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Expansion of spherical cavity of strain-softening materials with different elastic moduli of tension and compression*

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Abstract: An expansion theory of spherical cavities in strain-softening materials with different moduli of tension and compression was presented. For geomaterials, two controlling parameters were introduced to take into account the different moduli and strain-softening properties. By means of elastic theory with different moduli and stress-softening models, general solutions calculating Tresca and Mohr-Coulomb materials' stress and displacement fields of expansion of spherical cavity were derived. The effects caused by different elastic moduli in tensile and compression and strain-softening rates on stress and displacement fields and development of plastic zone of expansion of cavity were analyzed. The results show that the ultimate expansion pressure, stress and displacement fields and development of plastic zone vary with the different elastic moduli and strain-softening properties. If classical elastic theory is adopted and strain-softening properties are neglected, rather large errors may be the result.

Key words: Expansion of spherical cavity, Tresca material, Mohr-Coulomb material, Elastic theory with different moduli of tension and compression, Stress-dropping softening model

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INTRODUCTION

The theory of spherical cavity expansion has been widely applied to geotechnical problems such as the bearing capacity of deep foundations (Gibson, 1950; Vesic, 1972; Randolph *et al*., 1979), interpretations of pressuremeter tests (Gibson and Anderson, 1961; Vesic, 1972; Palmer and Mitchell, 1972; Hughes *et al*., 1977; Houlsby and Withers, 1988; Yu, 1990; Bolton and Whittle, 1999; Cao *et al*., 2001) and cone penetration tests (Gupta and Davison, 1986; Mayne, 1991; Collins *et al*., 1992; Chang *et al*., 2001; Burns and Mayne, 2002; Gupta, 2002).

Published solutions vary because of differences in the constitutive models used to describe the stress-strain relationship of the expanding mass. Most analyses were based on the elastic-perfectly plastic geotechnical materials (Hill, 1950; Chadwick, 1959; Gibson and Anderson, 1961; Vesic, 1972; Carter *et al*., 1986; Yu, 1990; Cao *et al*., 2001). Chadwick (1959) presented a derivation of the pressure expansion relationship for an elastic-perfectly plastic material while assuming Mohr-Coulomb criterion and flow rules. Vesic (1972) presented similarity solutions for limit pressures for a spherical cavity expansion in a cohesive frictional material. Collins and Yu (1996) provided an analytical study of the large-strain undrained expansion of spherical cavity in critical-state soil model. Cao *et al*.(2001) gave the solution of undrained cavity expansion in an elastic-perfectly plastic material with modified Cam clay. But most researchers assumed that the soil was an elastic-perfectly plastic material that had the same

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modulus in tension and compression. In fact, these assumptions mentioned are only suitable for ideal materials. For geomaterials significant errors may be the result.

Soil tensile property is different from compression property, which had been verified by triaxial compression and tensile tests (Gong, 1984). Moreover, the stress-strain relationship of geomaterials has apparent peak value, as shown graphically in Fig.1 (Jiang and Shen, 1996).

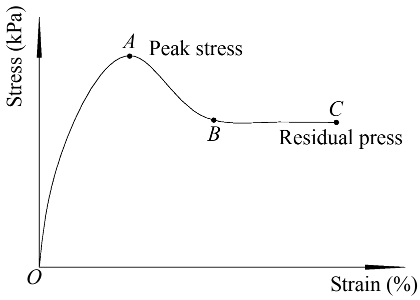

Fig.1 Representative strain-softening curve

From Fig.1 it is evident that: (1) When stress is below the peak stress, the material is in the elastic state at initial stage, so the stress-strain relationship can be simulated with linear elastic or nonlinear elastic model; (2) After stress peak value, the stress would decrease with the increase of strain, which is the phenomenon of strain softening; (3) When stress lowered to the residual stress, the material would be in perfectly plastic state. In the strain-softening zone the inner product of stress increment and strain increment is negative, that is to say, negative work, so numerical simulation results have no uniform value (Shen, 1982; Prevost and Hughes, 1984). In order to reflect strain-softening behavior of geomaterials, stress-dropping softening model is applied in this paper.

STRESS-DROPPING SOFTENING MODEL

In order to simulate the stress-strain relationship shown in Fig.1, linear functions in each phase and stress-dropping softening model (stress-dropping from peak stress to residual stress) as shown graphically in Fig.2 are applied to the expansion of the spherical cavity. For Mohr-Coulomb materials, yield surfaces in π plane are shown graphically in Fig.3. Point A and point B in Fig.3 are consistent with yield surfaces $F(\sigma_{ij})$=0 and $f(\sigma_{ij})$=0 respectively. When material is loaded to the initial yield surface (point A) and loading condition is satisfied, stress will drop to the subsequent yield surface (point B). Then plastic flow occurs and the material becomes failure finally.

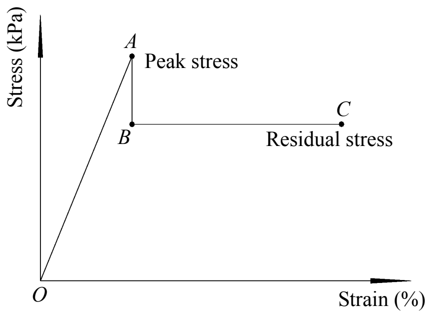

Fig.2 Stress-dropping softening model

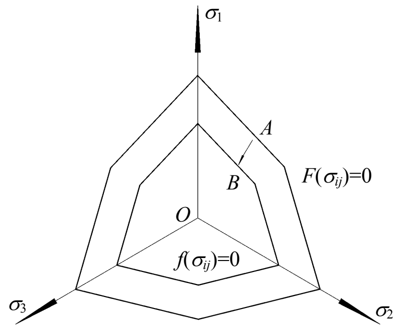

Fig.3 Stress-dropping model of Mohr-Coulomb materials in π plane

EXPANSION PROBLEM OF SPHERICAL CAVITY

For spherical cavity made of materials with different elastic moduli of tensile compression and strain-softening behavior, its expansion law is different from the ideal elastic-plastic material. When a spherical cavity of initial radius a_0 is expanded by uniformly distributed internal pressure p, it will be compressed in the radial direction and extended in the tangential direction. If the pressure increases to a critical value (p_c), a spherical zone around the cavity will pass into the yield state and damaged surface S_c will be formed. With the increase of the internal pressure, stress dropping happens and a plastic zone around the cavity is formed. This plastic zone will expand to a radius r_{max} until the pressure reaches an ultimate value p_u. Beyond this surface, the rest of the mass remains in a state of elastic equilibrium. As shown in Fig.4, a is the expanded radius and r_1 is the radius of damaged surface.

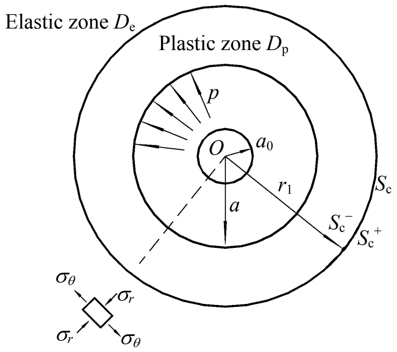

Fig.4 Expansion of spherical cavity

General solution

Apart from these assumptions of stress-strain relationship and different moduli in tension and compression, other assumptions are identical with those of Vesic (1972)'s spherical expansion theory.

Considering an element at a radial distance r from the cavity center, the equation of equilibrium can be expressed as follows:

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}+\frac{2}{r}(\sigma_r-\sigma_\theta)=0,\quad \tau_{r\theta}=0, \tag{1}$$

where r is the distance to the cavity center.

The equations of geometry are:

$$\varepsilon_r=\frac{\mathrm{d}u_r}{\mathrm{d}r},\ \varepsilon_\theta=\frac{u_r}{r}. \tag{2}$$

The coordination equation of deformation is:

$$r\frac{\mathrm{d}\varepsilon_\theta}{\mathrm{d}r}+\varepsilon_\theta-\varepsilon_r=0. \tag{3}$$

The constitutional equations of elastic stage, based on elastic theory of different moduli (АМБАРЦУЯН, 1986), are written as follows:

$$\begin{cases}\varepsilon_r=a_{11}\sigma_r+2a_{12}\sigma_\theta,\\ \varepsilon_\theta=(a_{22}+a_{12})\sigma_\theta+a_{12}\sigma_r,\end{cases} \tag{4}$$

in which $a_{ij}=f(E^+,E^-,\nu^+,\nu^-)$ and the superscript '+'and '−' of E and ν respectively represent compression and tension of soil.

For the expansion of spherical cavity, a_{11}, a_{22}, a_{12}, a_{21} can be obtained as follows:

$$\begin{cases}a_{11}=(E^+)^{-1},\\ a_{22}=(E^-)^{-1},\\ a_{12}=a_{21}=-\nu^-/E^-.\end{cases} \tag{5}$$

After introducing Eq.(5) into Eq.(4), Eq.(4) transforms into

$$\begin{cases}\varepsilon_r=\sigma_r/E^+-2\nu^+\sigma_\theta/E^+,\\ \varepsilon_\theta=(1-\nu^-)\sigma_\theta/E^--\nu^-\sigma_r/E^-.\end{cases} \tag{6}$$

At the plastic stage, Tresca yield criterion or Mohr-Couloumb yield criterion is applied to the problem of spherical cavity.

The initial yield function of Tresca materials takes the form:

$$F=\sigma_r-\sigma_\theta-2K=0\quad (\text{at } S_c^+), \tag{7a}$$

and the subsequent yield function is written as:

$$f=\sigma_r-\sigma_\theta-2\beta K=0\ \ (\text{in the plastic zone } D_p), \tag{7b}$$

where K is Tresca material constant, β is a controlling parameter of material softening degree. If β=1, the soil becomes an ideal solid.

The initial yield function of Mohr-Coulomb material takes the form:

$$F=(\sigma_r-\sigma_\theta)-(\sigma_r+\sigma_\theta)\sin\varphi-2c\cos\varphi=0, \tag{8a}$$

and the subsequent yield function can be written as:

$$f=(\sigma_r-\sigma_\theta)-(\sigma_r+\sigma_\theta)\sin\varphi_r-2c_r\cos\varphi_r=0, \tag{8b}$$

in which c, φ, c_r and φ_r are material cohesion, angle of internal friction, residual cohesion and residual angle of internal friction, respectively.

General solution of elastic-plastic analyses

1. Elastic zone D_e, $D_e=\{r|r\geq a, p<p_c\}$

Stress function $\psi(r)$ is chosen to make σ_r and σ_θ

become as follows:

$$\begin{cases} \sigma_r = 2\psi / r, \\ \sigma_\theta = \dfrac{\mathrm{d}\psi}{\mathrm{d}r} + \dfrac{\psi}{r}. \end{cases} \tag{9}$$

Introducing Eqs.(4), (5) and (9) into Eq.(3), the following expression can be obtained:

$$r^2 \frac{\mathrm{d}^2\psi}{\mathrm{d}r^2} + 2r\frac{\mathrm{d}\psi}{\mathrm{d}r} - 2\frac{a_{11} + a_{12}}{a_{22} + a_{12}}\psi = 0. \tag{10}$$

The following solution of the differential Eq.(10) is derived as follows:

$$\psi(r) = At^{(\lambda-1)/2} + Bt^{-(\lambda+1)/2}, \tag{11}$$

in which, both A and B are the integration constant, t is the function of r, and

$$\lambda = \sqrt{1 + 8\frac{a_{11} + a_{12}}{a_{22} + a_{12}}} = \sqrt{1 + 8\frac{E^-(1-\nu^+)}{E^+(1-\nu^-)}}. \tag{12}$$

By inserting Eq.(11) into Eq.(9), as well as the boundary condition, the following expression can be obtained

$$\begin{cases} \sigma_r = p(a/r)^{\alpha+3/2}, \\ \sigma_\theta = -\eta p(a/r)^{\alpha+3/2}, \end{cases} \tag{13}$$

where

$$\alpha = \sqrt{1/4 + 2E^-(1-\nu^+)/[E^+(1-\nu^-)]},$$

$$\eta = \sqrt{1/16 + E^-(1-\nu^+)/[2E^+(1-\nu^-)]} - 1/4.$$

Taking account of Eqs.(2), (6) and (13), the following expression can be obtained:

$$u_r = -\frac{(1-\nu^-)(\alpha - 1/2) + 2\nu^-}{2E^-} p(a/r)^{\alpha+3/2} r. \tag{14}$$

Eqs.(13) and (14) are respectively formulas of computing stress and displacement in the elastic zone $D_e=\{r|r\geq a,\ p<p_c\}$. For the plastic zone $D_p=\{r|r\geq r_1,\ p\geq p_c\}$, r_1 and p' substituted for a and p respectively, where r_1 is radius of the plastic zone D_p and p' is the radial stress of the yield surface S_c^+.

2. Plastic zone D_p, $D_p=\{r|a\leq r<r_1,\ p\geq p_c\}$

(1) Tresca material

When the internal pressure increases to the critical stress p_c, the plastic zone around the cavity will occur. At this moment the cavity has a radius r_1, where $r_1=a$.

Combining Eq.(13) with Eq.(7a), the following expression can be obtained:

$$p_c = 2K/(1+\eta). \tag{15}$$

Combining Eq.(1) with Eq.(7b), as well as the boundary condition, stress fields of the plastic zone are written as follows:

$$\sigma_r = p - 4\beta K \ln\frac{r}{a}, \tag{16a}$$

$$\sigma_\theta = p - 2\beta K\left(2\ln\frac{r}{a} + 1\right). \tag{16b}$$

When $\sigma_r=p_c=2K/(1+\eta)$, Eq.(16a) transforms into:

$$p = \frac{2}{1+\eta}K + 4\beta K \ln\frac{r}{a}. \tag{17}$$

This plastic zone will expand until the pressure reaches the ultimate value p_u. At this moment the plastic zone around the cavity will extend to a maximum radius r_{max}, so Eq.(17) transforms into

$$p_u = \frac{2}{1+\eta}K + 4\beta K \ln\frac{r_{max}}{a}. \tag{18}$$

The average plastic volumetric strain Δ of Tresca material is equal to zero (Vesic, 1972; Gong, 1999), so the change of volume of the cavity is equal to the change of volume of the elastic zone; this relationship can be written as follows:

$$a^3 - a_0^3 = r_{max}^3 - (r_{max} - u_r)^3. \tag{19}$$

Substituting Eq.(14) into Eq.(19), and ignoring higher order terms of u_r^3, Eq.(19) becomes

$$(r_{max}/a)^3 = (1 - a_0^3/a^3)/(3m - 3m^2), \tag{20}$$

where

$$m=[(1-\nu^-)(\alpha-1/2)+2\nu^-]K/[E^-(1+\eta)]. \quad (21)$$

After introducing Eq.(20) into Eq.(18), the ultimate pressure p_u can be obtained as follows:

$$p_u=\left(\frac{2}{1+\eta}+\frac{4}{3}\beta\ln\frac{1-a_0^3/a^3}{3m-3m^2}\right)K. \quad (22)$$

If β=1, $E^+=E^-$, $\nu^+=\nu^-$ and neglecting the higher powers of m^2 and a_0^2, Eqs.(20) and (21) respectively transform into:

$$\left(\frac{r_{max}}{a}\right)^3=\frac{E}{2(1+\nu)K}, \quad (23)$$

$$p_u=\frac{4}{3}\left(1+\ln\frac{E}{2(1+\nu)K}\right)K. \quad (24)$$

Eq.(24) is identical with the solution of ideal elastic-plastic material (Vesic, 1972; Gong, 1999).

In the elastic zone ($r\geq r_1$) by inserting Eq.(15) into Eq.(13), the following expression can be obtained:

$$\begin{cases}\sigma_r=\dfrac{2K}{1+\eta}\left(\dfrac{r_1}{r}\right)^{\alpha+3/2}, \\ \sigma_\theta=-\dfrac{2\eta K}{1+\eta}\left(\dfrac{r_1}{r}\right)^{\alpha+3/2}, \end{cases} \quad (25)$$

$$u_r=-\frac{[(1-\nu^-)(\alpha-1/2)+2\nu^-]K}{E^-(1+\eta)}(r_1/r)^{\alpha+3/2}r. \quad (26)$$

If it is further assumed that prior to the application of load, the following expressions could be derived as above.

For the elastic zone ($r\geq a$, $p\leq p_c$), the following expressions can be obtained:

$$\begin{cases}\sigma_r=(p-p_0)(a/r)^{\alpha+3/2}+p_0, \\ \sigma_\theta=-\eta(p-p_0)(a/r)^{\alpha+3/2}+p_0, \\ u_r=-\dfrac{(1-\nu^-)(\alpha-1/2)+2\nu^-}{2E^-}(p-p_0)(a/r)^{\alpha+3/2}r. \end{cases} \quad (27)$$

The critical pressure between the elastic zone and the plastic zone is:

$$p_c=2K/(1+\eta)+p_0. \quad (28)$$

Stresses in the plastic zone are shown as follows

$$\sigma_r=p-4\beta K\ln(r/a), \quad (29a)$$

$$\sigma_\theta=p-2\beta K[2\ln(r/a)+1]. \quad (29b)$$

The ultimate pressure is:

$$p_u=\left(\frac{2}{1+\eta}+\frac{4}{3}\beta\ln\frac{1-a_0^3/a^3}{3m-3m^2}\right)K+p_0. \quad (30)$$

(2) Mohr-Coulomb material

Being completely analogous to previous section, the critical yield stress by taking the yield function into account can be written as follows:

$$p_c=\frac{2c\cos\varphi}{1+\eta-(1-\eta)\sin\varphi}. \quad (31)$$

In the plastic zone ($a\leq r<r_1$), introducing Eq.(8b) into Eq.(1), the following equation can be obtained:

$$\frac{d\sigma_r}{dr}+\frac{4\sin\varphi_r}{1+\sin\varphi_r}\frac{\sigma_r}{r}+\frac{c_r\cos\varphi_r}{1+\sin\varphi_r}\frac{4}{r}=0. \quad (32)$$

The solution of the differential equation is:

$$\sigma_r=(p+c_r\cot\varphi_r)(a/r)^{\frac{4\sin\varphi_r}{1+\sin\varphi_r}}-c_r\cot\varphi_r, \quad (33a)$$

$$\sigma_\theta=\frac{1-\sin\varphi_r}{1+\sin\varphi_r}(p+c_r\cot\varphi_r)(a/r)^{\frac{4\sin\varphi_r}{1+\sin\varphi_r}}-c_r\cot\varphi_r-\frac{2c_r\cos\varphi_r}{1+\sin\varphi_r}. \quad (33b)$$

When $r=r_1$, the critical yield pressure can be shown as follows:

$$\sigma_{r_1}=p_c=\frac{2c\cos\varphi}{1+\eta-(1-\eta)\sin\varphi}.$$

For $r_1=r_{max}$, the ultimate pressure is written to be as follows:

$$p_{\mathrm{u}}=\left(\frac{2c\cos\varphi}{1+\eta-(1-\eta)\sin\varphi}+c_{\mathrm{r}}\cot\varphi_{\mathrm{r}}\right)(r_{\max}/a)^{\frac{4\sin\varphi_{\mathrm{r}}}{1+\sin\varphi_{\mathrm{r}}}}-c_{\mathrm{r}}\cot\varphi_{\mathrm{r}}. \quad (34)$$

Because plastic volumetric strain of Mohr-Coulomb material is not equal to zero, the average plastic volumetric strain Δ is used. A relationship stating that the change of volume of the cavity is equal to the change of volume of elastic zone plus the change of volume of the plastic zone is also used. In this way, the following expression can be obtained

$$a^3-a_0^3=r_{\max}^3-(r_{\max}-u_r)^3+(r_{\max}^3-a^3)\Delta. \quad (35)$$

Substituting Eq.(26) into Eq.(34), and ignoring higher order terms of u_r^3, Eq.(35) becomes:

$$\left(\frac{r_{\max}}{a}\right)^3=\frac{1}{\Delta+3n-3n^2}\left(1+\Delta-\frac{a_0^3}{a^3}\right), \quad (36)$$

where

$$n=\frac{[(1-\nu^-)(\alpha-1/2)+2\nu^-]c\cos\varphi}{E^-[1+\eta-(1-\eta)\sin\varphi]}. \quad (37)$$

Combining Eq.(33) with Eq.(34), the following expression can be obtained:

$$p_{\mathrm{u}}=\left[\frac{1}{\Delta+3n-3n^2}\left(1+\Delta-\frac{a_0^3}{a^3}\right)\right]^{\frac{4\sin\varphi_{\mathrm{r}}}{3(1+\sin\varphi_{\mathrm{r}})}}\cdot\left(\frac{2c\cos\varphi}{1+\eta-(1-\eta)\sin\varphi}+c_{\mathrm{r}}\cot\varphi_{\mathrm{r}}\right)-c_{\mathrm{r}}\cot\varphi_{\mathrm{r}}. \quad (38)$$

If $\varphi_{\mathrm{r}}=\varphi$, $c_{\mathrm{r}}=c$ and neglecting the small quantity of n^2 and $(a_0/a)^2$, Eq.(38) transforms into:

$$p_{\mathrm{u}}=\left[\frac{(1+\Delta)(3-\sin\varphi)E}{\Delta E(3-\sin\varphi)+2(1+\nu)c\cos\varphi}\right]^{\frac{4\sin\varphi}{3(1+\sin\varphi)}}\cdot\left(\frac{4c\cos\varphi}{3-\sin\varphi}+c\cot\varphi\right)-c\cot\varphi. \quad (39)$$

Δ mentioned above is the function of stress conditions in the plastic zone and cannot be determined unless these conditions are known and introduced into an equation such as Eq.(36). To bypass this difficulty, it is possible to use iteration as follows:

(i) Determine the stresses condition in the plastic zone with an assumed average volumetric strain Δ_1;

(ii) From stress conditions as obtained under (i) and from a volume change-stress relationship determine a revised volumetric strain Δ_2;

(iii) With the revised value of Δ_2 repeat the procedure under (i) and (ii). if the newly determined Δ_3 is not much different from the revised value Δ_2, a satisfactory solution can be obtained, and the ultimate cavity pressure can be determined from Δ_3 and other data of the problem. If Δ_3 and Δ_2 are still significantly different, the process should be repeated.

In the elastic zone ($r\geq r_1$), combining Eq.(13) with Eq.(31), the following equations can be obtained:

$$\sigma_r=\frac{2c\cos\varphi}{1+\eta-(1-\eta)\sin\varphi}\left(\frac{r_1}{r}\right)^{\alpha+3/2}, \quad (40a)$$

$$\sigma_\theta=\frac{-2\eta c\cos\varphi}{1+\eta-(1-\eta)\sin\varphi}\left(\frac{r_1}{r}\right)^{\alpha+3/2}, \quad (40b)$$

$$u_r=-\frac{[(1-\nu^-)(\alpha-1/2)+2\nu^-]c\cos\varphi}{E^-[1+\eta-(1-\eta)\sin\varphi]}\left(\frac{r_1}{r}\right)^{\alpha+3/2}r. \quad (41)$$

(3) Application limit of the closed solution

The solution for the pressure expansion curve is obtainable whenever the restriction of small strains is imposed. When this restriction is relaxed, only the solution for the limit condition at infinitely large deformation is obtainable in closed form. Numerical techniques must be used to obtain the entire pressure-expansion curve including the large portion. The use of the numerical procedure has not been pursued here.

The closed form solutions will have application in the following practical problems. The small strain solutions for the spherical cavity are applicable to the interpretation of pressuremeter tests. Limit solutions for the spherical cavity may have application in the determination of the end bearing capacity of deep foundations in cohesive frictional soil.

COMPARISON AND ANALYSES

In order to compare with the ideal elastic-plastic material and reflect the effects caused by different elastic moduli in tension and compression and strain softening rates on stress and displacement fields and development of the plastic zone, Tresca material is used as an example. It is assumed that $E^{+}=1.0\times10^{5}$ kPa, $K=10^{3}$ kPa, $\nu^{+}=0.3$, $a_0/a=0.5$. Parameters η and β are respectively regarded as controlling parameters for different moduli and strain-softening properties.

Ultimate cavity pressure p_u and the critical yield pressure p_c vary with parameters η and β

Fig.5 clearly demonstrates the normalized ultimate pressure p_u/K reduces with the decreases of softening parameter β and increases with the decrease of modulus parameter η. In Fig.6 the normalized critical yield pressure p_c/K reduces with the increase of modulus parameter η.

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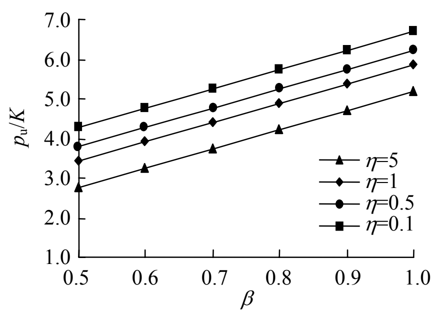

Fig.5 p_u/K vs β curves with different η

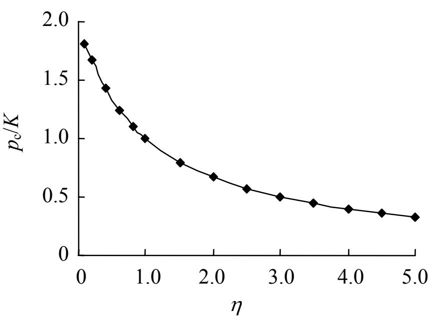

Fig.6 p_c/K vs η curve (β=1)

1. Development of the plastic zone

In Figs.7 and 8 the normalized cavity expansion pressure p/K is plotted against normalized cavity radius r_1/a. Fig.7 shows the normalized cavity expansion pressure p/K increases very rapidly with the increase of the softening parameter β. However in Fig.8 the normalized cavity expansion pressure p/K reduces with the modulus parameter η.

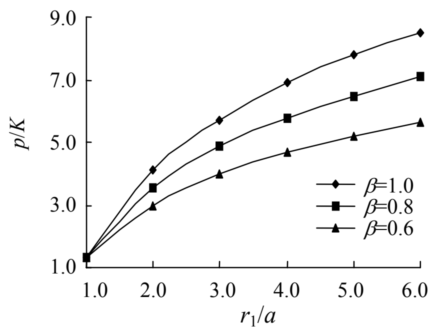

Fig.7 p/K vs r_1/a curves with different β (η=0.5)

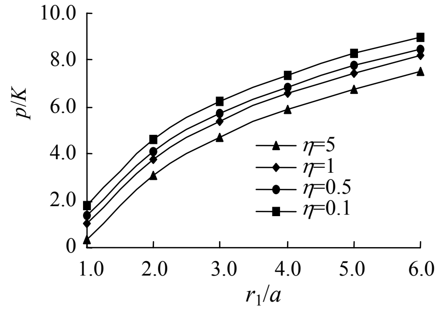

Fig.8 p/K vs r_1/a curves with different η (β=1)

2. Stress field

Fig.9 shows the relationship of the normalized internal pressure p/K and the normalized radial stress σ_r/K varying with softening parameter β. When the internal pressure is below the normalized critical pressure p_c/K, the relationship of p/K~σ_r/K is the same. But over the normalized critical pressure p_c/K, the curves of p/K~σ_r/K appear separated. From Fig.10 it is apparent that before the normalized critical pressure p_c/K, the curves will appear separated, however all the curves will trend towards one curve with the increase of the normalized normal radial stress σ_r/K.

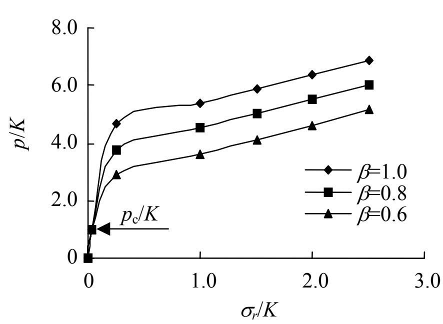

Fig.9 p/K~σ_r/K curves with different β (η=1, r_1/a=3.0)

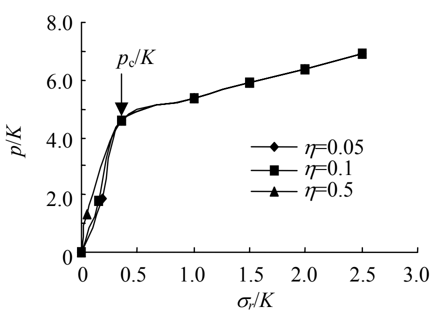

Fig.10 p/K~σ_r/K curves with different η (β=1, r_1/a=3.0)

CONCLUSION

From the analyses above, the ultimate pressure, stress field, strain field and development of the plastic zone of spherical cavity change with modulus parameter η and softening parameter β (or c_r, φ_r) for material with different elastic moduli of tensile compression and strain-softening behavior. Thus if the classical elastic theory (η=0.5) and traditional relationship of stress and strain (β=1.0 or $c_r=c$, $\varphi_r=\varphi$) are applied to engineering design and computation, significant errors for geomaterials such as rock and construction clay may occur.

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