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Numerical forecasting surge in a piping of compressor shops of gas pipeline network

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Abstract: This paper presents a method of forecasting stable operation of gas compressor unit (GCU) centrifugal supercharger (CFS) installed on a piping of compressor shops servicing gas pipelines. The stability of superchargers operation is assessed in relation to the phenomenon of surge. Solution of this problem amounts to the development and numerical analysis of a set of ordinary differential equations. The set describes transmission of gas through a compressor shop as a fluid dynamics model with lumped parameters. The proposed method is oriented to wide application by specialists working in the gas industry. The practical application of this method can use all-purpose programming and mathematical software available to specialists of gas companies.

Key words: Gas pipelines, Compressor station, Centrifugal supercharger (CFS), Surge, Numerical simulation
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INTRODUCTION

In the process of normal operation of a gas compressor unit (GCU) centrifugal supercharger (CFS), as the mass flow rate of natural gas being transmitted through the supercharger decreases, the pressure of gas after the CFS increases. A gradual decrease in the mass flow rate of gas leads to an abrupt change in the structure of flow in the GCU. Beyond this operating point, either a stall or a surge occurs in the CFS. Without regard to the nature of phenomena that occur, in accordance with the generally accepted terminology, the operating point on (current) performance curves of the CFS, at which point the initial axially symmetric flow of gas gets destabilized, is called a “surge point” (Cumpsty, 1989). The locus of surge points on CFS performance curves related to its various shaft speeds is defined as a surge limit line.

In case of surge, the averaged mass flow rate of natural gas in the entire ring-shaped volume of the CFS varies with time in such a way that the CFS operation mode generally alters, more or less, in phase, passing from the non-stall range to the stall range and back. A process of surge can be strong enough to form a reverse mass flow, with CFS-compressed gas reappearing at the CFS inlet. This phenomenon is peculiar to a full surge. Damage caused by a full surge relates to origination of lateral loads on parts of the CFS impeller and body, due to the nonaxisymmetric nature of surge. A surge produces significant fluid dynamics losses on CFS impeller vanes and enhances harmful effects (e.g. vibration of CFS parts).

This paper considers an option of numerical forecast for a full surge appearing in a system of “CFS group−adjacent connecting pipelines (CP) −anti-surge recirculation gas pipeline (ASRGP)”. At that, the stage of flow stall in the CFS and the stage of surge itself are not distinguished. Numerical analysis of the probability of surge in the system of “CFS group−CP−ASRGP” is generally made at optimal control of safe transmission of natural gas through gas pipeline networks (GPN). This allows to assess stability of the operating point forecasted position on CFS performance curves in terms of origination and development of an emergency situation. The proposed surge models can also be used for adjustment, testing and numerical evaluation of performance of hardware and software systems operated at compressor shops.

Works (Seleznev *et al*., 2005; Tirpak *et al*., 2003; Seleznev, 2007) give reasoning to the efficiency of applying high-accuracy computational fluid dynamics simulators (CFD-simulators) of complex extended pipeline networks for solution of problems related to improving safety and efficiency of natural gas transmission. The surge models described herein are designed for application as part of a CFD-simulator. To assure the internal consistency of the CFD-simulator, the projected surge models should be 1D. Safronov and Seleznev pioneered in developing such models of surge phenomena in gas transmission systems of compressor shops (CS) in the mid-1990s (Seleznev *et al*., 2005). The models were founded on the theoretical evidence and experimental studies described in (Kazakevich, 1974; Stepanoff, 1958).

Simulation of surge admits more simplifications than those admissible in numerical analysis of design modes of natural gas transmission through a GPN (Seleznev *et al*., 2005; Tirpak *et al*., 2003; Seleznev, 2007). The reason for this is complexity of the process under study and difficulties arising in developing its physical model. Regretfully, the phenomenon of surge is still far under explored. That is why surge models are largely qualitative and applied. GPN simulations use them only for estimative calculations of full surge parameters. The estimative calculations in this case mean obtainment of upper estimates. Such estimates are used for analysis of the worst scenarios of a full surge developing in CS piping, from an industrial safety standpoint. For example, wall roughness taken into account in the pipeline model would extend the stable operation range of the CFS. Accordingly, an assumption of absolutely smooth pipes implies estimates of the worst scenarios of full surge development. The list of the assumptions and simplifications admitted herein can be presented as follows:

(1) The pipelines making part of the simulated GPN have a short length and small changes in the elevation over the sea level;

(2) The pipelines making part of the simulated GPN are assumed to be absolutely smooth;

(3) For the purpose of simulating causes of surge in the pipeline system, it is assumed to have artificial throttles;

(4) Compared with the length of the simulated pipelines, the length of each throttle can be ignored;

(5) The flow of gas through CFS-adjacent connecting pipelines is assumed to be isoentropic;

(6) The flow of gas in the throttles is assumed to be adiabatic (Jason and Jerry, 2003);

(7) Simulating surge, we proceed from an assumption that natural gas conforms to the well-known equation of state of idealized gas: $P=Z\cdot\rho\cdot R\cdot T$, where P is the pressure of gas, Z is the gas-compressibility factor, ρ is the density of gas, R is the gas constant, T is the temperature of gas (TAGIF, 1990);

(8) The gas-compressibility factor Z is assumed to be constant along the length of the pipeline or throttle.

It should be noted that, in accordance with the first assumption, the mass forces affecting transmitted gas could be ignored.

DEVELOPING A MODEL OF CP

It is worth noting that, essentially, ASRGP is a CP. Therefore, all the mathematical manipulations carried out below for CP will be valid for ASRGP as well.

Subject to the above assumptions, simplifications and the required consistency of mathematical models within the CFD-simulator, for numerical analysis of surge phenomena, we shall use a simplified model of gas flow through cylindrical pipelines, that is, a lumped parameters model. Practical application of such a model is possible, because full-scale experiments have repeatedly showed that, in the event of a surge, gas in a particular pipeline system oscillates as a whole, while acoustic phenomena practically do not affect the nature of the process (Kazakevich, 1974). Let us consider the model in more detail.

The lumped parameters model in this case implies successive consideration of two processes. A conventional segregation of these processes becomes possible due to experimental data demonstrating that acoustic phenomena have an insignificant effect on the nature of gas motion in a pipe in the event of a surge.

The first process in our model is a gas volume moving as a whole through a cylindrical pipeline, i.e. the gas density throughout the CP (or its segment) length is assumed to be constant ($\rho_1 \approx \rho_2 = \rho$). From the practical standpoint, this assumption can be justified by small actual drops of gas pressure at the boundary cross-sections of the CP, which are evidenced by actual measurements taken at CS of gas transportation enterprises. Let us assume that the gas volume moves through the pipeline with acceleration. Then the gas pressure P_2 at the pipeline outlet will not be equal to the gas pressure P_1 at its inlet. The initial pressure will be less by the value of its loss due to the gas volume inertial force in the pipeline under study. The equation of natural gas momentum in a cylindrical pipeline is presented as follows:

$$L_{\mathrm{a}} \cdot \frac{\mathrm{d}Q}{\mathrm{d}t} = P_1 - P_2, \qquad (1)$$

where $L_{\mathrm{a}}=\rho l/f$ is the acoustic mass, l is the pipeline length between its boundary cross-sections 1 and 2, f is the pipeline cross-section area; $Q=fw$ is the gas volumetric flow rate, w is the velocity of gas; t is time.

Eq.(1) can be obtained by the spatial coordinate integration of the equation of one-component gas flow (Seleznev *et al.*, 2005):

$$\rho\frac{\partial Q}{\partial t} + \rho Q\frac{\partial w}{\partial x} = -f\left(\frac{\partial P}{\partial x} + g\rho\frac{\partial z_1}{\partial x}\right) - \frac{\sqrt{\pi}}{4}\lambda f^{-1.5}\rho Q^2,$$

where λ is the hydraulic friction coefficient of the throttle.

It should be noted that the gas velocity variation w along the pipeline length does not practically affect its acceleration, which enables us to admit that $\partial w/\partial x \approx 0$. Subject to the admitted simplifications, the considered equation of momentum will look as follows:

$$\frac{\rho}{f} \cdot \frac{\mathrm{d}Q}{\mathrm{d}t}\int_0^l \mathrm{d}x = -\int_{P_1}^{P_2} \mathrm{d}P.$$

Hence, Eq.(1) is obtained. This equation describes the so-called inertia of gas motion through the pipeline (Kazakevich, 1974).

The second process, analyzed in surge simulation, describes the acoustic phenomena related to the velocity of gas pressure variation within the volume of the simulated pipeline. The pipeline in this case is treated as a vessel containing gas whose density becomes instantly averaged throughout the volume. Proceeding from an assumption about the isentropic process of idealized gas flowing through CP and the constant density of gas throughout the volume, let us integrate the equation of continuity for one-component gas (Seleznev *et al.*, 2005)

$$\left(\frac{\partial(\rho f)}{\partial t} + \frac{\partial(\rho f w)}{\partial x} = 0\right) \Rightarrow \left(f\frac{\partial \rho}{\partial t} + \rho\frac{\partial Q}{\partial x} = 0\right),$$

spatially,

$$\left(f \cdot \frac{\mathrm{d}\rho}{\mathrm{d}t}\int_0^l \mathrm{d}x + \rho\int_{Q_1}^{Q_2} \mathrm{d}Q = 0\right) \Rightarrow \left(\frac{V}{\rho c_0^2} \cdot \frac{\mathrm{d}P}{\mathrm{d}t} = Q_1 - Q_2\right),$$

where $V=lf$ is the volume of the simulated pipeline between the boundary cross-sections, c_0 is the local velocity of sound.

In this case, to develop the equation, we use the well-known thermodynamic relation in respect of the isoentropic processes $\mathrm{d}P = (\partial P/\partial \rho)_S \mathrm{d}\rho = c_0^2 \mathrm{d}\rho$, where S is entropy. Then we can write:

$$C_{\mathrm{a}} \cdot \frac{\mathrm{d}P}{\mathrm{d}t} = Q_1 - Q_2, \qquad (2)$$

where $C_{\mathrm{a}}=V/(\rho c_0^2)=V/(\gamma R\rho T)=VZ/(\gamma P)$ is the acoustic flexibility.

Eqs.(1) and (2) describe, respectively, the inertia and volume-related properties of gas motion in a pipeline. Schematically, the process of gas flow in an actual CP is presented as "inertia" pipeline with a reservoir at one of its ends (Seleznev *et al.*, 2005) (Fig.1a). Then, for the motion of gas as a system depicted in Fig.1a, we have the following equations:

$$L_{\mathrm{a}1} \cdot \frac{\mathrm{d}Q_1}{\mathrm{d}t} = P_1 - P_2, \qquad (3\mathrm{a})$$

$$C_{\mathrm{a}2} \cdot \frac{\mathrm{d}P_2}{\mathrm{d}t} = Q_1 - Q_2, \qquad (3\mathrm{b})$$

where

$$\begin{cases} L_{\mathrm{a}1} = \rho_1 l K_{L1}/f, \\ C_{\mathrm{a}2} = lfK_{C2}/[\rho_2(c_0^2)_2] = lfZ_2K_{C2}/(\gamma P_2), \\ K_{L1} + K_{C2} = 1. \end{cases} \qquad (4)$$

In Eq.(4), K_{Li} (i=1, 2) are the reduction factors taking into account the influence that the inertia of gas with the acoustic mass L_{ai} in the CP have on the condition of self-excitation of gas volume oscillations in this CP (these factors were introduced by Kazakevich (1974), their recommended value for cylindrical pipelines being K_{Li}=0, 5, i=1, 2); K_{Ci} (i=1, 2) are the reduction factors taking into account the influence that the volume-related properties of gas with the acoustic flexibility C_{ai} in the CP have on the condition of self-excitation of gas volume oscillations in this CP (these factors were introduced by Kazakevich (1974), their recommended value for cylindrical pipelines being K_{Ci}=0, 5, i=1, 2).

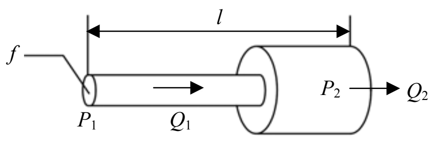
Let the reservoir be artificially concentrated at the left end of the pipeline (Fig.1b). Then, repeating the above reasoning, we obtain:

$$L_{a2} \cdot \frac{dQ_2}{dt} = P_1 - P_2, \quad (5a)$$

$$C_{a1} \cdot \frac{dP_1}{dt} = Q_1 - Q_2, \quad (5b)$$

where

$$\begin{cases} L_{a2} = \rho_2 l K_{L2} / f, \\ C_{a1} = lfK_{C1} / [\rho_1 (c_0^2)_1] = lfZ_1 K_{C1} / (\gamma P_1), \\ K_{L2} + K_{C1} = 1. \end{cases} \quad (6)$$



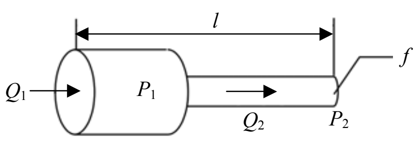


Fig.1 Schematic flow of gas in a pipeline
(a) Variant 1; (b) Variant 2

Deducting Eq.(5a) from Eq.(3a), and Eq.(5b) from Eq.(3b), subject to the admitted assumptions, we can write:

$$\rho_1 \frac{dQ_1}{dt} = \rho_2 \frac{dQ_2}{dt}; \quad \frac{1}{P_1} \cdot \frac{dP_1}{dt} = \frac{1}{P_2} \cdot \frac{dP_2}{dt}.$$

It follows from these equations that Eqs.(3) and (5) coincide. Accordingly, we can assert there are equations of motion of gas through CP, which is simultaneously characterized by inertia and volume-related properties. It follows from the above that, in this case, for simulation of a gas flow in CP, one equation of a volumetric flow rate of gas [either of Eqs.(3a) and (5a)] and one equation of gas pressure [either of Eqs.(3b) and (5b)] will suffice.

DEVELOPING A THROTTLE MODEL

As it was noted above, for simulation of the causes of surge in the pipeline, we assume that it is equipped with throttles. The throttles are used to simulate variation of a mass flow rate of gas in pipelines, affected by instant alteration of their cross-section areas at very short lengths. From the practical standpoint, throttles can simulate GPN valve control errors, consequences of natural disasters and third party actions, affecting the throughput capacity of the pipeline. Here we mean that a natural disaster (e.g., a landslide) may result in significant deformations of pipe walls. Furthermore, throttles are used to simulate functioning of blow-off valves.

To develop a model of gas flow through a throttle, let us consider its schematic representation (Fig.2). In a 1D case, a throttle can be simulated as a pipe segment with a reduced cross-section. Let the cross-section area of the throttle be equal to $f_{throttle}=k_{throttle}f$, where $k_{throttle} \in (0, 1]$ is the coefficient of reduction of the throttle cross-section.

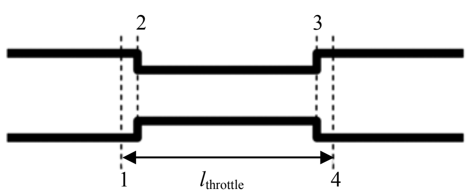


Fig.2 Schematic representation of the geometric structure of a throttle

Let us consider, in Fig.2, the segment between the cross-sections 1 and 4. We assume that the gas flow core has enough time to contract and expand at the segments [x_1, x_2] and [x_3, x_4], respectively (the subscripts here correspond to the numbers of points in Fig.2). At that, it is admitted that:

$$l_{\text{throttle}} \approx |x_3 - x_2|, \tag{7}$$

where l_{throttle} is the length of throttling segment.

Due to the small length of a throttle gas duct compared with the CP length, we here assume that the fluid dynamics processes instantaneously spread throughout the throttle. Then, at each moment of time, we can consider the process of gas transmission through the throttle as steady. Ignoring the gravity force (due to the short length of the throttle), let us consider a non-divergent form of the gas momentum equation in case of a steady flow mode (Seleznev *et al.*, 2005):

$$w\frac{\mathrm{d}w}{\mathrm{d}x} + \frac{1}{\rho}\cdot\frac{\mathrm{d}P}{\mathrm{d}x} + \frac{\lambda w^2\sqrt{\pi}}{4\sqrt{f}} = 0.$$

For determination of the hydraulic friction coefficient λ, the flow mode is assumed to be quadratic, which means that λ does not depend on gas flow parameters. The above equation transforms into a modification of the Bernoulli equation as applied to the process under study:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{w^2}{2}\right) + \frac{1}{\rho}\cdot\frac{\mathrm{d}P}{\mathrm{d}x} + \frac{\lambda w^2\sqrt{\pi}}{4\sqrt{f}} = 0. \tag{8}$$

Let us integrate Eq.(8) at the segment $[x_1, x_4]$, assuming that the process of gas flow through the throttle is adiabatic ($P=const\cdot\rho^{\gamma}$), and taking into account the continuity equation $\rho_1 w_1=\rho_4 w_4$:

$$\frac{Q_4^2}{2f^2}\left(1-\left(\frac{P_4}{P_1}\right)^{2/\gamma}\right) + \frac{1}{G}ZRT_1\left[\left(\frac{P_4}{P_1}\right)^{G} - 1\right] + \frac{\sqrt{\pi}\lambda l_{\text{throttle}} Q_4^2}{8k_{\text{throttle}}^{2.5} f^{2.5}}\left(1+\left(\frac{P_4}{P_1}\right)^{2/\gamma}\right) = 0, \tag{9a}$$

where $G=(\gamma-1)/\gamma$, γ is the adiabatic index.

Let us give a detailed explanation of how the third summand of the equation is obtained:

$$\int_1^4 \sqrt{\frac{\pi}{f(x)}}\frac{\lambda w^2}{4}\mathrm{d}x \approx \int_2^3 \sqrt{\frac{\pi}{f(x)}}\frac{\lambda w^2}{4}\mathrm{d}x = \sqrt{\frac{\pi}{f_{\text{throttle}}}}\frac{\lambda}{4}\int_2^3 w^2\mathrm{d}x$$
$$= \frac{\sqrt{\pi}\lambda J^2}{4f_{\text{throttle}}^{2.5}}\int_2^3 \rho^{-2}\mathrm{d}x \approx \frac{\sqrt{\pi}\lambda J^2}{4f_{\text{throttle}}^{2.5}}\int_1^4 \rho^{-2}\mathrm{d}x$$
$$\approx \frac{\sqrt{\pi}\lambda J^2 l_{\text{throttle}}}{8f_{\text{throttle}}^{2.5}}\left(\frac{1}{\rho_1^2}+\frac{1}{\rho_4^2}\right) = \frac{\sqrt{\pi}\lambda J^2 l_{\text{throttle}}}{8f_{\text{throttle}}^{2.5}\rho_4^2}(1+(P_4/P_1)^{2/\gamma}), \tag{10}$$

where $J=\rho(x)\cdot w(x)\cdot f(x)$ is the mass flow rate of gas.

For a steady flow, $J(x)$=constant.

Let us introduce indices "in_t" and "out_t" to denote the throttle inlet and outlet, after which, on the basis of Eq.(9), we can write:

$$Q_{\text{out\_t}}^2 = \frac{\frac{2}{G}f^2 ZRT_{\text{in\_t}}\left[1-\left(\frac{P_{\text{out\_t}}}{P_{\text{in\_t}}}\right)^{G}\right]}{\left[1-\left(\frac{P_{\text{out\_t}}}{P_{\text{in\_t}}}\right)^{2/\gamma}\right] + \sqrt{\frac{\pi}{f}}\frac{\lambda}{4}\frac{l_{\text{throttle}}}{k_{\text{throttle}}^{2.5}}\left[1+\left(\frac{P_{\text{out\_t}}}{P_{\text{in\_t}}}\right)^{2/\gamma}\right]}, \tag{11a}$$

$$Q_{\text{in\_t}}^2 = Q_{\text{out\_t}}^2\cdot\left(\frac{P_{\text{out\_t}}}{P_{\text{in\_t}}}\right)^{2/\gamma}. \tag{11b}$$

Regretfully, Eq.(11a) is not implicit with respect to the parameter $P_{\text{out\_t}}$, which calls for numerical methods for its calculation (given values of parameters $P_{\text{in\_t}}$, $Q_{\text{in\_t}}$ and $T_{\text{in\_t}}$ are known). To solve this problem, we can offer the following method of simplification.

Let us compare the first two summands in Eq.(8). The second summand in Eq.(8) can be transformed to:

$$\frac{1}{\rho}\cdot\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{1}{\bar{\rho}}\cdot\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{P}{\bar{\rho}}\right),$$

where $\bar{\rho}$ is a certain average value of the density ρ at the segment in question.

Subject to this formula, let us consider the sum of the specified summands in Eq.(8):

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{w^2}{2}\right) + \frac{1}{\rho}\cdot\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{w^2}{2}+\frac{P}{\bar{\rho}}\right).$$

The velocity of gas in pipelines has an order of magnitude of 10 m/s with a density of 40 kg/m^3 and pressure of 4×10^6 Pa. Substituting these values in the above ratio, we obtain that the first summand under the notation of the space derivative has an order of magnitude of 10^2 m^2/s^2, and the second is 10^5 m^2/s^2. It follows from the above reasoning that on a first approximation the summand $d(w^2)/dx$ is negligible versus the summand $(1/\rho)\cdot dP/dx$.

Now let us consider the transformations in Eq.(10). The last integral in these transformations can be valued as follows:

$$\int_2^3 \frac{dx}{\rho^2} \approx \int_1^4 \frac{dx}{\rho^2} \approx \frac{l_{\text{throttle}}}{\rho_1^2} \quad \text{or} \quad \int_2^3 \frac{dx}{\rho^2} \approx \int_1^4 \frac{dx}{\rho^2} \approx \frac{l_{\text{throttle}}}{\rho_4^2}.$$

Hence,

$$\int_1^4 \sqrt{\frac{\pi}{f}} \frac{\lambda}{4} w^2 dx \approx \sqrt{\pi} \frac{\lambda}{4} \frac{J^2}{f_{\text{throttle}}^{2.5}} \int_2^3 \rho^{-2} dx \approx \sqrt{\pi} \frac{\lambda}{4} \frac{Q_1^2 l_{\text{throttle}}}{f_{\text{throttle}}^{2.5}},$$

or

$$\int_1^4 \sqrt{\frac{\pi}{f}} \frac{\lambda}{4} w^2 dx \approx \sqrt{\pi} \frac{\lambda}{4} \frac{Q_4^2 l_{\text{throttle}}}{f_{\text{throttle}}^{2.5}}.$$

Then Eq.(9a) can be rewritten as

$$\frac{1}{G} ZRT_{\text{in\_t}} \left[\left(\frac{P_{\text{out\_t}}}{P_{\text{in\_t}}} \right)^G - 1 \right] + \frac{\sqrt{\pi} \lambda l_{\text{throttle}} Q_t^2}{4 k_{\text{throttle}}^{2.5} f^{2.5}} = 0, \quad (9b)$$

where Q_t can assume the value $Q_{\text{in\_t}}$ or $Q_{\text{out\_t}}$.

From the above equation, we obtain:

$$P_{\text{out\_t}} = P_{\text{in\_t}} \left[1 - \frac{\sqrt{\pi} \lambda l_{\text{throttle}} Q_t^2 G}{4 k_{\text{throttle}}^{2.5} f^{2.5} ZRT_{\text{in\_t}}} \right]^{1/G}.$$

Hence

$$\xi = \left[1 - \frac{\sqrt{\pi} \lambda l_{\text{throttle}} Q_t^2 G}{4 k_{\text{throttle}}^{2.5} f^{2.5} ZRT_{\text{in\_t}}} \right]^{1/G}, \quad (11c)$$

where the function ($\xi = P_{\text{out\_t}}/P_{\text{in\_t}}$) will be called the throttle performance curve.

DEVELOPING A CENTRIFUGAL SUPERCHARGER MODEL

A model of gas transmission through a CFS is described in detail in (Seleznev *et al.*, 2005; Tirpak *et al.*, 2003; Stepanoff, 1958; Kurz and Ohanian, 2003). This model relates gas parameters at the CFS inlet and outlet:

$$P_{\text{out\_CFS}} = \varepsilon(Q_{\text{in\_CFS}}) P_{\text{in\_CFS}}, \quad (12a)$$

$$T_{\text{out\_CFS}} = T_{\text{in\_CFS}} [\varepsilon(Q_{\text{in\_CFS}})]^{\sigma}, \quad (12b)$$

where $\varepsilon(Q_{\text{in\_CFS}})$ is the ratio of gas compression in the CFS, which is its nameplate performance curve (in general terms, $\varepsilon=\varepsilon(n, P_{\text{in\_CFS}}, T_{\text{in\_CFS}}, Q_{\text{in\_CFS}})$ (Seleznev *et al.*, 2005); $\sigma=(\gamma-1)/(\gamma\eta_p)=(\eta-1)/\eta=G/\eta_p$ (Stepanoff, 1958), where η_p is the polytropic efficiency of the CFS, η is the polytropic coefficient, n is the shaft speed of the CFS.

DEVELOPING A MODEL OF SURGE PHENOMENON IN A SYSTEM OF "CFS GROUP–CP–ASRGP"

Let us consider the application of the above models of CP (ASRGP), artificial throttles, and CFS in a numerical evaluation of the probability of surge in a system of "CFS group–CP–ASRGP". Let there be a hypothetical compressor shop (Fig.3) having the number N of paralleled CFS. To eliminate the surge arising in one or several systems "CFS-CP" of the CS, a common ASRGP equipped with a blow-off valve is used. It should be noted that such an arrangement is commonly used at compressor stations of Gazprom (Russia).

The simulation is made in order to evaluate the stability of operating points on the performance curves of all the CFS in the CS in the circumstances of unexpected accident-caused impairment of the throughput capacity of the CFS pipeline system. Initially, the CFS pipeline system operates in the design steady mode. A change in the throughput capacity is simulated in this case by closing artificial throttles at the inlet and/or outlet of the given pipeline system. Operation of the blow-off valve is simulated by opening the throttle denoted in Fig.3 as "throttle 3". In the open position, the cross-sectional area of the throttle coincides with the cross-sectional area of the adjacent pipeline.

Lengths of simulated pipelines are denoted in accordance with the indexes of their inlet or outlet boundary cross-sections (see the list of symbols) on

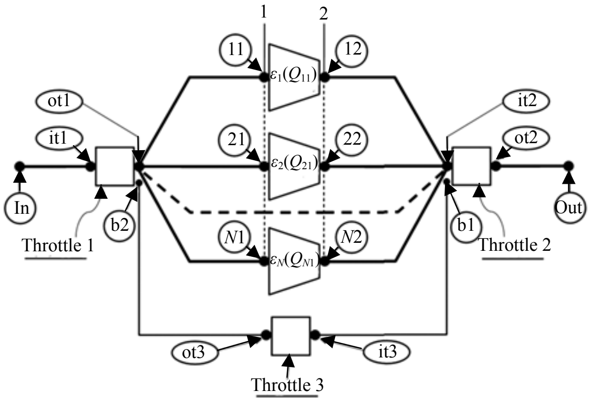


Fig.3 Schematic of index notations of parameters in the simulated system "CFS group–CP–ASRGP"

the condition that such denotation on the schematic is unique.

When simulating a surge in the system of "CFS group–CP–ASRGP", we assume that transient fluid dynamics processes are essential only at the segment of the system between the inlet and outlet boundary cross-sections denoted by "In" and "Out" on the diagram. The pressure and temperature in the inlet cross-section and the pressure in the outlet cross-section remain unchanged and are equal to the values obtained for the initial steady mode of gas transmission, which existed before the surge phenomena occurred.

When simulating operation of ASRGP, we assume that gas flows from the initial cross-section of the pipeline "b1" to its end cross-section "b2". No reverse flow is allowed here, as it would cause failure of the blow-off valve.

As is well known (Stepanoff, 1958), for the compressibility factors of gas at the CFS inlet and outlet, the following ratios are valid: $Z_{i1}/Z_{i2}\approx 1$, $i=1,\ldots,N$. Then, subject to the admitted assumption of the compressibility factors consistency along the length of the pipeline or throttle, we can write that

$$Z_{\mathrm{In}} = Z_{\mathrm{it1}} = Z_{\mathrm{ot1}} = Z_{\mathrm{b2}} = Z_{i1} = Z_{i2}$$
$$= Z_{\mathrm{b1}} = Z_{\mathrm{it2}} = Z_{\mathrm{ot2}} = Z_{\mathrm{Out}}, \qquad i = 1,\ldots,N.$$

Therefore, these factors are not time dependent.

On a first approximation, for simulation of a surge upon junction of gas flows, we assume that an instant isobaric mixing of gas flows takes place at the pipeline junction node. In actual CS, the joining flows are similar to each other in terms of the mass flow rate, for which reason the temperature at the junction node will be measured by simple averaging. We also assume that the gas flow transmitted through ARSPG to the pipeline junction node does not affect the overall change in the transmitted gas temperature in the flow junction zone at the inlet to the CFS group. Such assumption is justified since gas flows through ASRPG are much smaller than the main transmission flow. It should be also noted that, at the initial moment of ASRPG operation, the segment after the blow-off valve contains gas with the temperature close to the temperature of gas at the CS inlet.

Pursuant to recommendations in (Kazakevich, 1974), it is assumed that $K_{Li1}=K_{Li2}=K_{L\_\mathrm{it1}}=K_{L\_\mathrm{ot2}}=K_{L\_\mathrm{it3}}=K_{L\_\mathrm{ot3}}=K_L=0.5$ ($i=1,\ldots,N$) and $K_{Ci1}=K_{Ci2}=K_{C\_\mathrm{it3}}=K_{C\_\mathrm{ot3}}=K_C=0.5$ ($i=1,\ldots,N$) [Eqs.(4) and (6)]. It is also worth noting that in this case the equations: $P_{\mathrm{ot1}\_i}=P_{\mathrm{b2}}=P_{\mathrm{ot1}}$; $P_{\mathrm{it2}\_i}=P_{\mathrm{b1}}=P_{\mathrm{it2}}$ ($i=1,\ldots,N$) are satisfied.

A list of input data of the problem subject to solution looks as follows (Fig.3): lengths of all pipelines and artificial throttles in the system; cross-section areas of all pipelines in the system; the hydraulic friction coefficients $\lambda_{\mathrm{t}i}$ ($i=1,2,3$) for artificial throttles in the pipeline system; the gas constant R for the specified chemical composition of natural gas; the gas pressure P_{In} at the pipeline system inlet; the gas pressure P_{Out} at the pipeline system outlet; the gas temperature T_{In} at the pipeline system inlet; the natural gas adiabatic index γ for the initial pressure and temperature of gas at the system inlet; the polytrophic efficiency η_{p} for the specified types of CFS; the cross-section reduction coefficient $k_{\mathrm{t}i}$ ($i=1,2,3$) for the pipeline system throttles; the CFS shaft speeds n_i, $i=1,\ldots,N$; the CFS nameplate performance curve $\varepsilon_i(Q_{i1})$, $i=1,\ldots,N$; the gas-compressibility factor Z_{In}.

The CP inertia is described by the following equations:

(1) For suction CP in the CFS group [Eqs.(5a) and (6)],

$$\begin{cases} L_{\mathrm{a}i1}\dfrac{\mathrm{d}Q_{i1}}{\mathrm{d}t} = P_{\mathrm{ot1}} - P_{i1}, \\ L_{\mathrm{a}i1} = K_L \rho_{i1} l_{i1} / f_{i1}, \quad i=1,\ldots,N; \end{cases} \tag{13a}$$

(2) For delivery CP in the CFS group [Eqs.(3a) and (4)],

$$\begin{cases} L_{ai2}\dfrac{dQ_{i2}}{dt} = P_{i2} - P_{it2}, \\ L_{ai2} = K_L \rho_{i2} l_{i2} / f_{i2}, \quad i = 1,\ldots,N. \end{cases} \tag{13b}$$

Basing on conditions of the equality of mass flow rates of gas throughout the CFS ($\rho_{i1}Q_{i1}=\rho_{i2}Q_{i2}$, $i=1,\ldots,N$) (Seleznev *et al.*, 2005), the constancy of the gas-compressibility factor in analysis of CFS operation, and the dependencies in Eq.(12), we can write:

$$Q_{i2} = Q_{i1}[\varepsilon_i(Q_{i1})]^{\sigma-1},\ i = 1,\ldots,N. \tag{14}$$

Using an additional assumption of insignificant variation of the gas density with respect to the model of pipelines (the admitted simplifications in developing the pipeline model), we obtain:

$$\frac{d(\rho_{i1}Q_{i1})}{dt} = \frac{d(\rho_{i2}Q_{i2})}{dt},\ i = 1,\ldots,N,$$

hence

$$\rho_{i1}\frac{dQ_{i1}}{dt} = \rho_{i2}\frac{dQ_{i2}}{dt},\ i = 1,\ldots,N. \tag{15}$$

Let us multiply Eqs.(13a) by $\varepsilon_i(Q_{i1})$ and sum up them and the corresponding Eq.(13b). Then, subject to Eq.(15)

$$L_{ai}\frac{dQ_{i1}}{dt} = \varepsilon_i(Q_{i1})P_{ot1} - P_{it2},\ i = 1,\ldots,N, \tag{16}$$

where

$$L_{ai} = K_L \frac{l_{i1}}{f_{i1}}[\varepsilon_i(Q_{i1}) + A_i]\frac{P_{i1}}{Z_{In}RT_{i1}};\ A_i = \frac{l_{i2}}{l_{i1}}\frac{f_{i1}}{f_{i2}}.$$

Let us multiply Eq.(13a) by l_{i2}/f_{i2} and deduct them from the corresponding Eq.(13b) multiplied by l_{i1}/f_{i1}. Then, subject to Eqs.(12a) and (16)

$$P_{i1} = \frac{A_i P_{ot1} + P_{it2}}{A_i + \varepsilon_i(Q_{i1})},\ i = 1,\ldots,N. \tag{17}$$

The equations describing volume-related properties of the pipeline will be written as follows (Fig.3):

(1) For suction CP in the CFS group [Eqs.(5b) and (6)],

$$\begin{cases} C_{ai1}\dfrac{dP_{ot1}}{dt} = Q_{ot1\_i} - Q_{i1}, \\ C_{ai1} = \dfrac{K_C l_{i1} f_{i1}}{\gamma}\cdot\dfrac{Z_{In}}{P_{ot1}},\ i = 1,\ldots,N. \end{cases} \tag{18a}$$

(2) For delivery CP in the CFS group [Eqs.(3b), (4) and (14)],

$$\begin{cases} C_{ai2}\dfrac{dP_{it2}}{dt} = Q_{i1}[\varepsilon_i(Q_{i1})]^{\sigma-1} - Q_{it2\_i}, \\ C_{ai2} = \dfrac{K_C l_{i2} f_{i2}}{\gamma}\cdot\dfrac{Z_{In}}{P_{it2}},\ i = 1,\ldots,N. \end{cases} \tag{18b}$$

Let us sum up Eq.(18a) with each other and do the same with Eq.(18b) subject to the equations $\sum_{i=1}^{N} Q_{ot1\_i} = Q_{ot1} + Q_{b2}$ and $\sum_{i=1}^{N} Q_{it2\_i} = Q_{it2} + Q_{b1}$:

$$\frac{dP_{ot1}}{dt} = B_1\frac{P_{ot1}}{Z_{In}}\cdot\left[Q_{ot1} + Q_{b2} - \sum_{i=1}^{N} Q_{i1}\right], \tag{19a}$$

$$\frac{dP_{it2}}{dt} = B_2\frac{P_{it2}}{Z_{In}}\cdot\left[\sum_{i=1}^{N}\{Q_{i1}[\varepsilon_i(Q_{i1})]^{\sigma-1}\} - Q_{it2} - Q_{b1}\right], \tag{19b}$$

where

$$B_j = \gamma\left[K_C\sum_{i=1}^{N}(l_{ij}f_{ij})\right]^{-1},\quad j = 1,2.$$

For further reasoning, we use the formulas for calculation of gas temperature in the pipeline system (Fig.3), obtained in accordance with the agreed statement of the problem:

$$T_{it1} = T_{In}\cdot\left(\frac{P_{it1}}{P_{In}}\right)^G;$$

$$T_{otj} = T_{itj}\cdot\left(\frac{P_{otj}}{P_{itj}}\right)^G,\quad j = 1,2,3;$$

$$T_{i1} = T_{ot1}\cdot\left(\frac{P_{i1}}{P_{ot1}}\right)^G,\ T_{i2} = T_{i1}\left[\varepsilon_i(Q_{i1})\right]^{\sigma},\ i = 1,\ldots,N;$$

$$T_{it2} = \frac{1}{N}\sum_{i=1}^{N}\left[T_{i2}\left(\frac{P_{it2}}{P_{i2}}\right)^G\right];$$

$$T_{\rm b1}=T_{\rm it2};\ \ T_{\rm Out}=T_{\rm ot2}\left(\frac{P_{\rm Out}}{P_{\rm ot2}}\right)^{G};\ \ T_{\rm it3}=T_{\rm b1}\left(\frac{P_{\rm it3}}{P_{\rm b1}}\right)^{G};$$

$$T_{\rm b2}=T_{\rm ot1},$$

or

$$T_{\rm it1}=T_{\rm In}\cdot\left(\frac{P_{\rm it1}}{P_{\rm In}}\right)^{G};\quad T_{\rm ot1}=T_{\rm b2}=T_{\rm In}\cdot\left(\frac{P_{\rm ot1}}{P_{\rm In}}\right)^{G};$$

$$T_{i1}=T_{\rm In}\cdot\left(\frac{P_{i1}}{P_{\rm In}}\right)^{G},\ T_{i2}=T_{i1}\left[\varepsilon_i(Q_{i1})\right]^{\sigma},\quad i=1,\ldots,N;$$

$$T_{\rm it2}=T_{\rm b1}=T_{\rm In}\cdot\left(\frac{P_{\rm it2}}{P_{\rm In}}\right)^{G}\frac{1}{N}\sum_{i=1}^{N}\left[\varepsilon_i(Q_{i1})\right]^{\sigma-G};$$

$$T_{\rm ot2}=T_{\rm In}\cdot\left(\frac{P_{\rm ot2}}{P_{\rm In}}\right)^{G}\frac{1}{N}\sum_{i=1}^{N}\left[\varepsilon_i(Q_{i1})\right]^{\sigma-G};$$

$$T_{\rm Out}=T_{\rm In}\cdot\left(\frac{P_{\rm Out}}{P_{\rm In}}\right)^{G}\frac{1}{N}\sum_{i=1}^{N}\left[\varepsilon_i(Q_{i1})\right]^{\sigma-G};$$

$$T_{\rm it3}=T_{\rm In}\cdot\left(\frac{P_{\rm it3}}{P_{\rm In}}\right)^{G}\frac{1}{N}\sum_{i=1}^{N}\left[\varepsilon_i(Q_{i1})\right]^{\sigma-G};$$

$$T_{\rm ot3}=T_{\rm In}\cdot\left(\frac{P_{\rm ot3}}{P_{\rm In}}\right)^{G}\frac{1}{N}\sum_{i=1}^{N}\left[\varepsilon_i(Q_{i1})\right]^{\sigma-G}. \tag{20}$$

Let us introduce functions $\xi_{t1}(Q_{\rm it1}, P_{\rm ot1})$, $\xi_{t2}(Q_{\rm ot1}, P_{\rm it2}, Q_{i1})$, and $\xi_{t3}(Q_{\rm ot3}, P_{\rm it3}, Q_{i1})$, as performance curves of the pipeline system throttles (Fig.3), with

$$P_{\rm ot1}=\xi_{t1}(Q_{\rm it1},P_{\rm ot1})\cdot P_{\rm it1}, \tag{21a}$$

$$P_{\rm ot2}=\xi_{t2}(Q_{\rm ot2},P_{\rm it2},Q_{i1})\cdot P_{\rm it2}, \tag{21b}$$

$$P_{\rm ot3}=\xi_{t3}(Q_{\rm ot3},P_{\rm it3},Q_{i1})\cdot P_{\rm it3}. \tag{21c}$$

These curves can be constructed by solving corresponding non-linear algebraic equations [Eq.(11a)]:

$$\frac{\dfrac{2f_{\rm In}^2Z_{\rm In}RT_{\rm In}}{G}\cdot\left(\dfrac{P_{\rm ot1}}{P_{\rm In}}\right)^{G}\dfrac{[1-(\xi_{t1})^{G}]}{(\xi_{t1})^{G}}}{\left[1-(\xi_{t1})^{2/\gamma}\right]+\sqrt{\dfrac{\pi}{f_{\rm In}}}\dfrac{\lambda_{t1}}{4}\cdot\dfrac{l_{t1}}{k_{t1}^{2.5}}\left[1+(\xi_{t1})^{2/\gamma}\right]}-\frac{Q_{\rm it1}^2}{(\xi_{t1})^{2/\gamma}}=0, \tag{22a}$$

$$\frac{\dfrac{2f_{\rm Out}^2Z_{\rm In}RT_{\rm In}}{GN}\cdot\left(\dfrac{P_{\rm it2}}{P_{\rm In}}\right)^{G}[1-(\xi_{t2})^{G}]\sum_{i=1}^{N}[\varepsilon_i(Q_{i1})]^{\sigma-G}}{[1-(\xi_{t2})^{2/\gamma}]+\sqrt{\dfrac{\pi}{f_{\rm Out}}}\dfrac{\lambda_{t2}}{4}\cdot\dfrac{l_{t2}}{k_{t2}^{2.5}}[1+(\xi_{t2})^{2/\gamma}]}-Q_{\rm ot2}^2=0, \tag{22b}$$

$$\frac{\dfrac{2f_{\rm b1}^2Z_{\rm In}RT_{\rm In}}{GN}\cdot\left(\dfrac{P_{\rm it3}}{P_{\rm In}}\right)^{G}[1-(\xi_{t3})^{G}]\sum_{i=1}^{N}[\varepsilon_i(Q_{i1})]^{\sigma-G}}{[1-(\xi_{t3})^{2/\gamma}]+\sqrt{\dfrac{\pi}{f_{\rm b1}}}\dfrac{\lambda_{t3}}{4}\cdot\dfrac{l_{t3}}{k_{t3}^{2.5}}[1+(\xi_{t3})^{2/\gamma}]}-Q_{\rm ot3}^2=0. \tag{22c}$$

Subject to Eqs.(11b), (21a) and (21b), Eq.(19) can be represented as follows:

$$\frac{{\rm d}P_{\rm ot1}}{{\rm d}t}=B_1\frac{P_{\rm ot1}}{Z_{\rm In}}\left[Q_{\rm it1}\cdot\left[\xi_{t1}(Q_{\rm it1},P_{\rm ot1})\right]^{-\frac{1}{\gamma}}+Q_{\rm b2}-\sum_{i=1}^{N}Q_{i1}\right], \tag{23a}$$

$$\frac{{\rm d}P_{\rm it2}}{{\rm d}t}=B_2\frac{P_{\rm it2}}{Z_{\rm In}}\left[\sum_{i=1}^{N}\left[Q_{i1}\cdot\left[\varepsilon_i(Q_{i1})\right]^{\sigma-1}\right]-Q_{\rm ot2}\cdot\left[\xi_{t2}(Q_{\rm ot2},P_{\rm it2},Q_{i1})\right]^{\frac{1}{\gamma}}-Q_{b1}\right]. \tag{23b}$$

With respect to natural gas CP transmission at the pipeline system inlet, we can write [Fig.3 and Eqs.(5a), (6), (21a)]:

$$L_{\rm a\_it1}\frac{{\rm d}Q_{\rm it1}}{{\rm d}t}=P_{\rm In}-\frac{P_{\rm ot1}}{\xi_{t1}(Q_{\rm it1},P_{\rm ot1})}, \tag{24}$$

where

$$L_{\rm a\_it1}=D_{\rm In}\cdot\left(\frac{\xi_{t1}(Q_{\rm it1},P_{\rm ot1})}{P_{\rm ot1}}\right)^{G-1},\ D_{\rm In}=\frac{K_Ll_{\rm In}}{f_{\rm In}}\cdot\frac{(P_{\rm In})^G}{Z_{\rm In}RT_{\rm In}}.$$

By analogy, with respect to natural gas transmission through CP, at the pipeline system outlet we obtain [Fig.3 and Eqs.(3a), (4), (21b)]:

$$L_{\rm a\_ot2}\frac{{\rm d}Q_{\rm ot2}}{{\rm d}t}=\xi_{t2}(Q_{\rm ot2},P_{\rm it2},Q_{i1})\cdot P_{\rm it2}-P_{\rm Out}, \tag{25}$$

where

$$L_{\mathrm{a\_ot2}} = D_{\mathrm{Out}} \frac{N \cdot \left(\xi_{t2}(Q_{\mathrm{ot2}}, P_{\mathrm{it2}}, Q_{i1}) \cdot P_{\mathrm{it2}}\right)^{1-G}}{\sum_{i=1}^{N} \left[\varepsilon_i(Q_{i1})\right]^{\sigma-G}},$$

$$D_{\mathrm{Out}} = \frac{K_L l_{\mathrm{Out}}}{f_{\mathrm{Out}}} \cdot \frac{(P_{\mathrm{In}})^G}{Z_{\mathrm{In}} R T_{\mathrm{In}}}.$$

The set of equations for description of transmission of gas through ASRGP will look as follows [Fig.3 and Eqs.(3)~(6), (11b), (21c)]:

$$L_{\mathrm{a\_b1}} \frac{\mathrm{d}Q_{\mathrm{b1}}}{\mathrm{d}t} = P_{\mathrm{it2}} - P_{\mathrm{it3}}, \quad (26\mathrm{a})$$

$$\frac{\mathrm{d}P_{\mathrm{it3}}}{\mathrm{d}t} = B_{\mathrm{it3}} \frac{P_{\mathrm{it3}}}{Z_{\mathrm{In}}} \left[Q_{\mathrm{b1}} - Q_{\mathrm{ot3}} [\xi_{t3}(Q_{\mathrm{ot3}}, P_{\mathrm{it3}}, Q_{i1})]^{1/\gamma}\right], \quad (26\mathrm{b})$$

$$L_{\mathrm{a\_b2}} \frac{\mathrm{d}Q_{\mathrm{b2}}}{\mathrm{d}t} = \xi_{t3}(Q_{\mathrm{ot3}}, P_{\mathrm{it3}}, Q_{i1}) P_{\mathrm{it3}} - P_{\mathrm{ot1}}, \quad (26\mathrm{c})$$

$$\frac{\mathrm{d}P_{\mathrm{ot1}}}{\mathrm{d}t} = B_{\mathrm{ot3}} \frac{P_{\mathrm{ot1}}}{Z_{\mathrm{In}}} \left[Q_{\mathrm{ot3}} - Q_{\mathrm{b2}}\right], \quad (26\mathrm{d}^*)$$

where

$$L_{\mathrm{a\_b1}} = D_{\mathrm{b1}} \frac{N \cdot (P_{\mathrm{it2}})^{1-G}}{\sum_{i=1}^{N} [\varepsilon_i(Q_{i1})]^{\sigma-G}}; \; D_{\mathrm{b1}} = \frac{K_L l_{\mathrm{it3}}}{f_{\mathrm{it3}}} \cdot \frac{(P_{\mathrm{In}})^G}{Z_{\mathrm{In}} R T_{\mathrm{In}}};$$

$$L_{\mathrm{a\_b2}} = D_{\mathrm{b2}} (P_{\mathrm{ot1}})^{1-G}; \; D_{\mathrm{b2}} = \frac{K_L l_{\mathrm{ot3}}}{f_{\mathrm{ot3}}} \cdot \frac{(P_{\mathrm{In}})^G}{Z_{\mathrm{In}} R T_{\mathrm{In}}};$$

$$B_{\mathrm{it3}} = \frac{\gamma}{K_C l_{\mathrm{it3}} f_{\mathrm{it3}}}; \; B_{\mathrm{ot3}} = \frac{\gamma}{K_C l_{\mathrm{ot3}} f_{\mathrm{ot3}}}.$$

Let us deduct Eq.(23a) from Eq.(26d*), and we obtain:

$$Q_{\mathrm{ot3}} = \frac{B_1}{B_{\mathrm{ot3}}} \left[Q_{\mathrm{it1}} \cdot [\xi_{t1}(Q_{\mathrm{it1}}, P_{\mathrm{ot1}})]^{-\frac{1}{\gamma}} + Q_{\mathrm{b2}} - \sum_{i=1}^{N} Q_{i1}\right] + Q_{\mathrm{b2}}. \quad (26\mathrm{d})$$

Accordingly, the behavior of the pipeline system shown in Fig.3 can be described by the set of (N+7) ordinary non-linear differential equations. These equations are written for (N+7) phase variables: the volumetric flow rate of gas Q_{it1} at the inlet of throttle 1; the gas pressure P_{ot1} at the outlet of throttle 1; the volumetric flow rates of gas at the inlet of all of the CFS Q_{i1} (i=1,…,N); the gas pressure P_{it2} at the inlet of throttle 2; the volumetric flow rate of gas Q_{ot2} at the outlet of throttle 2; the volumetric flow rate of gas Q_{b1} at the inlet of the anti-surge recirculation gas pipeline; the gas pressure P_{it3} at the inlet of throttle 3; the volumetric flow rate of gas Q_{b2} at the outlet of the anti-surge recirculation gas pipeline. The resulting set of ordinary differential equations (SODE) looks as follows [Eqs.(12), (16), (17), (23)~(26)]:

$$\frac{\mathrm{d}Q_{\mathrm{it1}}}{\mathrm{d}t} = \frac{1}{L_{\mathrm{a\_it1}}} \left[P_{\mathrm{In}} - \frac{P_{\mathrm{ot1}}}{\xi_{t1}(Q_{\mathrm{it1}}, P_{\mathrm{ot1}})}\right], \quad (27\mathrm{a})$$

$$\frac{\mathrm{d}P_{\mathrm{ot1}}}{\mathrm{d}t} = B_1 \frac{P_{\mathrm{ot1}}}{Z_{\mathrm{In}}} \left[Q_{\mathrm{it1}} \cdot [\xi_{t1}(Q_{\mathrm{it1}}, P_{\mathrm{ot1}})]^{-\frac{1}{\gamma}} + Q_{\mathrm{b2}} - \sum_{i=1}^{N} Q_{i1}\right], \quad (27\mathrm{b})$$

$$\frac{\mathrm{d}Q_{i1}}{\mathrm{d}t} = \frac{1}{L_{\mathrm{a}i}} \cdot \left[\varepsilon_i(Q_{i1}) \cdot P_{\mathrm{ot1}} - P_{\mathrm{it2}}\right], \quad i = 1, \ldots, N, \quad (27\mathrm{c})$$

$$\frac{\mathrm{d}P_{\mathrm{it2}}}{\mathrm{d}t} = B_2 \frac{P_{\mathrm{it2}}}{Z_{\mathrm{In}}} \left[\sum_{i=1}^{N} \left\{Q_{i1} \cdot \left[\varepsilon_i(Q_{i1})\right]^{\sigma-1}\right\} - Q_{\mathrm{ot2}} \cdot \left[\xi_{t2}(Q_{\mathrm{ot2}}, P_{\mathrm{it2}}, Q_{i1})\right]^{\frac{1}{\gamma}} - Q_{\mathrm{b1}}\right], \quad (27\mathrm{d})$$

$$\frac{\mathrm{d}Q_{\mathrm{ot2}}}{\mathrm{d}t} = \frac{1}{L_{\mathrm{a\_ot2}}} \left[\xi_{t2}(Q_{\mathrm{ot2}}, P_{\mathrm{it2}}, Q_{i1}) \cdot P_{\mathrm{it2}} - P_{\mathrm{Out}}\right], \quad (27\mathrm{e})$$

$$\frac{\mathrm{d}Q_{\mathrm{b1}}}{\mathrm{d}t} = \frac{1}{L_{\mathrm{a\_b1}}} \cdot \left[P_{\mathrm{it2}} - P_{\mathrm{it3}}\right], \quad (27\mathrm{f})$$

$$\frac{\mathrm{d}P_{\mathrm{it3}}}{\mathrm{d}t} = B_{\mathrm{it3}} \frac{P_{\mathrm{it3}}}{Z_{\mathrm{In}}} \left[Q_{\mathrm{b1}} - Q_{\mathrm{ot3}} \cdot \left[\xi_{t3}(Q_{\mathrm{ot3}}, P_{\mathrm{it3}}, Q_{i1})\right]^{1/\gamma}\right], \quad (27\mathrm{g})$$

$$\frac{\mathrm{d}Q_{\mathrm{b2}}}{\mathrm{d}t} = \frac{\xi_{t3}(Q_{\mathrm{ot3}}, P_{\mathrm{it3}}, Q_{i1}) \cdot P_{\mathrm{it3}} - P_{\mathrm{ot1}}}{L_{\mathrm{a\_b2}}}, \quad (27\mathrm{h})$$

where

$$A_i = \frac{l_{i2}}{l_{i1}} \cdot \frac{f_{i1}}{f_{i2}}; \; B_j = \gamma \cdot \left[K_C \sum_{i=1}^{N} (l_{ij} f_{ij})\right]^{-1}, \quad j = 1, 2;$$

$$G = (\gamma - 1)/\gamma; \; \sigma = (\eta - 1)/\eta; \; B_{\mathrm{it3}} = \gamma/(K_C l_{\mathrm{it3}} f_{\mathrm{it3}});$$

$$B_{\mathrm{ot3}} = \gamma/(K_C l_{\mathrm{ot3}} f_{\mathrm{ot3}}); \; D_{\mathrm{In}} = \frac{K_L l_{\mathrm{In}}}{f_{\mathrm{In}}} \cdot \frac{(P_{\mathrm{In}})^G}{Z_{\mathrm{In}} R T_{\mathrm{In}}};$$

$$D_{\mathrm{Out}} = \frac{K_L l_{\mathrm{Out}}}{f_{\mathrm{Out}}} \cdot \frac{(P_{\mathrm{In}})^G}{Z_{\mathrm{In}} R T_{\mathrm{In}}}; \; D_{\mathrm{b1}} = \frac{K_L l_{\mathrm{it3}}}{f_{\mathrm{it3}}} \cdot \frac{(P_{\mathrm{In}})^G}{Z_{\mathrm{In}} R T_{\mathrm{In}}};$$

$$D_{\mathrm{b2}} = \frac{K_L l_{\mathrm{ot3}}}{f_{\mathrm{ot3}}} \cdot \frac{(P_{\mathrm{In}})^G}{Z_{\mathrm{In}} R T_{\mathrm{In}}}; P_{i1} = \frac{A_i P_{\mathrm{ot1}} + P_{\mathrm{it2}}}{A_i + \varepsilon_i(Q_{i1})}, \quad i = 1, \ldots, N,$$

$$L_{\mathrm{a\_it1}} = D_{\mathrm{In}} \cdot \left(\frac{\xi_{t1}(Q_{\mathrm{it1}}, P_{\mathrm{ot1}})}{P_{\mathrm{ot1}}}\right)^{G-1};$$

$$L_{\mathrm{a\_ot2}} = D_{\mathrm{Out}} \frac{N \cdot [\xi_{t2}(Q_{\mathrm{ot2}}, P_{\mathrm{it2}}, Q_{i1}) \cdot P_{\mathrm{it2}}]^{1-G}}{\sum_{i=1}^{N} [\varepsilon_i(Q_{i1})]^{\sigma-G}};$$

$$L_{\mathrm{a\_b1}} = D_{\mathrm{b1}} \frac{N \cdot (P_{\mathrm{it2}})^{1-G}}{\sum_{i=1}^{N} [\varepsilon_i(Q_{i1})]^{\sigma-G}}; \ L_{\mathrm{a\_b2}} = D_{\mathrm{b2}} \cdot (P_{\mathrm{ot1}})^{1-G};$$

$$L_{\mathrm{a}i} = K_L \frac{l_{i1}}{f_{i1}} [\varepsilon_i(Q_{i1}) + A_i] \frac{(P_{\mathrm{In}})^G}{Z_{\mathrm{In}} R T_{\mathrm{In}}} \cdot (P_{i1})^{1-G};$$

$$Q_{\mathrm{ot3}} = \frac{B_1}{B_{\mathrm{ot3}}} \left[Q_{\mathrm{it1}} \cdot [\xi_{t1}(Q_{\mathrm{it1}}, P_{\mathrm{ot1}})]^{-\frac{1}{\gamma}} + Q_{\mathrm{b2}} - \sum_{i=1}^{N} Q_{i1} \right] + Q_{\mathrm{b2}}.$$

It should be noted here that, provided the equation (k_{t3}=0) is admitted, ASRGP is excluded from analysis of the pipeline system. Such an approach fully corresponds to the concept of projecting the worse scenarios of emergency situation development, because an additional damping of the gas volume in the system is excluded in this case. Therefore, Eqs.(27f)~(27h) are excluded from the SODE Eq.(27), and the gas flow parameters Q_{b1} and Q_{b2} are put to zero. The equations (k_{t1}=0) and (k_{t2}=0) are not admitted in surge simulation (the above).

Characteristics of the throttles are determined as a result of solution of the non-linear algebraic Eq.(22). To make analytical calculation of the parameters ξ possible, we can use the simplified method offered earlier. To determine the dependence $\xi_{t1}(Q_{\mathrm{it1}}, P_{\mathrm{ot1}})$, let us apply Eq.(9b). Taking into account T_{it1}= $T_{\mathrm{In}} \cdot (P_{\mathrm{it1}}/P_{\mathrm{In}})^G$ [Eq.(20)], this equation can be rewritten as follows:

$$\frac{Z_{\mathrm{In}} R T_{\mathrm{In}}}{G P_{\mathrm{In}}^G} \cdot [P_{\mathrm{ot1}}^G - P_{\mathrm{it1}}^G] + \frac{\sqrt{\pi} \lambda_{t1} l_{t1}}{4 k_{t1}^{2.5} f_{\mathrm{In}}^{2.5}} Q_{\mathrm{it1}}^2 = 0,$$

$$P_{\mathrm{it1}} = \left(P_{\mathrm{ot1}}^G + \frac{\sqrt{\pi} G \lambda_{t1} l_{t1} P_{\mathrm{In}}^G}{4 k_{t1}^{2.5} f_{\mathrm{In}}^{2.5} Z_{\mathrm{In}} R T_{\mathrm{In}}} Q_{\mathrm{it1}}^2 \right)^{1/G},$$

$$\xi_{t1}(Q_{\mathrm{it1}}, P_{\mathrm{ot1}}) = P_{\mathrm{ot1}} \left(P_{\mathrm{ot1}}^G + \frac{\sqrt{\pi} G \lambda_{t1} l_{t1} P_{\mathrm{In}}^G}{4 k_{t1}^{2.5} f_{\mathrm{In}}^{2.5} Z_{\mathrm{In}} R T_{\mathrm{In}}} Q_{\mathrm{it1}}^2 \right)^{-1/G}. \quad (28a)$$

To determine $\xi_{t2}(Q_{\mathrm{ot2}}, P_{\mathrm{it1}}, Q_{i1})$ and $\xi_{t3}(Q_{\mathrm{ot3}}, P_{\mathrm{it3}}, Q_{i1})$, we use Eq.(11c) ($Q_t$=$Q_{\mathrm{out\_t}}$) subject to the formulas T_{it2}=$T_{\mathrm{In}} \cdot (P_{\mathrm{it2}}/P_{\mathrm{In}})^G \cdot N^{-1} \cdot \sum_{i=1}^{N} [\varepsilon_i(Q_{i1})]^{\sigma-G}$ and T_{it3}=$T_{\mathrm{In}} \cdot (P_{\mathrm{it3}}/P_{\mathrm{In}})^G \cdot N^{-1} \cdot \sum_{i=1}^{N} [\varepsilon_i(Q_{i1})]^{\sigma-G}$ [Eq.(20)]:

$$\xi_{t2}(Q_{\mathrm{ot2}}, P_{\mathrm{it2}}, Q_{i1}) = \left[1 - \frac{\sqrt{\pi} N G \lambda_{t2} l_{t2} P_{\mathrm{In}}^G}{4 k_{t2}^{2.5} f_{\mathrm{Out}}^{2.5} Z_{\mathrm{In}} R T_{\mathrm{In}}} Q_{\mathrm{ot2}}^2 \left(P_{\mathrm{it2}}^G \cdot \sum_{i=1}^{N} [\varepsilon_i(Q_{i1})]^{\sigma-G} \right)^{-1} \right]^{1/G}, \quad (28b)$$

$$\xi_{t3}(Q_{\mathrm{ot3}}, P_{\mathrm{it3}}, Q_{i1}) = \left[1 - \frac{\sqrt{\pi} N G \lambda_{t3} l_{t3} P_{\mathrm{In}}^G}{4 k_{t3}^{2.5} f_{\mathrm{ot3}}^{2.5} Z_{\mathrm{In}} R T_{\mathrm{In}}} Q_{\mathrm{ot3}}^2 \left(P_{\mathrm{it3}}^G \cdot \sum_{i=1}^{N} [\varepsilon_i(Q_{i1})]^{\sigma-G} \right)^{-1} \right]^{1/G}. \quad (28c)$$

The problem of simulating surge phenomena in the system of "CFS group–CP–ASRGP" in this case is reduced to solution of the Cauchy problem. To solve it, we need to integrate the SODE Eq.(28) with specified initial conditions:

$$\begin{cases} Q_{\mathrm{it1}}(0) = Q_{\mathrm{it1}}^{\mathrm{specified}}; \quad P_{\mathrm{ot1}}(0) = P_{\mathrm{ot1}}^{\mathrm{specified}}; \\ Q_{i1}(0) = Q_{i1}^{\mathrm{specified}}, \quad i = 1, \ldots, N; \\ P_{\mathrm{it2}}(0) = P_{\mathrm{it2}}^{\mathrm{specified}}; \quad Q_{\mathrm{ot2}}(0) = Q_{\mathrm{ot2}}^{\mathrm{specified}}; \\ Q_{\mathrm{b1}}(0) = Q_{\mathrm{b1}}^{\mathrm{specified}}; \quad P_{\mathrm{it3}}(0) = P_{\mathrm{it3}}^{\mathrm{specified}}; \\ Q_{\mathrm{b2}}(0) = Q_{\mathrm{b2}}^{\mathrm{specified}}. \end{cases} \quad (29)$$

The initial conditions are specified using values of parameters of the set steady mode of gas transmission through the simulated pipeline system (Fig.3). The determination of parameters of steady modes of CS operation is described in detail in (Seleznev *et al.*, 2005).

To solve the SODE Eq.(27), in this case we use the widely known explicit Runge-Kutta method based on the Dorman and Prince formulas (Kahaner *et al.*, 1989; Hamming, 1962). An example of simulation of a surge in the pipeline system shown in Fig.3 is given in Fig.4. If, as a result of solution of the SODE Eq.(27), the target functions prove to be periodical, then it is safe to say that a surge has occurred in the simulated system of "CFS-group–GP–ASRGP". If the target functions tend to a certain limit value on the phase plane, it is concluded that the transient process, which has arisen in the simulated pipeline, is attenuating. The system is returning to stable functioning. What is meant here is that the target functions tend to a certain limit value provided that the numerical experiment in time is continued. After that, the admissibility of the selected operation for the given time step is recognized. Otherwise, the selected operation is considered inadmissible, and determination of another control actions is resumed.

EXAMPLES OF NUMERICAL FORECASTING OF SURGE PHENOMENA

The next examples were prepared by A.S. Komissarov under a scientific management of the given paper's authors. Under consideration, there is a model pipeline system, which includes one operating supercharger. Key parameters of its connecting pipelines are specified in Fig.4.

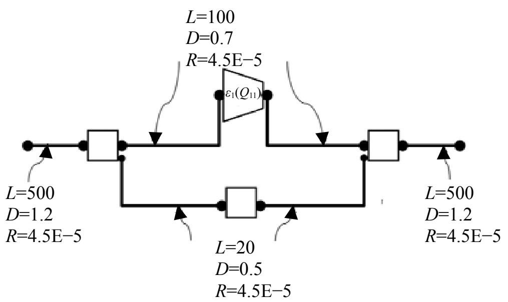


Fig.4 Key parameters of pipelines of the model system (*L*=length (m), *D*=diameter (m), *R*=roughness (m))

Roughness values of connecting pipelines presented in Fig.4 are necessary for evaluation of an initial condition (an initial mode of gas transmission) of the model system.

For an efficient friction factor we shall use a dependence $\lambda=\zeta_{\text{total}}\cdot D/l$ (Ziegler, 1998), where D is a diameter of a pipeline containing a model throttle (valve); l is the model throttle (valve) length (l=1 m). For the description of the function ζ_{total} a formula is used:

$$\zeta_{\text{total}}(k_{\text{throttle}}) = 783.989536 \cdot \left(\frac{1}{20000}\right)^{k_{\text{throttle}}}. \quad (30)$$

Eq.(30) approximates a tabulated function $\zeta_{\text{total}}(k_{\text{throttle}})$ (Table 1) for a cylindrical valve.

For a supercharger in the model pipeline system, let us take the centrifugal supercharger CFS-650-22-2 (Seleznev *et al*., 2005). Key parameters, which are necessary for the simulation of the given CFS, are shown in Tables 2 and 3.

Table 1 Tabulated function $\zeta_{\text{total}}(k_{\text{throttle}})$ for a cylindrical valve

k_{throttle}	ζ_{total}
0.11	275
0.19	95.3
0.35	20.7
0.52	6.15
0.69	1.84
0.85	0.31
0.93	0.05

Table 2 Key parameters for the simulation of the given CFS

Parameter	Value
Gas constant (N·m/(kg·K))	518
Temperature of gas at the input (*K*)	288
Compressibility factor	0.888
Nominal shaft speed (min^{-1})	3700

Table 3 Technological restrictions for the simulation of the given CFS

Technological restrictions	Value
Minimal admissible reduced volumetric flow rate (m^3/min)	450
Maximal admissible reduced volumetric flow rate (m^3/min)	810
Minimum allowed speed of a supercharger shaft (min^{-1})	2590
Maximum allowed speed of a supercharger shaft (min^{-1})	4070

An example of a unstable state of model pipeline system and ways of leading the system out of the given state is proposed.

Initial conditions for the SODE solution look as follows:

$Q_{\text{it1}}(0)$=9.98799 m^3/s; $P_{\text{ot1}}(0)$=5060142.94 Pa;
$Q_{i1}(0)$=10.03144 m^3/s; $P_{\text{it2}}(0)$=7406926.63 Pa;
$Q_{\text{ot2}}(0)$=7.84571 m^3/s; $Q_{\text{b1}}(0)$=0;
$P_{\text{it3}}(0)$=7406926.63 Pa; $Q_{\text{b2}}(0)$=0;
$P_{\text{ot3}}(0)$=5060142.94 Pa.

Boundary conditions for this SODE are presented by further values:

P_{In}=5066250.0 Pa; T_{In}=288.15 K; P_{Out}=7402124.4 Pa.

The CFS shaft operating speed is n=3700 min^{-1}. The cross-section reduction factors for model throttles 1 and 3 can be determined like this: $k_{t1}(t)=1$, $k_{t3}(t)=0$ for $t\in[0, \infty)$. A process of throttling (for a model throttle 2) is established by a law in which value k_{t2} decreases from 0.574 up to 0.409 in a time interval from 60 to 240 s.

The calculation results are shown in Fig.5.

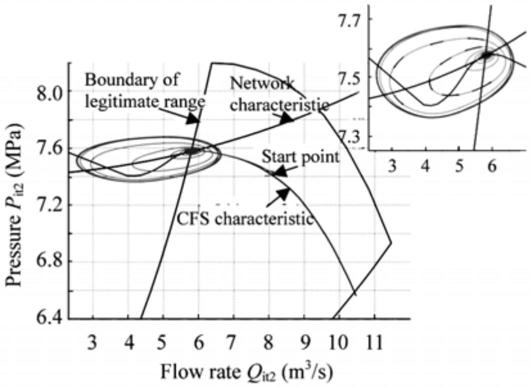


Fig.5 The calculation results on a phase plane "pressure of gas-flow rate of gas" in a point before throttle 2

From Fig.5 it is visible that at throttling the CFS operating point shifts from equilibrium state up along the head-capacity curve beyond legitimate range. The results of the solution in this example show that by the moment when the operating point reaches an ascending segment of the performance curve, the model pipeline system gets unstabilized, and the trajectory of CFS operating point on the phase plane transforms in a certain limit cycle. Thus, continuous oscillations of fluid dynamics parameters take place in the model system (Fig.4), which corresponds to the moment of surge occurrence. As an example in this case, oscillations frequency is approximately equal to 1.8 Hz. It can be explained by high-cube transmitted gas (38.5 m^3) in the charge pipeline.

To lead out of surge we shall perform turning on the anti-surge recirculation gas pipeline by opening model throttle 3 on 28% linearly for 5 s, starting with 360 s of calculation.

The obtained solutions of the SODE are given in Figs.6~8. The results of numerical experiments show that head, created by a supercharger at gas recirculation, decreases as the gas volumetric flow rate increases, in consequence of which the operating point comes back to a legitimate range on a descending segment of the performance curve of the CFS. Thus,

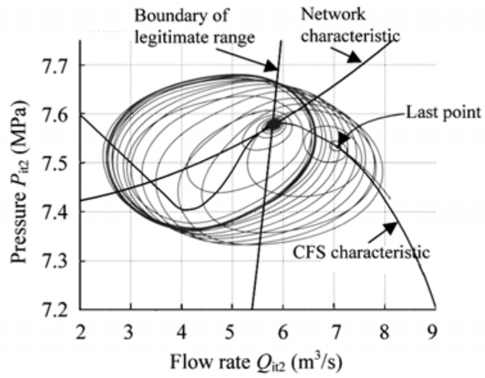


Fig.6 The calculation results on a phase plane "pressure of gas-flow rate of gas" in a point before model throttle 2

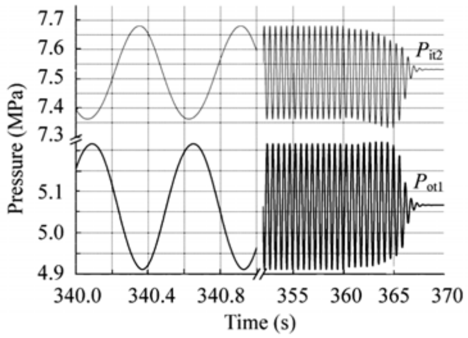


Fig.7 The calculation results: pressure of gas in computational points depending on time

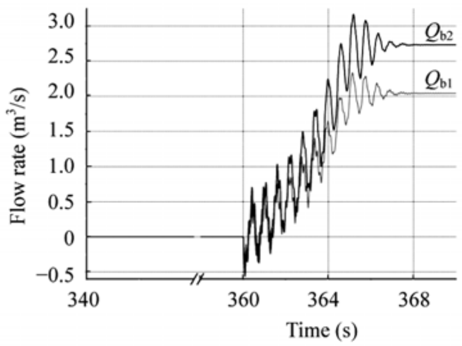


Fig.8 The calculation results: flow rate of gas in computational points of the anti-surge recirculation gas pipeline depending on time

the CFS operating point final position corresponds to equilibrium state of the model pipeline system (pressure and flow rate oscillations of the transmitted gas attenuate and stop). In Figs.6~8 the transient process related to the beginning of anti-surge recirculation gas pipeline performance is submitted.

In Fig.5 we can observe an increase of the gas flow rate throughout model throttle 3 at the very point of the throttle opening, and it proves that the obtained solution corresponds to the essence of the going on physical process.

CONCLUSION

This paper offers a numerical method of forecasting stable operation of GCU CFS installed on pipeline systems of compressor shops servicing gas trunklines. The method is designed for working out guidelines for optimal operation of gas trunklines, as well as for adjustment, testing and efficiency evaluation of anti-surge hardware and software systems. The practical application of this method does not require a profound knowledge of computational mechanics and can use all-purpose programming and mathematical software available to employees of gas companies.

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