \xi_k^U$, the i th objective is considered neutral.

In current literature, no specific approaches are available to determine the degree of satisfaction in the context of vague set theory. Instead, the valuation mainly relies on DMs' subjective judgment, resulting in arbitrariness in a decision-making process. Generally speaking, a DM can choose any value in $[0, 1]$ for the lower bound of satisfaction and upper bound of dissatisfaction based on his/her subjective judgment as long as the lower bound of satisfaction is greater than the upper bound of dissatisfaction.

Intrigued by max-max/min-max and max-min/min-min approaches in the context of fuzzy multi-objective decision making (Li, 2002), three methods, tailored for different DM's natures, are proposed to determine upper and lower bounds. It is expected that subjectivity and arbitrariness can be effectively contained by following one or more of the three methods (depending on the nature of a DM) in determining the upper and lower bounds of satisfaction and dissatisfaction.

1. Conservative or risk-averse DMs

$$\xi_k^U = \max_{1 \leq i \leq m} \{ \max_{1 \leq j \leq n} \{ \mu_{ij}^k \} \}, \quad \xi_k^L = \min_{1 \leq i \leq m} \{ \max_{1 \leq j \leq n} \{ \mu_{ij}^k \} \}.$$

This approach guarantees that the maximum is attained and makes the supporting set as small as possible, but the opposing set as large as possible. Therefore, this is a conservative approach and may be taken by risk-averse DMs.

2. Optimistic or risk-taking DMs

$$\xi_k^U = \max_{1 \leq i \leq m} \{ \min_{1 \leq j \leq n} \{ \mu_{ij}^k \} \}, \quad \xi_k^L = \min_{1 \leq i \leq m} \{ \min_{1 \leq j \leq n} \{ \mu_{ij}^k \} \}.$$

This method ensures that the minimum is achieved and makes the supporting set as large as possible, but the opposing set as small as possible. Hence, this is an optimistic approach and is appropriate for risk-taking DMs.

To mitigate the extremities of the aforemen-

tioned two approaches, we provide another combinatorial method.

3. Combinatorial approach

The aforementioned approaches are either too conservative or too optimistic. By taking a compromise between them, a middle approach is established as follows:

$$\xi_k^U = \max_{1 \leq i \leq m} \{ \alpha \max_{1 \leq j \leq n} \{ \mu_{ij}^k \} + (1-\alpha) \min_{1 \leq j \leq n} \{ \mu_{ij}^k \} \}, \quad \alpha \in [0, 1],$$

$$\xi_k^L = \min_{1 \leq i \leq m} \{ (1-\beta) \max_{1 \leq j \leq n} \{ \mu_{ij}^k \} + \beta \min_{1 \leq j \leq n} \{ \mu_{ij}^k \} \}, \quad \beta \in [0, 1].$$

In these two formulas, α and $1-\alpha$ are the corresponding weights for "Conservative DMs" and "Optimistic DMs" in determining the lower bound of satisfaction, respectively, and β and $1-\beta$ are the corresponding weights for "Optimistic DMs" and "Conservative DMs" in computing the upper bound of dissatisfaction, respectively. The larger α is and the smaller β is, the more conservative the DM is. Otherwise, the DM is more optimistic.

When α, β assume the following values,

$$\alpha = \frac{\sum_{j=1}^n \mu_{ij}^k - n \min_{1 \leq j \leq n} \{ \mu_{ij}^k \}}{(n-1) \left[\max_{1 \leq j \leq n} \{ \mu_{ij}^k \} - \min_{1 \leq j \leq n} \{ \mu_{ij}^k \} \right]},$$

$$\beta = \frac{n \max_{1 \leq j \leq n} \{ \mu_{ij}^k \} - \sum_{j=1}^n \mu_{ij}^k}{(n-1) \left[\max_{1 \leq j \leq n} \{ \mu_{ij}^k \} - \min_{1 \leq j \leq n} \{ \mu_{ij}^k \} \right]}.$$

We have

$$\xi_k^U = \max_{1 \leq i \leq m} \{ \xi_{ki}^U \}, \quad \xi_k^L = \min_{1 \leq i \leq m} \{ \xi_{ki}^L \}. \tag{7}$$

where $\xi_{ki}^U = \left(\sum_{j=1}^n \mu_{ij}^k - \min_{1 \leq j \leq n} \{ \mu_{ij}^k \} \right) / (n-1)$, and

$\xi_{ki}^L = \left(\sum_{j=1}^n \mu_{ij}^k - \max_{1 \leq j \leq n} \{ \mu_{ij}^k \} \right) / (n-1)$. Given this in-

formation, the supporting, opposing, and neutral sets of the j th plan for DM P_k can be defined as follows.

Definition 2

(1) The supporting set of the j th plan for DM P_k is determined by

$$S_j^k = \{ f_i \in \mathbf{f}_{P_k}(x) \mid \mu_{ij}^k \geq \xi_k^U \}.$$

(2) The opposing set of the j th plan for DM P_k is

determined by

$$O_j^k = \{f_i \in f_{P_k}(x) \mid \mu_{ij}^k \leq \xi_k^L\}.$$

(3) The neutral set of the j th plan for DM P_k is determined by

$$N_j^k = \{f_i \in f_{P_k}(x) \mid \xi_k^L < \mu_{ij}^k < \xi_k^U\}.$$

For any plan x_j , the degree to which it satisfies DM P_k 's requirements in the m objectives can be denoted by a vague value

$$E(x_j^k) = [t(x_j^k), 1 - f(x_j^k)], \tag{8}$$

where $t(x_j^k) = \sum_{i^k \in S_j^k} \omega_{i^k}$ and $f(x_j^k) = \sum_{i^k \in O_j^k} \omega_{i^k}$.

For a vague value $E(x_j^k) = [t(x_j^k), 1 - f(x_j^k)]$, different score functions are proposed in the literature. Each score function has its own advantages and disadvantages [For details, readers are referred to (Liu, 2004)]. In this article, a new score function is proposed as follows:

$$S[E(x_j^k)] = t(x_j^k) - f(x_j^k) - \frac{1}{4} \left\{ [1 - t(x_j^k) - f(x_j^k)] + [1 - |t(x_j^k) - 0.5| - |0.5 - f(x_j^k)|] \right\}. \tag{9}$$

This score function is designed to take an integrative approach to accounting for the information contained in the supporting, opposing, and neutral sets, where $t(x_j^k)$ captures the information in the supporting set, $f(x_j^k)$ represents the opposing set, and $1 - t(x_j^k) - f(x_j^k)$ depicts the impacts of the neutral set. In addition, this score function is monotonic and ensures that a same value is assumed when $t(x_j^k) = f(x_j^k)$, indicating that two bidding plans cannot be differentiated when their supporting and opposing sets are of equal size. In this case, another score function, $S'[E(x_j^k)] = 1 - f(x_j^k)$, should be introduced to compare these two plans.

DMs are more satisfied with a plan when the value of $S[E(x_j^k)]$ increases. According to the score function value, DM P_k 's ranking for the plans can be

easily obtained. The new score function, along with a novel method presented earlier in this article to determine the degree of satisfaction, takes DMs' preferences into account and makes the supporting set and opposing set more clearly so that the DM group can select the best plan more expeditiously.

As each judge (DM) in the group may play a different role in selecting the final bidding plan, a weight $\bar{\lambda}_k$ is assigned to P_k to reflect its importance level in the deliberation process, then the weight vector of the DM group is

$$\bar{\lambda} = [\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_l]^T. \tag{10}$$

Eq.(10) can be standardized as follows

$$\lambda_k = \bar{\lambda}_k / \sum_{k=1}^l \bar{\lambda}_k. \tag{11}$$

After calculating the sequence of each plan for every DM, we can aggregate the rankings by using a Borda function. Given each DM's weight, the weighted Borda function value of the whole group for each bidding plan can be calculated by

$$b(x_j) = \sum_{k=1}^l \lambda_k b_k(x_j). \tag{12}$$

A multi-objective group decision-making algorithm for bidding purchase based on vague sets

The steps of the algorithm are now formulated below.

Step 1: Convert each DM's decision matrix F^k to a relative membership grade matrix U^k .

Step 2: For each plan x_j , calculate the upper bound of dissatisfaction and the lower bound of satisfaction, then the corresponding supporting, opposing, and neutral sets can be obtained.

Step 3: Calculate each DM's vague value and score function value for each bidding plan, then rank the bidding plans for each DM.

Step 4: Calculate the Borda function value for each bidding plan, and select the best one accordingly.

AN ILLUSTRATIVE EXAMPLE

The problem under consideration is to select an equipment supplier via bidding purchase in a college.

Assuming there are four experts (P_1, P_2, P_3, P_4) in the judge panel, three bidding plans (x_1, x_2, x_3) and five indicators (f_1, \dots, f_5). The judge panel will conduct comprehensive evaluations on the three bidding plans based on these five factors to select an optimal bidding plan. Among the five indicators, f_1 is a fuzzy linguistic variable, and the other four indicators are numerical scores assessed by the experts in the scale of 100. Each DM's assessment for the three bidding plans as per the five indicators are given in Table 3 and Table 4 furnishes each DM's weight assignment for the five indicators.

Table 3 Experts'raw assessments for each bidding plan with respect to each indicator

		f_1	f_2	f_3	f_4	f_5
p_1	x_1	Better	80	81	85	80
	x_2	Good	82	90	83	87
	x_3	Extremely good	76	86	87	81
p_2	x_1	Better	88	65	76	80
	x_2	Good	96	85	85	88
	x_3	Moderate	94	74	81	93
p_3	x_1	Very good	76	86	84	80
	x_2	Very good	84	76	85	90
	x_3	Better	84	83	83	76
p_4	x_1	Very good	70	90	75	80
	x_2	Better	85	74	85	77
	x_3	Better	86	82	84	85

Table 4 Weights of the five indicators by each expert

	f_1	f_2	f_3	f_4	f_5
P_1	9.2	5.0	7.2	8.5	4.6
P_2	8.8	4.6	6.6	8.1	3.1
P_3	9.0	5.3	8.1	8.2	5.0
P_4	9.4	5.1	7.9	8.7	4.2

The weight vectors can be standardized as per Eq.(6) as shown below

$$\begin{aligned} \omega^1 &= [0.27, 0.14, 0.21, 0.25, 0.13]^T, \\ \omega^2 &= [0.28, 0.15, 0.21, 0.26, 0.10]^T, \\ \omega^3 &= [0.25, 0.15, 0.23, 0.23, 0.14]^T, \\ \omega^4 &= [0.27, 0.14, 0.22, 0.25, 0.12]^T. \end{aligned}$$

Next, the ranking scheme is illustrated for DM

P_1 .

Step 1: From Table 3, we can extract DM P_1 's decision matrix F^1 and, then convert it to a relative membership grade matrix by Table 2 and Eq.(3) ($\theta=1$).

$$F^1 = \begin{matrix} & x_1 & x_2 & x_3 \\ \begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{matrix} & \begin{bmatrix} \text{better} & \text{very good} & \text{extremely good} \\ 80 & 82 & 76 \\ 81 & 90 & 86 \\ 85 & 83 & 87 \\ 80 & 87 & 81 \end{bmatrix} \end{matrix},$$

$$U^1 = \begin{matrix} & x_1 & x_2 & x_3 \\ \begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{matrix} & \begin{bmatrix} 0.80 & 0.90 & 0.975 \\ 0.98 & 1.00 & 0.93 \\ 0.90 & 1.00 & 0.96 \\ 0.98 & 0.95 & 1.00 \\ 0.92 & 1.00 & 0.93 \end{bmatrix} \end{matrix}.$$

Step 2: Calculate the supporting set and opposing set for each bidding plan based on U^1 . According to Eq.(7), $\xi_1^U=0.99$, $\xi_1^L=0.85$. Then $S_1^1=\phi$, $O_1^1=\{f_1\}$, $S_2^1=\{f_2, f_3, f_5\}$, $O_2^1=\phi$, $S_3^1=\{f_4\}$, $O_3^1=\phi$.

Step 3: Calculate the vague value for each bidding plan: $t(x_1^1)=0$, $f(x_1^1)=0.27$, $t(x_2^1)=0.48$, $f(x_2^1)=0$, $t(x_3^1)=0.25$, $f(x_3^1)=0$, then $S[E(x_1^1)]=-0.52$, $S[E(x_2^1)]=0.23$, $S[E(x_3^1)]=0$. We can see that the ranking of these three bidding plans for P_1 is $x_2 \succ x_3 \succ x_1$.

In a similar fashion, the rankings for the other three DMs P_2, P_3, P_4 are obtained and listed as follows: $x_2 \succ x_1 \succ x_3$, $x_2 \succ x_1 \succ x_3$, $x_1 \succ x_3 \succ x_2$, respectively.

Step 4: Assume that the four experts assume equal power in determining the final bidding plan and, hence, are assigned the same weight, that is, $\bar{\lambda}=[1,1,1,1]^T$, it is standardized by Eq.(11)

$$\lambda = [0.25, 0.25, 0.25, 0.25]^T.$$

Using the Borda function to aggregate all DMs' individual assessments, we obtain the group's Borda function values as

$$b(x_2) = 1.5, \quad b(x_1) = 1.0, \quad b(x_3) = 0.5.$$

The result shows that x_2 is the best choice, and the ranking for the three bidding plans is $x_2 \succ x_1 \succ x_3$.

CONCLUSION

A vague-set-based fuzzy multi-objective decision-making method is developed for bidding purchase, and a new way is put forward to determine the lower bound of satisfaction and the upper bound of dissatisfaction, and a new score function is designed to calculate vague values for bidding purchase. This approach allows a DM to express its assessments as either linguistic variables or specific crisp values. An illustrative example demonstrates how this approach may be applied.

Significant issues remain open. For instance, the model does not consider different sets of evaluating indicators in the decision-making process. In addition, linguistic variables need to be expressed by fuzzy sets more effectively. These remaining issues warrant further research in this area.

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