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Modeling of D-STATCOM in distribution systems load flow

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Abstract: This paper presents modeling of Distribution STATCOM (D-STATCOM) in load flow calculations for the steady-state voltage compensation. An accurate model for D-STATCOM is derived to use in load flow calculations. The rating of this device as well as the direction of required reactive power injection for voltage compensation in the desired value (1 p.u.) is derived and discussed analytically and mathematically by the phasor diagram method. Furthermore, an efficient method for node and line identification used in load flow calculations is presented. The validity of the proposed model is examined by using two standard distribution systems consisting of 33 and 69 nodes, respectively. The best location of D-STATCOM for under voltage problem mitigation approach in the distribution networks is determined. The results validate the proposed model for D-STATCOM in large distribution systems.

Key words: Distribution system, D-STATCOM, Voltage compensation, Load flow
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INTRODUCTION

Providing demanding power to the entire load while maintaining voltage magnitude at an acceptable range is one of the major system constraints. There are two principal conventional means of controlling voltage in distribution systems: series voltage regulators and shunt capacitors. Conventional series voltage regulators are commonly used for voltage regulation in distribution system (Bishop *et al*., 1994; Gu and Rizy, 1996; Kojovic, 2006). These devices cannot generate reactive power and by its operation only force the source to generate requested reactive power. Furthermore, they have quite slow response and these operations are step by step (Ramsay *et al*., 1996). Shunt capacitors can supply reactive power to the system. Reactive power output of a capacitor is proportional to the square of the system voltage which may reduce its effectiveness in high and low voltages. Hence, for improvement of capacitors in different loading conditions, their constructions are generally the combination of fixed and switched capacitors. Therefore, they are not capable to generate continuously variable reactive power. Another difficulty associated with the application of distribution capacitors is the natural oscillatory behavior of capacitors when they are used in the same circuit with inductive components. This, sometimes results in the well-known phenomena of ferroresonance and/or self-excitation of induction machinery (Ramsay *et al*., 1996). Hence, when regulators that operate by adjusting their taps to maintain predetermined set point voltage levels are coupled with capacitors that are switched on and off to regulate voltage, the voltage swings can cause power quality problems for customers.

With the improvements in current and voltage handling capabilities of the power electronics devices that have allowed for the development of Flexible AC Transmission System (FACTS), the possibility has arisen in using different types of controllers for efficient shunt and series compensation. It should be mentioned that FACTS devices respond quickly to the changes in network condition. The concept of FACTS devices was originally developed for transmission systems, but similar idea has been started to be applied to distribution systems. Distribution STATCOM (D-STATCOM) is a shunt connected voltage source converter which has been utilized to compensate power quality problems such as unbalanced load, voltage sag, voltage fluctuation and voltage unbalance (Haque, 2001; Sensarma *et al*., 2001; Masdi *et al*., 2004; Blazic and Papic, 2004; Xu *et al*., 2005; Mariun *et al*., 2006) which occur in short duration in millisecond range. In this duration, D-STATCOM can inject both active and reactive power to the system for compensation of sensitive loads, and active power injection into the system must be provided by energy storage system (Haque, 2001). Almost all of the models reported for D-STATCOM utilized in a two-bus distribution system consist of a sensitive load and the source. Then, the effects of D-STATCOM modeling on compensation of power quality problems of sensitive loads are considered. However, the effects of D-STATCOM on large distribution system and other loads in distribution have not been considered. Also, the impacts of D-STATCOM are dynamically considered in a short duration but not for a long term. In (Ramsay *et al*., 1996), the application of D-STATCOM for distribution voltage regulation on long, voltage-limited feeders is considered. But the analysis is limited to a 1-lateral distribution system in two specific cases including distributed load and lumped load. Furthermore, they have used a simple model of STATCOM in the transmission systems in order to model the D-STATCOM in the distribution systems.

In this paper, D-STATCOM is utilized for the improvement of another aspect of power quality, i.e. voltage compensation in long term. Since this device is utilized in steady-state condition for long term, because of limited capacity of energy storage system, it cannot inject active power to the system for long term. Therefore, a suitable model for D-STATCOM has been proposed in load flow program, which is applicable in large distribution systems. Also, the rating and direction of reactive power which must be exchanged by D-STATCOM for voltage compensation in the desired value (1 p.u.) are derived and discussed analytically and mathematically by using phasor diagram method. Then, the effects of D-STATCOM on voltage improvement at other nodes are considered and the best location of D-STATCOM for under voltage problem mitigation in the distribution network is determined. Load flow is an important method for analysis, operation and planning studies of any power system in a steady-state condition (Haque, 1996; Ghosh and Das, 1999; Ma *et al*., 2002). In this paper an efficient method for node and line identification utilized in load flow has been proposed. Two standard distribution systems consisting of 33 and 69 nodes are considered and the D-STATCOM model is applied to load flow. The results reveal the effectiveness of the proposed model for the D-STATCOM in large distribution systems.

Section 2 presents the steady-state modeling of D-STATCOM. In Section 3, the radial distribution system load flow method has been briefly discussed and an efficient method for node and line identification used in load flow calculations is presented. Model of D-STATCOM in load flow is presented in Section 4. In Section 5, the results associated with application of D-STATCOM model in 33-bus and 69-bus standard distribution systems are presented and discussed. Finally, Section 6 summarizes the main points and results of this paper.

STEADY-STATE MODELING OF D-STATCOM

Distribution STATic COMpensator

D-STATCOM is a shunt device that injects or absorbs both active and reactive current. Its diagrams are shown in Fig.1. In Fig.1a, it can be seen that D-STATCOM consists of energy storage and voltage source converter. In this model, D-STATCOM is capable of injecting active power in addition to reactive power. Since energy storage has a capacity limit, it cannot inject active power for a long term for voltage regulation purpose. Therefore, for the steady-state application, D-STATCOM consists of a small DC capacitor and a voltage source converter, and the steady-state power exchange between D-STATCOM and the AC system is reactive power (Fig.1b).

Steady-state modeling of D-STATCOM

The single line diagram of two buses of a distribution system and its phasor diagram are shown in

Fig.2 and Fig.3, respectively. Generally, voltage of buses in the system is less than 1 p.u. and it is desired to compensate voltage of interested bus (V_j) to 1 p.u. by using D-STATCOM.

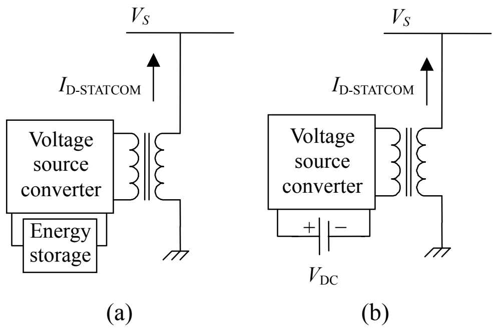

Fig.1 A typical model of D-STATCOM. (a) Active and reactive power exchange; (b) Only reactive power exchange

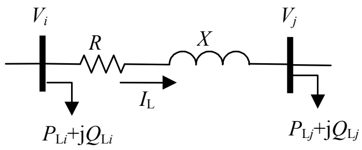

Fig.2 Single line diagram of two buses of a distribution system. Subscript 'L' in P_L and Q_L refers to the load connected to each bus

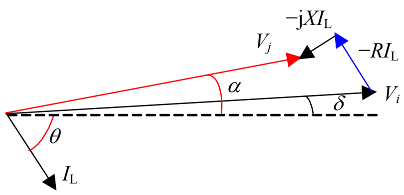

Fig.3 Phasor diagram of voltages and current of the system shown in Fig.2

In Fig.2, the relationship between voltage and current can be written as:

$$V_j\angle\alpha = V_i\angle\delta - ZI_L\angle\theta, \tag{1}$$

where $V_j\angle\alpha$ and $V_i\angle\delta$ are the voltage of buses j and i before compensation respectively, $Z=R+jX$ is the impedance between buses i and j, $I_L\angle\theta$ is the current flow in line. Voltage $V_i\angle\delta$ and current $I_L\angle\theta$ are derived from the load flow calculations.

As noted earlier, in this paper, D-STATCOM is used for voltage regulation in the steady-state condition and can inject only reactive power to the system. Consequently, $I_{D-STATCOM}$ must be kept in quadrature with voltage of the system. By installing D-STATCOM in distribution system, all nodes voltage, especially the neighboring nodes of D-STATCOM location, and branches current of the network, change in the steady-state condition. The schematic diagram of buses i and j of the distribution systems, when D-STATCOM is installed for voltage regulation in bus j, is shown in Fig.4. The phasor diagram of these buses with D-STATCOM effects is shown in Fig.5. Voltage of bus j changes from V_j to $V_{j\text{new}}$ when D-STATCOM is used. For simplicity, the angle of voltage V_i', i.e. δ', is assumed to be zero.

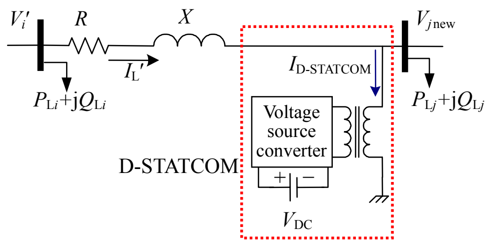

Fig.4 Single line diagram of two buses of a distribution system with D-STATCOM consideration

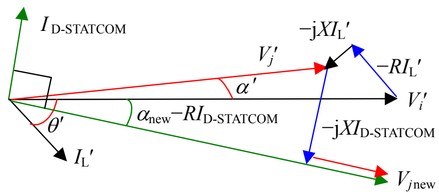

Fig.5 Phasor diagram of voltages and currents of the system shown in Fig.4

It can be seen from Figs.4 and 5 that:

$$\angle I_{D-STATCOM} = \pi/2 + \alpha_{new}, \quad \alpha_{new} < 0, \tag{2}$$

$$V_{j\text{new}}\angle\alpha_{new} = V_i'\angle\delta' - (R + jX)I_L'\angle\theta' - (R + jX)I_{D-STATCOM}\angle(\alpha_{new} + \pi/2), \tag{3}$$

where $I_{D-STATCOM}\angle(\pi/2 + \alpha_{new})$ is the injected current by D-STATCOM, $V_{j\text{new}}\angle\alpha_{new}$ is the voltage of bus j after compensation by D-STATCOM, $V_i'\angle\delta'$ is the voltage of bus i after D-STATCOM installation, $I_L'\angle\theta'$ is the current flow in line after D-STATCOM installation. Voltage $V_i'\angle\delta'$ and current $I_L'\angle\theta'$ are derived from the load flow calculations.

Separating the real and imaginary parts of Eq.(3) yields:

$$V_{j\text{new}}\cos\alpha_{\text{new}}=\text{Re}(V_i'\angle\delta')+XI_{\text{D-STATCOM}}\sin(\pi/2+\alpha_{\text{new}})$$
$$-\text{Re}(ZI_{\text{L}}'\angle\theta')-RI_{\text{D-STATCOM}}\cos(\pi/2+\alpha_{\text{new}}),\quad(4)$$
$$V_{j\text{new}}\sin\alpha_{\text{new}}=\text{Im}(V_i'\angle\delta')-XI_{\text{D-STATCOM}}\cos(\pi/2+\alpha_{\text{new}})$$
$$-\text{Im}(ZI_{\text{L}}'\angle\theta')-RI_{\text{D-STATCOM}}\sin(\pi/2+\alpha_{\text{new}}).\quad(5)$$

Using the notations below:

$$a_1=\text{Re}(V_i'\angle\delta')-\text{Re}(ZI_{\text{L}}'\angle\theta'),$$
$$a_2=\text{Im}(V_i'\angle\delta')-\text{Im}(ZI_{\text{L}}'\angle\theta'),$$
$$b=V_{j\text{new}},\ c_1=-R,\ c_2=-X,$$
$$x_1=I_{\text{D-STATCOM}},\ x_2=\alpha_{\text{new}},$$

Eqs.(6) and (7) are obtained from Eqs.(4) and (5), respectively:

$$b\cos x_2=a_1-c_1x_1\sin x_2-c_2x_1\cos x_2,\quad(6)$$
$$b\sin x_2=a_2-c_2x_1\sin x_2+c_1x_1\cos x_2,\quad(7)$$

where a_1, a_2, c_1 and c_2 are constants, b is the magnitude of compensated voltage (e.g. 1 p.u.), x_1, x_2 are variables to be determined. Rearranging Eqs.(6) and (7) for x_1 yields:

$$x_1=\frac{b\cos x_2-a_1}{-c_1\sin x_2-c_2\cos x_2},\quad(8)$$

and

$$x_1=\frac{b\sin x_2-a_2}{-c_2\sin x_2+c_1\cos x_2}.\quad(9)$$

By equating Eqs.(8) and (9), it can be shown that

$$(a_1c_2-a_2c_1)\sin x_2+(-a_1c_1-a_2c_2)\cos x_2+bc_1=0,\quad(10)$$

Considering x=sinx_2, the following equation will be derived from Eq.(10):

$$(k_1^2+k_2^2)x^2+(2k_1bc_1)x+(b^2c_1^2-k_2^2)=0,\quad(11)$$

where

$$k_1=a_1c_2-a_2c_1,\ k_2=a_1c_1+a_2c_2.$$

Therefore

$$x=(-B\pm\sqrt{\Delta})/(2A),\quad(12)$$

where

$$\Delta=B^2-4AC,$$
$$A=k_1^2+k_2^2,\ B=2k_1bc_1,\ C=b^2c_1^2-k_2^2.$$

After identifying x, x_2=α_{new} (angle of corrected voltage) is defined as:

$$x_2=\arcsin x,\quad(13)$$

thus, x_1=$I_{\text{D-STATCOM}}$ is defined by Eq.(8) or Eq.(9).

It can be seen from Eq.(12) that there are two roots for x and therefore, two values are calculated for x_2 and x_1, but only one is acceptable. To determine the correct answer, these roots are examined under boundary conditions in the load flow results:

If b=$V_{j\text{new}}$=V_j, then x_1=$I_{\text{D-STATCOM}}$=0 and x_1=α_{new}=α.

After testing these conditions on load flow results, $x=(-B+\sqrt{\Delta})/(2A)$ is selected as the correct answer for Eq.(11) and then x_2 and x_1 are calculated from Eqs.(13) and (8), respectively.

Finally, injected reactive power by D-STATCOM can be written as:

$$\text{j}Q_{\text{D-STATCOM}}=\boldsymbol{V}_{j\text{new}}\boldsymbol{I}^*_{\text{D-STATCOM}},\quad(14)$$

where

$$\boldsymbol{V}_{j\text{new}}=V_{j\text{new}}\angle\alpha_{\text{new}},$$
$$\boldsymbol{I}_{\text{D-STATCOM}}=I_{\text{D-STATCOM}}\angle(\alpha_{\text{new}}+\pi/2),$$

and * denotes conjugate of complex variable.

RADIAL DISTRIBUTION SYSTEM LOAD FLOW

Load flow is an important and basic method for analysis, operation and planning studies of any power system in a steady-state condition. By using load flow, it can be determined which variables exceed their limits, and thus efficient corrective solutions such as shunt, series and other compensation techniques must be taken to stir the state variables within an acceptable and secured operating zone. Most distribution systems are fed at one point and have a radial structure. Several methods have been developed based on the concept of doing backward/forward sweeps of radial network (Haque, 1996; Ghosh and Das, 1999; Ma *et al.*, 2002). An efficient and simple load flow method based on backward/forward sweeps is used in this paper (Ghosh and Das, 1999). However, the equation presented for calculating current flow in (Ghosh and Das, 1999) has additional calculation operation. In this paper two equations are presented for current flow without additional calculation operation. Also, an efficient method for node and line identification utilized in the load flow is presented and described below.

Node and branch numbering

For a radial distribution system, the number of branches n_b and the number of buses n are related through

$$n=n_b+1. \tag{15}$$

The node numbering process is started at 0 for the source node and is increased for other nodes. The advantage of this numbering process is that the number of any node and that of its upstream branches, are always the same. This reduces the amount of calculation. Fig.6 shows the single-line diagram of a balanced radial distribution feeder with 15 buses and thus 14 lines concluding node and branch numbering. It is observed from the figure that the assigned number of each node and that of its upstream branches are the same.

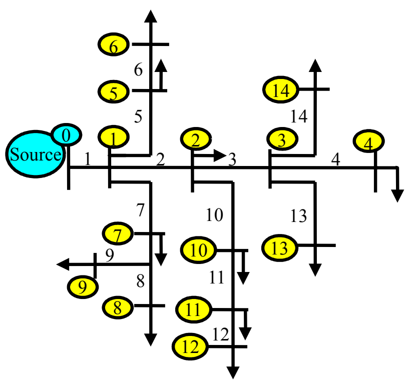

Fig.6 Single-line diagram of 15-bus distribution system

Load flow equations

It is assumed that the three-phase radial distribution system is balanced. Based on node and branch numbering procedure, voltage of node i can be expressed as:

$$V(i)=V(i-1)-I(i)Z(i), \tag{16}$$

where $V(i)$ and $V(i-1)$ are the voltage of nodes i and $i-1$ respectively, $Z(i)$ is the impedance of line i, $I(i)$ is the current flow in line i.

Since the voltage of source node is known, Eq.(16) can be used in the forward sweeps to determine the voltage of other nodes in the distribution systems.

The load current of node i, $I_L(i)$, can be written as:

$$I_L(i)=\frac{P_L(i)-Q_L(i)}{V^*(i)}, \tag{17}$$

where $P_L(i)$ and $Q_L(i)$ are active and reactive power of load connected to node i, respectively.

The current through a branch i, i.e. $I(i)$, equals $I_L(i)$ plus the sum of the branch currents connected to this line:

$$I(i)=I_L(i)+\sum_{j\in\beta_i}I(j), \tag{18}$$

where β_i is the set consisting of all branches connected to node i. Thus, β_i is empty for each end node. As a result, $I(i)$ connected to the end nod i can be expressed as:

$$I(i)=I_L(i). \tag{19}$$

It can be seen for calculating current of each branch, all branches connected to it must be determined. Also, Eqs.(18) and (19) must be utilized in backward sweeps from all end nodes toward the source node. Therefore, end nodes and branches of the path connected to the source node and adjacent downstream nodes of an interested node must be determined. The algorithm of determination of these parameters is described in the next subsection.

Determination of related parameters

It is assumed that the distribution system has n nodes. Parameters to be determined are the end nodes and branches of the paths connected to the source node, and the adjacent downstream nodes of an interested node. The stages of algorithm are as below:

(1) At the first step, a node and branch matrix $\boldsymbol{M}$ is produced, which has n columns (equals the num-

ber of nodes) and $n-1$ rows (equals the number of branches). It is obvious that all lines have a sending-end node and a receiving-end node. In each row of $\boldsymbol{M}$, a column corresponding to a sending-end node is equal to 1 and a column corresponding to a receiving-end node is equal to −1 while the other elements of this row are 0. Thus, each row contains only one 1, one −1 and the rest elements are 0.

(2) In this step, the branches of the path connecting node i to the source node are determined. During the algorithm description, in order to make it illustrative, an example is followed, in which the branches of the path connecting node 4 to the source node are determined while the algorithm is presented step by step. For this approach, at first, in the ith column of $\boldsymbol{M}$ (4th column), an element equal to 1 is determined. Then, the row which the determined element (1) belongs to is determined. This row is the first branch of interested path. According to the node and branch numbering described above, this row is always equal to i (4). Then, in the determined row i, we find an element equal to −1 and the column which −1 belongs to is found (Column 3). After that, in this column, an element equal to 1 is determined and the number of the row of its location in $\boldsymbol{M}$ is the second element of the interested path (3). This search continues until the algorithm reaches a column which has no element equal to 1 (source node). Table 1 shows the node and branch matrix $\boldsymbol{M}$ of the system shown in Fig.6. The process of branches determination located between node 4 and the source node is shown in Table 1. Following the mentioned process, branches 4, 3, 2 and 1 are respectively determined as the branches connecting node 4 to the source node.

(3) In this step, all end nodes in distribution system are determined. For this purpose, we must find columns which have only one element equal to 1 but no element equal to −1. For example, it can be seen from Table 1 that the nodes 4, 6, 8, 9, 12, 13 and 14 are the end nodes which satisfy the above-mentioned condition.

(4) The last step is to determine the adjacent downstream nodes of an interested node. After identification of the end nodes (Step 3), for each end node, the path connecting it to the source node must be determined and ordered increasingly. Then, for all the paths including the interested node, the first element after the interested node is an adjacent node of it. Eliminating the nodes which are repeated more than one time among the adjacent nodes, we reach the set β used in Eq.(18). This is done for all nodes in the distribution system. Table 2 shows all the paths of the end nodes and the method for achieving the nodes connected to node 2. In all rows of the table, node 2 is found and the first element after node 2 is selected. It can be seen that node 3 is repeated 3 times. Thus, set β for node 2 includes nodes 3 and 10.

Table 1 Matrix node and branch for the 15-bus distribution system

Line No.	Node No. 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	−1	1	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	−1	1	0	0	0	0	0	0	0	0	0	0
5	0	−1	0	0	0	1	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	−1	1	0	0	0	0	0	0	0	0
7	0	−1	1	0	0	0	0	1	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	−1	1	0	0	0	0	0	0
9	0	0	0	0	0	0	0	−1	0	1	0	0	0	0	0
10	0	0	−1	0	0	0	0	0	0	0	1	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	−1	1	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	−1	1	0	0
13	0	0	0	−1	0	0	0	0	0	0	0	0	0	1	0
14	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	1

The colored lines indicate the path connecting node 4 to the source node

Table 2 Path of the end nodes to the source and the method for determination of downstream nodes connected to node 2

End node	Path of the end node to the source node ordered increasingly				
4	1	2*	3**	4	–
6	1	5	6	–	–
8	1	7	8	–	–
9	1	7	9	–	–
12	1	2*	10**	11	12
13	1	2*	3**	13	–
14	1	2*	3**	14	–

* Interested node; ** The node connected to node 2

Backward/forward sweeps in load flow and criterion of convergence

Initially, a constant voltage of all nodes is assumed to be (1 p.u.∠0). Then all load currents are computed by using Eq.(17). After that, branch currents are computed by using Eq.(18) or Eq.(19) in backward sweeps. Thereafter, voltage of each node is calculated by Eq.(16) in forward sweeps. Once the new values of voltages of all nodes are computed, the convergence criterion of the solution is checked. If it does not converge, then load currents are computed using the most recent values of voltages and the whole process is repeated. The convergence criterion is that, in successive iterations the maximum difference in voltage magnitudes must be less than 1×10^{-5} p.u.

MODELING OF D-STATCOM IN LOAD FLOW

For modeling D-STATACOM in load flow calculations, in any iteration in forward sweep, it is first assumed that the voltage magnitude in the node where D-STATACOM is located be 1 p.u. Then, the phase angle of voltage in the compensated node and the reactive power injection of D-STATCOM are calculated by Eqs.(13) and (14), respectively. Then, the new magnitude and phase angle of the compensated node are utilized to determine the voltage of D-STATCOM located downstream nodes in the forward sweep of load flow. If the reactive power calculated from Eq.(14) is greater than the maximum reactive power rating of D-STATACOM, the injected reactive power of D-STATCOM is set to its maximum rating and is considered as a negative constant value in load model in node j, and the load flow program is solved in a normal way as if there is no D-STATCOM. The updated voltage of nodes and injected reactive power by D-STATACOM are used to determine the load currents by Eq.(17) in the next backward sweep of load flow. These procedures are continued until the load flow is converged.

SIMULATION RESULTS

Two distribution systems consisting of 33 and 69 buses respectively are selected and the proposed models associated with distribution D-FACTS devices are applied to the load flow program for simulation. The results obtained in these systems are briefly summarized in this section.

Thirty-three-bus test system

The single line diagram of the 12.66 kV, 33-bus, 4-lateral radial distribution system is shown in Fig.7. The data of the system are obtained from (Baran and Wu, 1989b). The load of the system is considered as (3715+2300j) kVA. A summary of load flow solution before D-STATCOM installation is presented in Table 3. It is assumed that the upper and lower limits of the voltage magnitude are 1.05 p.u. and 0.95 p.u., respectively. It can be seen that 18 out of 33 nodes of the distribution system (54.54%) have under voltage problem.

In order to indicate and compare the effects of D-STATCOM implementing in the distribution system, different locations are selected for the installation of D-STATCOM. Also, the rating of D-STATCOM is obtained from the load flow calculations. For this purpose, nodes 17 and 32 as the end nodes,

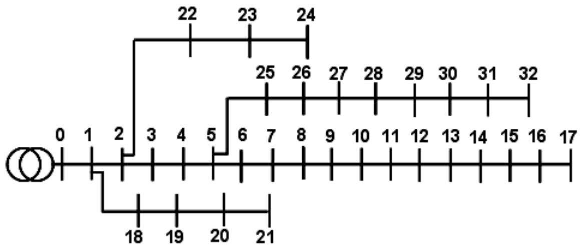

Fig.7 Single line diagram of 33-bus distribution system

Table 3 Voltage magnitude and phase angle in 33-bus distribution system without implementing D-STATCOM

Node No.	Voltage magnitude (p.u.)	Phase angle (°)	Node No.	Voltage magnitude (p.u.)	Phase angle (°)
0	1.0000	0	17	0.9171	−0.4865
1	0.9971	0.0145	18	0.9965	0.0036
2	0.9867	0.0995	19	0.9929	−0.0633
3	0.9792	0.1646	20	0.9922	−0.0827
4	0.9719	0.2307	21	0.9916	−0.1030
5	0.9535	0.1371	22	0.9935	−0.0156
6	0.9501	−0.0913	23	0.9869	−0.1018
7	0.9452	−0.0556	24	0.9837	−0.1443
8	0.9390	−0.1281	25	0.9516	0.1762
9	0.9332	−0.1901	26	0.9491	0.2319
10	0.9323	−0.1829	27	0.9376	0.3129
11	0.9309	−0.1715	28	0.9295	0.3901
12	0.9248	−0.2620	28	0.9259	0.4945
13	0.9225	−0.3400	30	0.9218	0.4108
14	0.9211	−0.3774	31	0.9209	0.3880
15	0.9197	−0.4004	32	0.9206	0.3803
16	0.9177	−0.4770			

nodes 12 and 28 in the laterals, and nodes 3 and 5 in the main feeder of the distribution system are selected. In fact, D-STATCOM is utilized to compensate voltage at selected nodes to 1 p.u. and to improve the voltage of other nodes in the system. Also, the effect of capacity constraint in D-STATCOM for voltage compensation is studied. It should be noted that only one D-STATCOM is used at the time when performing load flow calculations.

Table 4 shows the results of load flow calculations with D-STATCOM consideration for compensation of voltage in the selected nodes. This table shows that D-STATCOM improves the voltage of both nearby downstream nodes and nearby upstream nodes, especially the nodes located between D-STATCOM and the source. D-STATCOM installation in node 32 (the end node), for example, causes the voltage of node 32 to regulate to 1 p.u., and additionally it can improve the voltage of nodes 26~31 which are located between D-STATCOM and the source, and also the nodes 7~11 which are located in nearby feeder. In other words, D-STATCOM installation in line 32 mitigates under voltage problem of 12 out of 33 nodes (36.36%). Similarly, the same results are achieved by D-STATCOM installation in line 17. In the next stage of simulation, the effect of D-STATCOM installation in the laterals of distribution system is considered. For this purpose, D-STATCOM is installed in node 12. The result shows that D-STATCOM installation in this node strongly improves the voltage of neighboring downstream nodes (13~17), and the voltage of neighboring upstream nodes (7~11, 26 and 27) and therefore mitigates under voltage problem of these nodes. Similar result is obtained by D-STATCOM installation in line 28. Afterwards, the effect of D-STATCOM installation in the main feeder of the distribution system is investigated. It is observed from Table 4 that when D-STATCOM is installed in node 5, under voltage problem in all nodes are mitigated. Also, the effect of D-STATCOM installation in each node of the distribution system is studied and the results are shown in Table 5. This table includes Rate of Under Voltage Mitigated Nodes (RUVMN) and the amount of injected reactive power by D-STATCOM. It can be seen from Table 5 that under voltage problem of the system is mitigated considerably by using D-STATCOM. Table 5 shows that the best locations for D-STATCOM installation to mitigate under voltage problem are nodes 27, 7, 6, 26, 25 and 5, respectively, which have *RUVMN* equal to 54.54%. The suggested locations for D-STATCOM are ordered in terms of *RUVMN* as well as the required reactive power for compensation.

Table 4 Voltage magnitude in 33-bus distribution system with D-STATCOM consideration

Node No.	The node where D-STATCOM is installed					
	3	5	12	28	17	32
1	0.9990	0.9985	0.9977	0.9980	0.9974	0.9977
2	0.9993	0.9962	0.9907	0.9927	0.9892	0.9909
3	1.0000	0.9948	0.9858	0.9891	0.9833	0.9861
4	0.9928	0.9940	0.9811	0.9858	0.9775	0.9815
5	0.9749	1.0000	0.9731	0.9827	0.9654	0.9735
6	0.9715	0.9967	0.9790	0.9793	0.9676	0.9701
7	0.9668	0.9921	0.9774	0.9747	0.9647	0.9654
8	0.9607	0.9862	0.9820	0.9686	0.9649	0.9593
9	0.9550	0.9807	0.9874	0.9630	0.9657	0.9537
10	0.9542	0.9799	0.9876	0.9622	0.9654	0.9528
11	0.9528	0.9784	0.9880	0.9607	0.9649	0.9514
12	0.9468	0.9727	1.0000	0.9548	0.9694	0.9454
13	0.9446	0.9705	0.9979	0.9526	0.9738	0.9432
14	0.9432	0.9692	0.9966	0.9513	0.9772	0.9418
15	0.9419	0.9679	0.9954	0.9500	0.9810	0.9405
16	0.9399	0.9660	0.9935	0.9480	0.9952	0.9385
17	0.9393	0.9654	0.9929	0.9474	1.0000	0.9379
18	0.9985	0.9980	0.9972	0.9975	0.9969	0.9972
19	0.9949	0.9944	0.9936	0.9939	0.9933	0.9936
20	0.9942	0.9937	0.9929	0.9932	0.9926	0.9929
21	0.9936	0.9931	0.9922	0.9926	0.9920	0.9923
22	0.9955	0.9950	0.9942	0.9945	0.9939	0.9942
23	0.9889	0.9884	0.9876	0.9879	0.9873	0.9876
24	0.9856	0.9852	0.9843	0.9846	0.9841	0.9843
25	0.9730	0.9982	0.9712	0.9830	0.9635	0.9731
26	0.9705	0.9957	0.9687	0.9835	0.9610	0.9726
27	0.9594	0.9849	0.9576	0.9926	0.9498	0.9751
28	0.9514	0.9771	0.9496	1.0000	0.9417	0.9773
29	0.9479	0.9737	0.9461	0.9967	0.9382	0.9776
30	0.9439	0.9698	0.9420	0.9929	0.9341	0.9879
31	0.9430	0.9689	0.9411	0.9920	0.9332	0.9924
32	0.9427	0.9687	0.9409	0.9918	0.9329	1.0000

The results of Tables 4 and 5 are achieved based on the assumption that D-STATCOM has no capacity limit for reactive power injection to voltage compensation. In order to study the effect of capacity constraint in D-STATCOM, it is assumed that the maximum injected reactive power by D-STATCOM is 2

Table 5 *RUVMN* and injected reactive power by D-STATCOM in 33-bus distribution system

Node No.	*RUVMN* (%)	*RPR* (MVA)	Node No.	*RUVMN* (%)	*RPR* (MVA)
1	3.03	10.0780	17	36.36	1.5127
2	12.12	7.2307	18	0	2.7431
3	24.24	6.9701	19	0	0.7266
4	45.45	6.7793	20	0	0.6113
5	54.54	5.4139	21	0	0.4530
6	54.54	3.9683	22	0	2.9199
7	54.54	3.9627	23	0	1.9744
8	48.48	3.3537	24	0	1.4917
9	45.45	2.9656	25	54.54	5.2641
10	45.45	2.9635	26	54.54	5.0729
11	45.45	2.9554	27	54.54	3.9150
12	39.39	2.4935	28	45.45	3.4664
13	39.39	2.2376	29	45.45	3.3865
14	39.39	2.0910	30	36.36	2.8227
15	39.39	1.9656	31	36.36	2.6433
16	39.39	1.5979	32	36.36	2.3879

RUVMN: rate of under voltage mitigated nodes; *RPR*: reactive power rating

Table 6 *RUVMN* and injected reactive power by 2-MVA D-STATCOM in 33-bus distribution system

Node No.	*RUVMN* (%)	*RPR* (MVA)	Node No.	*RUVMN* (%)	*RPR* (MVA)
1	0	2	17	36.36	1.5127
2	3.03	2	18	0	2
3	6.06	2	19	0	0.7266
4	6.06	2	20	0	0.6113
5	18.18	2	21	0	0.4530
6	24.24	2	22	0	2
7	30.30	2	23	0	1.9744
8	39.39	2	24	0	1.4917
9	39.39	2	25	18.18	2
10	39.39	2	26	21.21	2
11	39.39	2	27	33.33	2
12	39.39	2	28	33.33	2
13	39.39	2	29	33.33	2
14	39.39	2	30	33.33	2
15	39.39	1.9656	31	33.33	2
16	39.39	1.5980	32	33.33	2

RUVMN: rate of under voltage mitigated nodes; *RPR*: reactive power rating

MVA. Table 6 shows that the ability of D-STATCOM in voltage compensation is decreased when its reactive power rating is limited. This table shows that *RUVMN* is decreased in many places as compared with Table 5. Also the usefulness of D-STATCOM decreases much more when the difference between required reactive power and maximum rating of reactive power of D-STATCOM becomes greater. For example, 2-MVA D-STATCOM installation at nodes 5, 6, 25 and 26 causes *RUVMN* to decrease from 54.54% (Table 5) to 18.18%, 24.24%, 18.18% and 21.21% (Table 6), respectively. Table 5 shows that the best locations for 2-MVA D-STATCOM installation for voltage compensation are nodes 8~16 which have *RUVMN* equal to 39.39%.

Sixty-nine-bus test system

The 12.66 kV, 69-bus, 8-lateral radial distribution system based on the new node numbering with few modifications in active and reactive power demand is considered as another test system. The data of this system are obtained from (Baran and Wu, 1989a). New and basic node numbering is presented in Table 7. The upper and lower limits of voltage magnitude are considered as 1.05 p.u. and 0.95 p.u., respectively. A summary of load flow solution before D-STATCOM installation shows that 18 out of 69 nodes of distribution system (26.08%) have under voltage problem. These nodes consist of the nodes numbering from 18 to 26 and 45 to 53 (based on new node numbering).

Table 7 shows the result of D-STATCOM installation in each location of 69-bus distribution system. This table includes *RUVMN*, and the injected reactive power by unlimited D-STATCOM and 2-MVA D-STATCOM. It shows that the unlimited D-STATCOM is more effective as compared with the 2-MVA D-STATCOM. Based on the result shown in Table 7, the best locations for unlimited D-STATCOM are nodes 53, 45, 52, 46, 47, 51, 50, 48 and 49, due to *RUVMN* is equal to 26.08% in these locations and the best locations for 2-MVA D-STATCOM are nodes 45~53 which have *RUVMN* equal to 14.49%. The suggested locations for unlimited D-STATCOM are ordered in terms of *RUVMN* as well as required reactive power for compensation. Besides, Table 7 shows that the ability of D-STATCOM in voltage compensation is decreased when its reactive power rating is limited. For example, *RUVMN* in nodes 45~53 decreases from 26.08% to 14.49%.

Table 7 *RUVMN* and injected reactive power by unlimited D-STATCOM and 2-MVA STATCOM in 69-bus distribution system

Node No.		*RUVMN* (%)		*RPR* (MVA)		Node No.		*RUVMN* (%)		*RPR* (MVA)		Node No.		*RUVMN* (%)		*RPR* (MVA)	
New	Basic	Un-limited	Limited 2 MVA	Un-limited	Limited 2 MVA	New	Basic	Un-limited	Limited 2 MVA	Un-limited	Limited 2 MVA	New	Basic	Un-limited	Limited 2 MVA	Un-limited	Limited 2 MVA
1	1	0	0	4.7915	2	24	24	13.04	13.04	3.7551	2	47	48	26.08	14.49	8.6200	2
2	2	0	0	4.7914	2	25	25	13.04	13.04	3.6642	2	48	49	26.08	14.49	9.3691	2
3	3	0	0	4.6050	2	26	26	13.04	13.04	3.6134	2	49	50	26.08	14.49	9.4194	2
4	4	2.90	1.45	4.8671	2	27	27	0	0	0.9551	0.9550	50	51	26.08	14.49	9.2725	2
5	5	13.04	13.04	7.9479	2	28	28	0	0	0.1420	0.1419	51	52	26.08	14.49	9.0603	2
6	6	14.49	13.04	8.2681	2	29	29	0	0	0.1445	0.1444	52	53	26.08	14.49	8.2025	2
7	7	15.94	13.04	8.2985	2	30	30	0	0	0.1447	0.1447	53	54	26.08	14.49	6.9529	2
8	8	15.94	13.04	8.2943	2	31	31	0	0	0.1455	0.1455	54	55	14.49	13.04	6.4088	2
9	9	14.49	13.04	6.9829	2	32	32	0	0	0.1458	0.1457	55	56	14.49	13.04	6.4002	2
10	10	14.49	13.04	6.8038	2	33	33	0	0	0.1238	0.1237	56	57	14.49	13.04	5.1527	2
11	11	14.49	13.04	6.1306	2	34	34	0	0	0.0959	0.0959	57	58	14.49	13.04	5.1472	2
12	12	14.49	13.04	5.3282	2	35	35	0	0	2.4762	2	58	27e	0	0	1.0388	1.0388
13	13	13.04	13.04	4.8499	2	36	36	0	0	1.0558	1.0558	59	28e	0	0	0.2430	0.2430
14	14	13.04	13.04	4.5540	2	37	37	0	0	0.9109	0.9108	60	65	0	0	0.2276	0.2276
15	15	13.04	13.04	4.5147	2	38	38	0	0	0.8260	0.8260	61	66	0	0	0.2254	0.2254
16	16	13.04	13.04	4.4327	2	39	40	14.49	13.04	7.5676	2	62	67	0	0	0.2250	0.2250
17	17	13.04	13.04	4.4313	2	40	41	14.49	13.04	6.3160	2	63	68	0	0	0.1575	0.1575
18	18	13.04	13.04	4.3294	2	41	42	17.39	13.04	7.8592	2	64	69	0	0	0.1509	0.1509
19	19	13.04	13.04	4.2695	2	42	43	17.39	13.04	7.4959	2	65	70	0	0	0.1502	0.1502
20	20	13.04	13.04	4.1806	2	43	44	18.84	13.04	7.1341	2	66	88	0	0	0.1500	0.1500
21	21	13.04	13.04	4.1756	2	44	45	21.73	13.04	6.8792	2	67	89	0	0	0.1472	0.1472
22	22	13.04	13.04	4.1183	2	45	46	26.08	14.49	7.6515	2	68	90	0	0	0.1471	0.1471
23	23	13.04	13.04	4.0006	2	46	47	26.08	14.49	8.2642	2						

RUVMN: rate of under voltage mitigated nodes; *RPR*: reactive power rating

By comparing the effects of D-STATCOM installation in these two cases, it is concluded that the performance of this device in the system as well as their effectiveness is similar.

CONCLUSION

In this paper, the model of Distribution STATCOM (D-STATCOM) in load flow program is derived. In this model, the rating and direction of reactive power injection designated as D-STATCOM for the voltage compensation in desired value (1 p.u.) are derived and discussed analytically and mathematically by using phasor diagram method. The proposed model for D-STATCOM is applied to load flow calculations in 33- and 69-bus test systems. Moreover, the best locations of D-STATCOM for under voltage problem mitigation approach in the test systems are derived. The results presented in this paper indicated that D-STATCOM is an effective device for under voltage problem mitigation, and that the ability of D-STATCOM in voltage compensation is decreased when its reactive power rating is limited. The results indicated that the proposed model is valid for D-STATCOM in large distribution systems.

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