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Modeling and control of a small solar fuel cell hybrid energy system*

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Abstract: This paper describes a solar photovoltaic fuel cell (PVEC) hybrid generation system consisting of a photovoltaic (PV) generator, a proton exchange membrane fuel cell (PEMFC), an electrolyser, a supercapacitor, a storage gas tank and power conditioning unit (PCU). The load is supplied from the PV generator with a fuel cell working in parallel. Excess PV energy when available is converted to hydrogen using an electrolyser for later use in the fuel cell. The individual mathematical model for each component is presented. Control strategy for the system is described. MATLAB/Simulink is used for the simulation of this highly nonlinear hybrid energy system. The simulation results are shown in the paper.

Key words: Photovoltaic (PV), Fuel cell, Electrolyser, Maximum power point tracking
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INTRODUCTION

Photovoltaic (PV) power is an intermittent source of energy for dealing with the variations in solar radiation level over the day. This phenomenon may cause operational problems such as excessive frequency deviations, spinning reserve increase, etc. at a central control center in a power utility. One solution is to store electrical energy in batteries and accumulators but this can only be achieved in limited amount and can have direct impact on the environment. Another solution for the long term storage problem can be reached by converting the electrical energy into hydrogen using water electrolysis and store hydrogen for later use in fuel cells. Main characteristics of fuel cells include high efficiency, near zero emissions, and low noise levels. Presently, prototype solar cells in the range of 10~50 kW are available for testing and demonstration, while prototypes of proton exchange membrane fuel cells (PEMFCs) are now in the development and testing phase.

So far as several solar hydrogen projects were developed over the past decade around the world. At the residential level, Hollmuller *et al.*(2000) reported a single-family home in Switzerland has operated a 5 kW PV array since 1991 to produce electrolytic H_2 which is then utilized to power a stove, laundry machine, and minivan. At the building level, Barthels *et al.*(1998) introduced a study of a solar-hydrogen powered library in Germany outlines the challenges to systems engineering optimization in a stand-alone configuration. Lehman *et al.*(1997) reported that a marine laboratory at Humboldt State University has been powered successfully by solar-hydrogen for many years in a stand-alone configuration. However, because of commercial secret or other reasons, these research institutes provide little technical information. It is significant to carry on this research on our own.

In this paper, a combination of PV and fuel cell hybrid energy system for stationary applications is provided. The system includes solar panels and a PEMFC system working in parallel, a supercapacitor, power conditioning units to smooth voltage fluctuations, and storage tanks for the compressed hydrogen. In this hybrid energy system, if the solar radiation level is high enough, the PV array powers the load

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and the excess power is stored in hydrogen by the electrolyser. Otherwise, the fuel cell is switched on to generate electricity to complement any shortfall in solar radiation. A forward-facing simulation software for photovoltaic fuel cell (PVEC) hybrid system based on MATLAB/Simulink is developed and the simulation models of PVFC hybrid system are also built, and are used as a main method to study PVFC control strategy.

Basically, the hybrid system will be operating under transient conditions most of the time due to the intermittent nature of solar radiations. Therefore, the objective of this work is to model the system and to conduct transient analysis of its operation under varying solar conditions. The suggested system will be dimensioned to provide electricity in stand-alone mode. The work includes modelling and computer simulations using available commercial software to study the transient behavior of the hybrid energy system. Simulation results are also shown in the paper.

CONFIGURATION

As shown in Fig.1, PVFC hybrid power system includes a photovoltaic (PV) generator, a proton exchange membrane fuel cell (PEMFC), an electrolyser, a supercapacitor, a storage gas tank and power conditioning unit (PCU). In this hybrid power system, the fuel cell is used to produce power if the load power exceeds that produced from the PV generator. It can also function as an emergency generator, if the PV generator system fails. An electrolyser converters electricity into chemical energy which produces hydrogen which can be stored in pressure tanks and used for fuel cell later. A supercapacitor is an energy storage device constructed like that of a battery and is used for the safe operation of the fuel cell component and also to supply power during transient load conditions. In addition, a control system is required to monitor and guide the operation of the components of the system.

SIMULATION MODELS FOR PVFC HYBRID SYSTEM

One of the goals of this paper is to simulate the operation of PVFC hybrid energy system as accurately as possible so that realistic optimal control strategies can be found. To achieve this aim, one needs a set of relatively detailed models. In this section, the individual mathematical model for each component was developed in MATLAB, and the model, simulation and control of the systems was developed using Simulink.

Photovoltaic array model

The current-voltage characteristics of a photovoltaic cell are characterized by a parametric diode model proposed by Roger and Maguin (1982) and Green (1981). A detailed approach to PV cell module or array modelling based on a mathematical description of the equivalent electrical circuit of a PV cell is given in (Ro and Rahman, 1998).

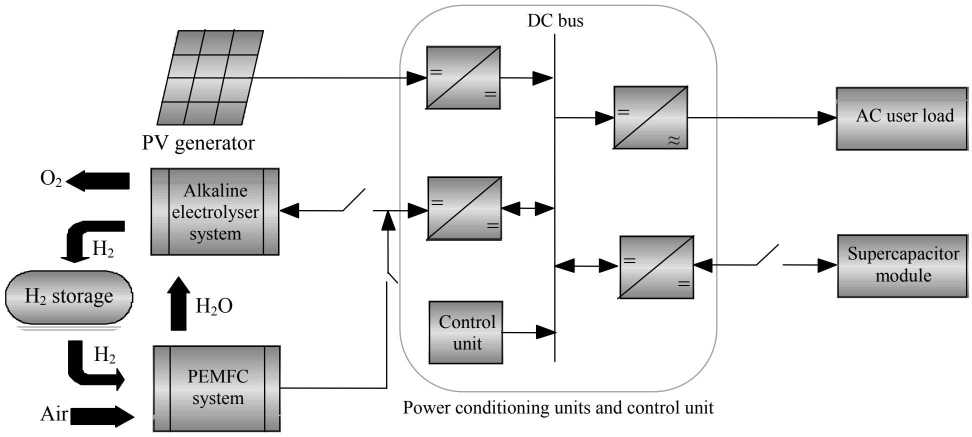

Fig.1 Block diagram of the PVFC hybrid power system

$$I_{\mathrm{PV}}=I_{\mathrm{ph}}-I_{\mathrm{o}}\left[\exp\left\{\frac{q(V_{\mathrm{PV}}+I_{\mathrm{PV}}R_{\mathrm{s}})}{AKT}\right\}-1\right]-\frac{V_{\mathrm{PV}}+I_{\mathrm{PV}}R_{\mathrm{s}}}{R_{\mathrm{sh}}}, \quad (1)$$

$$I_{\mathrm{o}}=I_{\mathrm{or}}\left(\frac{T}{T_{\mathrm{r}}}\right)^{3}\exp\left\{\frac{qE_{\mathrm{G}}}{KA}\left(\frac{1}{T_{\mathrm{r}}}-\frac{1}{T}\right)\right\}, \quad (2)$$

$$I_{\mathrm{ph}}=\{I_{\mathrm{scr}}+k_{\mathrm{i}}(T-T_{\mathrm{r}})\}\lambda/100, \quad (3)$$

$$I_{\mathrm{PV}}\left(1+\frac{R_{\mathrm{s}}}{R_{\mathrm{sh}}}\right)=n_{\mathrm{p}}I_{\mathrm{ph}}-n_{\mathrm{p}}I_{\mathrm{o}}\left[\exp\left\{\frac{q(V_{\mathrm{PV}}+I_{\mathrm{PV}}R_{\mathrm{s}})}{AKT}\right\}-1\right]-\frac{(V_{\mathrm{PV}}+I_{\mathrm{PV}})/n_{\mathrm{s}}}{R_{\mathrm{sh}}}, \quad (4)$$

$$P_{\mathrm{PV}}=n_{\mathrm{p}}I_{\mathrm{ph}}V_{\mathrm{PV}}-n_{\mathrm{p}}I_{\mathrm{o}}\left[\exp\left\{\frac{q(V_{\mathrm{PV}}+I_{\mathrm{PV}}R_{\mathrm{s}})}{AKT}\right\}-1\right]V_{\mathrm{PV}}-\frac{((V_{\mathrm{PV}}+I_{\mathrm{PV}})/n_{\mathrm{s}})V_{\mathrm{PV}}}{R_{\mathrm{sh}}}, \quad (5)$$

where I_{PV} is the PV cell output current, I_{ph} is light current, I_{o} is diode reverse saturation current, I_{or} is saturation current, I_{scr} is the short-circuit current, A is the ideality factor, E_{G} is the band gap for silicon, K is Boltzmann's constant, k_{i} is the short-circuit current temperature coefficient at I_{scr}. q is the electron charge, R_{s} and R_{sh} are the series resistance and shunt resistance respectively. T is the PV cell temperature. T_{r} is the reference temperature. V_{PV} is the PV cell output voltage and I_{PV} is the PV cell output current. λ is the irradiance.

Fuel cell model

The *U-I* characteristics of a PEM fuel cell presented in Fig.2 can be divided into three regions, which are governed by different overvoltages. Activation overvoltage dominates at low current densities. The middle of the region is governed by the ohmic losses and bending down of the polarization curves due to the concentration overvoltage. The cell voltage model was studied empirically and physically by many researchers. In this study, a generalized electrical model of PEMFC is developed by using a combination of physical and empirical modelling techniques developed by Mann *et al.*(2000) and Amphlett *et al.*(1995).

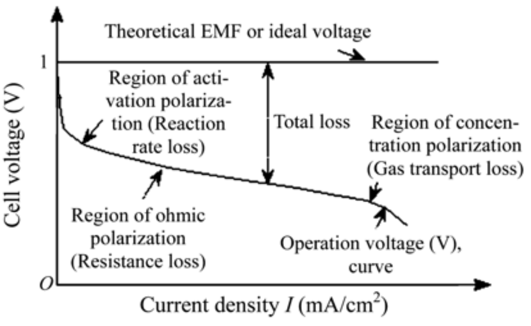

Fig.2 Typical *U-I* characteristic of a fuel cell

The output voltage of each cell was defined with Eq.(6) as a function of the thermodynamic equilibrium potential (E) corresponding to the overall chemical reaction expressed in Eq.(7), the activation overvoltage (η_{act}) and the ohmic overvoltage (η_{ohm}):

$$P_{\mathrm{fc}}=nV_{\mathrm{fc}}I_{\mathrm{fc}}=nI_{\mathrm{fc}}(E_{\mathrm{Nernst}}-\eta_{\mathrm{act}}+\eta_{\mathrm{ohm}}), \quad (6)$$

$$\mathrm{H_2\ (g)}+\frac{1}{2}\mathrm{O_2\ (g)}\rightarrow \mathrm{H_2O\ (l)}+\mathrm{e}^-, \quad (7)$$

$$E_{\mathrm{Nernst}}=1.229-0.85\times10^{-3}(T-298.15)+4.308\times10^{-5}T[\ln(p_{\mathrm{H_2}})+0.5\ln(p_{\mathrm{O_2}})], \quad (8)$$

$$\eta_{\mathrm{act}}=\xi_1+\xi_2T+\xi_3T[\ln(c_{\mathrm{O_2}})]+\xi_4T[\ln(i)], \quad (9)$$

$$\eta_{\mathrm{ohm}}=-i(R^{\mathrm{electronic}}+R^{\mathrm{proton}})=-iR^{\mathrm{internal}}, \quad (10)$$

$$R^{\mathrm{proton}}=r_{\mathrm{m}}l/A, \quad (11)$$

$$r_{\mathrm{m}}=\frac{181.6[1+0.03(i/A)+0.062(T/303)^2(i/A)^{2.5}}{[\lambda_{\mathrm{m}}-0.634-3(i/A)]\exp(4.18[(T-303)/T])}, \quad (12)$$

where P_{fc} is the stack power, n is the cell number, V_{fc} is cell voltage, I_{fc} is cell current, E is the open circuit voltage, η_{act} is the activation overvoltage, η_{ohm} is the ohmic overvoltage, T is the temperature, $p_{\mathrm{H_2}}$ is the hydrogen reactant partial pressure, $p_{\mathrm{O_2}}$ is the oxygen reactant partial pressure, A is the active cell area, i is the current, $c_{\mathrm{O_2}}$ is oxygen concentration at the cathode, r_{m} is the membrane specific resistivity for the flow of hydrated protons, l is the thickness of the polymer membrane, λ_{m} is membrane water content, ξ_i are empirical coefficients for calculating activation overvoltage. According to Eqs.(6)~(12), the electricity model is applicable for PEMFC stack of various configurations and operating conditions.

Electrolyser model

The electrolyser model is based on a combination of fundamental thermodynamics, heat transfer theory, and empirical electrochemical relationships. The electrode kinetics of an electrolyser cell can be modelled using empirical voltage-current (*U-I*) relationships. Empirical *U-I* models for electrolyzers were suggested by Hug *et al.*(1993) and Vanhanen *et al.*(1996), to mention a few. The basic form of the *U-I* curve for a known operation temperature is shown below (Ulleberg, 2003):

$$U = U_{\text{rev}} + \frac{r}{A} I + s \log\left(\frac{t}{A} I + 1\right), \tag{13}$$

where U is operation cell voltage, U_{rev} is reversible cell voltage, r is ohmic resistance of electrolyte, s, t are coefficients for over-voltage on electrodes, A is area of electrode, I is current through cell.

Fig.3 shows the plot of the voltage for an alkaline water electrolyser cell vs the current density at high and low operation temperatures. The difference between the two *U-I* curves is mainly due to the temperature dependence of the over-voltages.

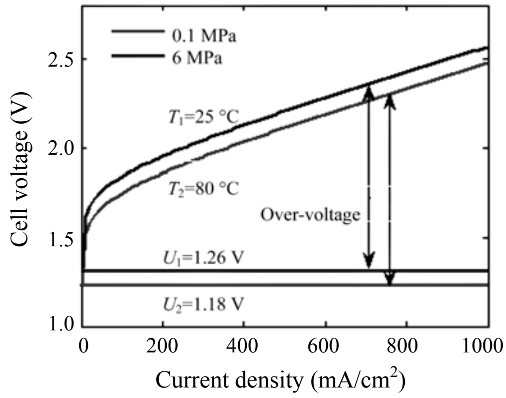

Fig.3 Typical *U-I* characteristic of a electrolyser cell

The actual flow rates of hydrogen and oxygen production or water consumption in an electrolyser cell can be calculated by:

$$\dot{m}_{\text{H}_2} = 2\dot{m}_{\text{O}_2} = \dot{m}_{\text{H}_2\text{O}} = \eta_{\text{F}} \frac{N_{\text{s}} I}{nF}, \tag{14}$$

where $\dot{m}_{\text{H}_2}$ is hydrogen production rate, $\dot{m}_{\text{O}_2}$ is oxygen production rate, $\dot{m}_{\text{H}_2\text{O}}$ is water consumption rate, η_{F} is Faraday's efficiency, N_{s} is number of series cells, I is input current to electrolyser, n is number of electrons per mole, F is Faraday's constant.

Supercapacitor model

A supercapacitor is an energy storage device with a construction similar to that of a battery. It has two electrodes immersed in an electrolyte with a separator between the electrodes. Burke (2000) proposed that the surface area of the electrode materials used in supercapacitor is 500~2000 m^2/g, which is much greater than that used in conventional capacitor electrodes. Therefore, a supercapacitor can store greater amounts of energy than conventional capacitors, and can deliver more power than batteries. The voltage-current relationship of the supercapacitor is expressed by:

$$U_{\text{SCap}} = ESR \times I_{\text{SCap}} + \frac{1}{C}\int_0^t (I_{\text{SCap}} - I_{\text{dis}})\mathrm{d}t + U_{\text{SCap}}(0), \tag{15}$$

where *ESR* is the internal equivalent series resistance. U_{SCap} is the voltage of the capacitor. I_{SCap} is the current of the capacitor. I_{dis} is the self-discharge current. And $U_{\text{SCap}}(0)$ is the initial voltage across the capacitance C.

Power conditioning unit model

Vosen and Keller (1999) provided a mathematical model for the power conditioning units that is based on empirical efficiency curves for electrical converters (DC/DC) or inverters (DC/AC). Three different models which are used to describe the PCU as an energy converster are linear, quadratic, and piecewise linear models. The quadratic model is the best model among all the models to represent all PCUs in the system study because its parameters have physical meaning of the PCU and are very simple for programming (Abd El-Aal *et al.*, 2006).

The output power in this model can be represented as:

$$P_{\text{out}} = \begin{cases} C_0 + C_1 P_{\text{in}} + C_2 P_{\text{in}}^2, & P_{\text{standby}} \le P_{\text{in}} \le P_{\text{in,max}}; \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

The parameters of the model (C_i) are extracted from experimental data using the least-squares method.

Gas storage model

The hydrogen storage model is based on the ideal gas law. The mathematical model for the hydrogen pressure p in a storage tank can be calculated from:

$$p = nRT / V, \tag{17}$$

where p is the hydrogen pressure inside the tank, n is the number of moles, R is the universal gas constant, T is the temperature of the gas and V is the volume of the tank.

CONTROL DESIGN AND SIMULATION RESULTS

Control strategy for PVFC

The input data such as solar radiation and ambient temperature are fed in to calculate the amount of energy that can be generated by the PV generator. The value of PV output power (P_{PV}) is compared with the load demand energy (P_{L}) to determine the distribution of energy flow between the storage unit and the load. Surplus PV energy is stored in the form of hydrogen by the electrolyser (P_{el}) and deficit energy can be taken from hydrogen by the fuel cell (P_{fc}) back-up generator.

A flow diagram of the control strategy for the PVFC hybrid system is shown in Fig.4.

According to Fig.4, the control strategy is based on the following three different cases:

(1) If $P_{\mathrm{PV}}>P_{\mathrm{L}}$, then $P_{\mathrm{el}}=P_{\mathrm{PV}}-P_{\mathrm{L}}$. That is, if the radiation level is high enough, the PV array empowers the load and the excess power is stored in hydrogen by the electrolyser.

(2) If $P_{\mathrm{PV}}<P_{\mathrm{L}}$ and $P_{\mathrm{L}}-P_{\mathrm{PV}}\leq P_{\mathrm{fc}}$, then $P_{\mathrm{L}}-P_{\mathrm{PV}}=P_{\mathrm{fc}}$. That is, if the PV generator cannot power the load, then the load is connected directly to the PV generator and the fuel cell is switched on.

(3) If $P_{\mathrm{PV}}<P_{\mathrm{L}}$ and $P_{\mathrm{L}}-P_{\mathrm{PV}}>P_{\mathrm{fc}}$, then $P_{\mathrm{L}}=P_{\mathrm{fc}}=0$ and $P_{\mathrm{el}}=P_{\mathrm{PV}}$. That is, if the PV generator cannot power the load and the fuel cell fails to start, then the load and fuel cell are disconnected and the electrolyser is connected directly to the PV generator, then the load is connected according to the conditions in Eq.(1) or Eq.(2).

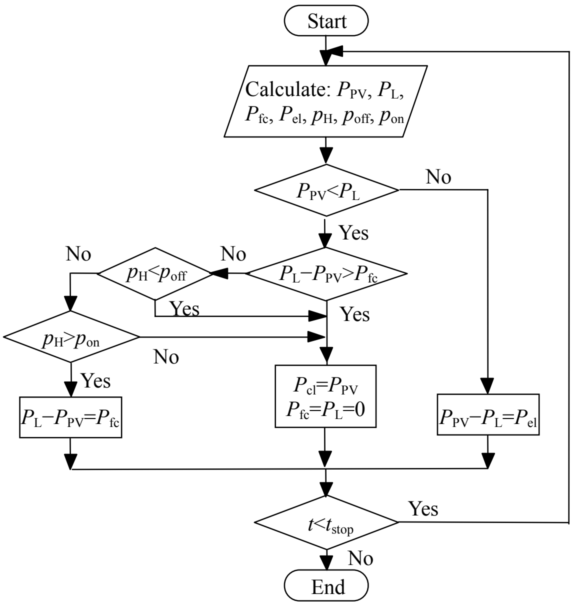

Fig.4 Basic flow diagram of the control strategy for the PVFC system

Simulation results

The main technical parameters of the PVFC parts are illustrated in Table 1 and are used in the simulation and calculation in the paper.

The performance of the PVFC hybrid power system is simulated and calculated by simulation software based on the MATLAB/Simulink, and the results are shown as Figs.5~7.

Two classes of input data are necessary to define the system simulation model. The first class comprises the technical characteristics, and these will be defined for the simulation model. This class was shown in detail in Table 1. The second class of data consists of weather data and user load demand. All these data are stored in files and are called when needed by the used components within the simulation system.

Figs.5~7 show simulation results for a day of the system in one summer day. From Fig.5, it can be seen that the solar radiation varies according to the time. During the period of low solar radiation, the state of charge for hydrogen storage in the tank drops because the hydrogen production is lower than consumption. The fuel cell is switched on. When the solar radiation is high enough, the PV array powers the load and the excess power is stored in hydrogen by the electrolyser. Form 11:00 to 17:00, the user load is at a constant low and the radiation is high. All the power from the PV could be used to run the electrolyser, which gives an ideal situation for high hydrogen production. The fluctuations in the pressure of

Table 1 The technical parameters of PVFC

Technical parameters	Value
Photovoltaic module	
Electron charge (C)	1.610×10^{-19}
Series resistance (Ω)	5×10^{-5}
Shunt resistance (Ω)	5×10^{5}
Boltzmann's constant (Nm/°K)	13805×10^{-23}
Ideality factor	1.6
Reference temperature (°K)	301.18
NOCT (°C)	47±2
PEMFC system	
Electrical rated power (kW)	2
Membrane water content	8
Empirical coefficient (ξ_1)	–0.9514
Empirical coefficient (ξ_2)	0.00312
Empirical coefficient (ξ_3)	7.4×10^{-5}
Empirical coefficient (ξ_4)	–0.000187
Width×depth×height (cm^3)	110×70×172
Permissible ambient temperature (°C)	15~25
Electrolyser	
Operating pressure (bar)	30
Ambient temperature (°C)	2~40
Electrolyte temperature (max) (°C)	85
Maximum power (kW)	3.6
Current density (max) (mA/cm^2)	400
Hydrogen production (Nm^3/h)	0.8
Oxygen production (Nm^3/h)	0.4
Supercapacitor module	
Operating voltage (V)	29~8
Maximum voltage (V)	32
Minimum voltage (V)	8
Maximum power (kW)	17.5
Width×depth×height (mm^3)	110×70×172
Internal ohmic resistance (mΩ)	12~18
Operating temperature (°C)	–50~50
Leakage current at 28 V (mA)	5

the hydrogen storage vary with the hydrogen produced by the electrolyser and the hydrogen consumed by the fuel cell (Fig.7). The power required by the electrolyser varied, depending on the excess power available on the busbar. From Fig.6 and Fig.7, from 17:00 to 24:00, it can be seen that there is less power available from the PV, and the electrolyser is not switched on. At the same time, the fuel cell is switched on to power the load with the PV generator.

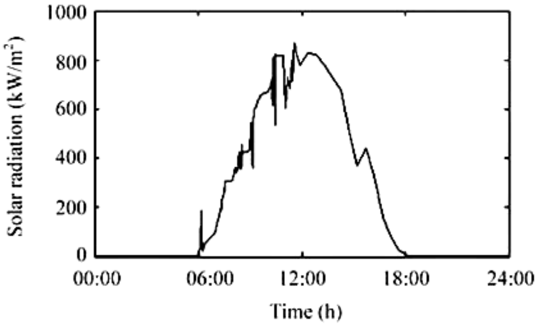

Fig.5 Solar radiation

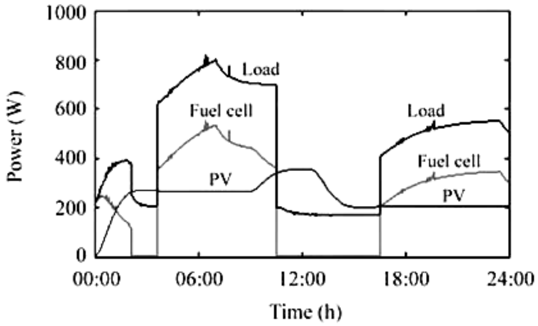

Fig.6 Output power of PV, fuel cell and load

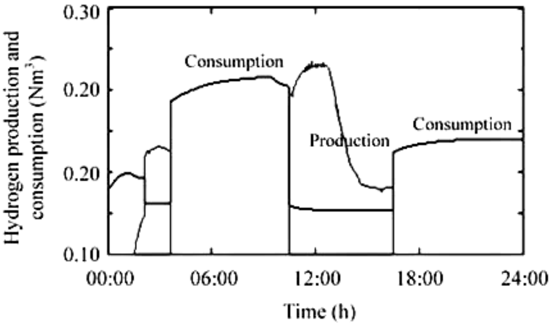

Fig.7 Hydrogen production and consumption

CONCLUSION

A small PVFCl hybrid energy system for stand-alone operation is proposed in this paper. In this study, the individual mathematical model for each component was developed in MATLAB, and the model, simulation and control of the systems was developed using Simulink. With solar radiation and temperature data as input and with equations from the literature and information from manufactures specifications, this paper is expected to provide guidelines for sizing the PV and fuel cell that are to be coupled; to develop a simulation model for the designer to evaluate performance with weather data for the system. The results of simulation showed that the system performance is excellent and that the mathematical models and control techniques developed in this paper can also be used to design and control the other different topologies of PVFC hybrid energy system.

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