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Analysis of 3D curve expansion of conical cam with oscillating tapered roller follower*

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Abstract: This paper focuses on the analysis of running conditions and machining processes of conical cam with oscillating follower. We point out the common errors existing in the design and machining of the widely used plane expansion method of conical cam trough-out line. We show that the motion can be divided into two parts, i.e. the oscillating motion of oscillating bar and the rotary motion of oscillating bar relative to the conical cam. By increasing the rotary motion of oscillating bar, the motion path of tapered roller on oscillating bar (i.e. contour surface of conical cam) can be expanded on the cylinder. Based on these analyses, we present a creative and effective designing and machining method for 3D curve expansion of conical cam with oscillating follower.

Key words: Oscillating follower, Conical cam, Tapered roller plane expansion, 3D curve expansion
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INTRODUCTION

With the development of science and technology, many kinds of devices with cam have been used widely in various mechanical equipments. One important part of those devices, for example weaving machine, is to meet different demands of respective applications by changing rotary motion or reciprocating linear motion into complicated motion with the curve-out line (or called trough-out line) of cam.

The shapes of cams can be roughly divided into two categories, planar cam and spatial cam. The designing and machining of planar cam are simple because of its planar curve-out line, whereas the outline of spatial cam is fairly complicated. Therefore the design and machining of oscillating follower conical cam can be markedly difficult. Nevertheless there is significant demand to replace cylindrical roller with tapered roller, since this not only would improve the characteristic of frictions between roller and cam, but is also convenient for many other applications e.g. the adjustment of clearance between roller and the trough of the trough-out line cam.

Lee and Lee (2007) proposed an interference-free toolpath generating method for five-axis machining of a spatial cam. Yan and Chen (1994) derived equations for the surface geometry of roller gear cams with cylindrical rollers. Yin *et al*.(2002) used the Web-based remote design system to design spatial cam mechanisms based on mathematical models. Many researchers (Yan and Chen, 1994; Lin and Tsai, 1996; Tsay and Lin, 1996) integrated the activities for design and manufacturing of the variable pitch lead screw and roller gear cam. In the gouging study of tool orientation, Choi *et al*.(1993) proposed a method to generate optimal cutter location data for free-form surface. The optimization problem is for-

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mulated as a 2D constrained minimization problem. There are three constraints: joint limits, gouging and collisions on cutter location. Lee (1998) presented a method for finding the admissible tool orientation by considering both local gouging and rear gouging. Ge *et al.*(2006) deduced the contour surface equation of spatial cam based on pure rolling condition of roller on the cam contour surface. Li and Yin (2003) proposed that the contour surface of cylindrical cam could be expanded from linear surface. Grant and Soni (1999) worked out the analytic expression for cam contour surface of conical cam mechanism with oscillating tapered roller follower in 3D space by applying the theory of envelopes for one parameter family of surfaces. They also investigated the method for numerical control machining. All these studies are limited to the questions as to how to establish the parameters for 3D geometrical model. The information for design and machining is still mostly at the stage of theoretical analysis, which has serious limitation for its practical application.

By focusing on conical cam mechanism, this paper presents that the motion can be divided into two categories, i.e. the oscillating motion of the oscillating bar and the rotary motion of oscillating bar relative to conical cam. With the expansion of the rotary motion, we propose a new "3D curve expansion" model of conical cam with oscillating follower. This new method has led to a creative and effective way for design and machining for practical use.

ANALYSIS OF MACHINING METHOD FOR CONICAL CAM WITH OSCILLATING FOLLOWER

Conical cams are generally divided into two categories, i.e. cam with translating follower and cam with oscillating follower. Design and machining for trough of cam with translating follower could be made based on its expanded plane figure (Xiao *et al.*, 2005), but this technique is not applicable for design and machining for trough of conical cam with oscillating follower.

The 3D model of conical cam mechanism with oscillating follower is shown in Fig.1. As shown in this figure, the Cartesian coordinating frame is set up based on the following conditions: Oscillating axis of oscillating bar is perpendicular to and intersects with generatrix of conical cam; Oscillating axis of oscillating bar intersects with rotary axis of conical cam in different spaces; Line *MN* is the common perpendicular line of these two axes, with *M* as the point of intersection between common perpendicular and oscillating axis of oscillating bar being the origin of coordinate frames; Oscillating axis of oscillating bar is the *Z*-axis and the common perpendicular is the *Y*-axis.

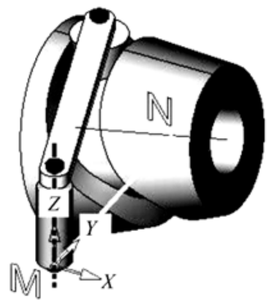

Fig.1 Conical cam mechanism with oscillating follower

Generating cutting, usually adopted in machining of conical cam trough, means that conical cam meeting the demand of design is produced in the following way: replace tapered roller i.e. follower of conical cam mechanism by cutter with same figure and dimension and then simulate the correlative motion relationship under the practical running condition of conical cam and tapered roller follower in the way of numerical control.

If design and machining were based on the expanded plane curve, it would not meet the design demands for the theoretical trough-out line of conical cam with oscillating follower and machining of generating cutting of trough. This is because the expanded plane curve is a 2D curve. Through two-step linkage numerical control machining, there can be no more than two linkage motions in the machining process: One is the rotary motion of conical cam, and the other is the cutter's motion along rotary axis of conical cam. However in practice, there should be at least three motions in generating cutting of trough of conical cam with oscillating follower (as shown in Fig.1): one is the φ-rotation of conical cam, the other two are motions along *X*-direction and *Y*-direction of cutter (i.e. tapered roller as shown in Fig.1). These two motions consist the arched oscillating motion of tapered roller follower. The following problems will certainly exist if design and machining were calculated based on plane expansion:

(1) Interference of follower. During the course of two-step linkage machining, when the tapered roller i.e. follower of conical cam mechanism was replaced by a cutter and the machining was made in the way of numerical control, the central line of the cutter would always go across the rotary axis of the conical cam. When this happens, the central line of tapered roller would deviate from the rotary axis (*MN*) of conical cam during the process in most occasions. This interference would form the black area as shown in the right of Fig.2b.

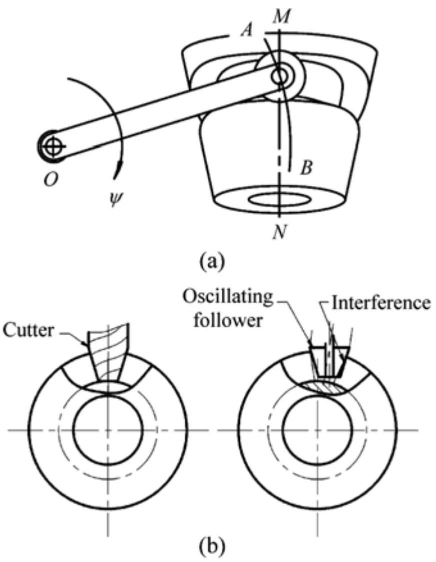

Fig.2 Interference of the follower. (a) Working condition of cutter; (b) Working condition of roller

(2) Error in motion path. During the course of machining, in the rotary motion of conical cam, the angle between cutter and the start position should be equal to the rotary angle of conical cam. But during the working process of machinery, the oscillating follower i.e. tapered roller would deviate from the rotary axis of conical cam, so the angle between oscillating follower and start position could not be equal to the rotary angle of conical cam. As shown in Fig.3, when the rotary angle of conical cam was 50°, the angle between cutter and start position was also 50°. However, when the machinery rotated to this position, as the oscillating motion of conical cam deviated from the central line, the angle between the top center of roller and start position would not be 50°. This could lead to a deviation between the intended motion path of follower and the practical motion path.

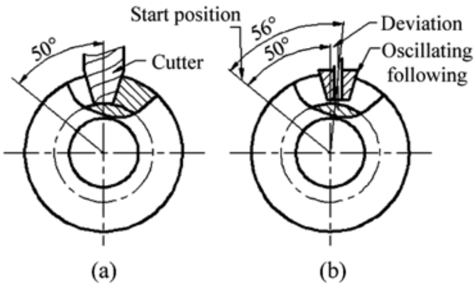

Fig.3 Deviation of the follower. (a) Cutter position; (b) Roller position

Plane expansion method cannot be applied to the design and machining of trough of conical cam with oscillating follower, as the error in follower motion path may even lead to the dead-lock phenomenon, which inevitably exists in practice. Therefore tri-axial machining method is necessary for generating required cutting.

Simply using the software of CAD/CAM is not enough to fulfill the requirement of developing design and machining method for the trough of conical cam with oscillating follower. It is necessary to analyze the practical process of the machinery. Since the motion path of the body of conical cam with oscillating follower is a segment of plane arc, and is rolling around conical cam at the same time, the theoretical trough-out line of conical cam with oscillating follower should be synthesized with these two circular paths. For the sake of design and machining, we can expand the rotary motion relative to conical cam, but keep the arched motion of conical cam body as an invariable. Based on these considerations, we propose a new design and machining method of trough of conical cam with oscillating follower. We name this new method as “3D curve expansion” method.

DESIGN OF 3D CURVE EXPANSION

Through the above analysis, we believe that we should use a method to control expansion according to the rotary motion of conical cam when we design method for managing contour surface of conical cam machinery.

Expansion of conical cam rotary motion

Oscillating tapered roller follower stays in a plane and reciprocates motion along the arc of oscillating bar. According to the above analysis, this motion can be synthesized by motion along *X*-direction and motion along *Y*-direction, and the synthesis of the two motions should result in the reciprocating oscillating motion of conical cam follower along the arc for which radius is equal to l as length of oscillating bar.

As shown in Fig.4, line *MN* is the common perpendicular line of these two axes. *M* and *N* are the perpendicular points and conical cam is rotated around its axis. During the rotary course, the distance between each point of conical cam and *N* is invariable, whereas its position is constantly changing with φ (i.e. conical cam rotary angle). When we apply the kinematical inversion widely used in the design of conical cam contour surface, tapered roller follower and oscillating bar frame should reverse every round around *N* and *YZ*-plane if we keep the conical cam motionless.

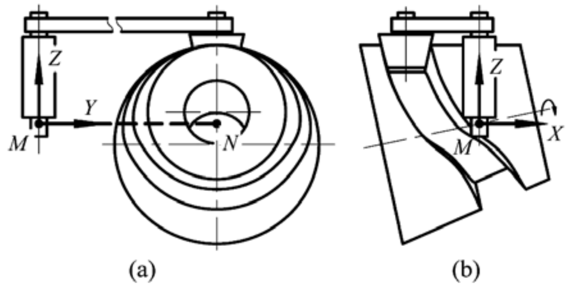

Fig.4 Conical cam mechanism with oscillating follower
(a) Front view; (b) Left view

Following the double dot line (Fig.5), the rotation of machine frame is expanded to a line and is perpendicular to the surface, on which the oscillating bar is oscillating. As shown in Fig.6, direction of the line (i.e. φ-direction of conical cam rotary angle) is paralleled to *Z*-axis in the coordinating frame. Arched motion of tapered roller can then be expanded to cylinder motion, which is the synthesis of motions in both *X*-direction and *Y*-direction, as shown in Fig.6.

Design of known oscillating-angle curve

Oscillating motion of oscillating bar is driven by the rotation of conical cam. Based on demands of motion path of machining, a general relationship can be deduced with conical cam rotary angle as parameters in the following equation:

$$\psi=f(\varphi), \tag{1}$$

where φ is rotary angle of conical cam, degree; ψ is rotary angle of oscillating bar, degree.

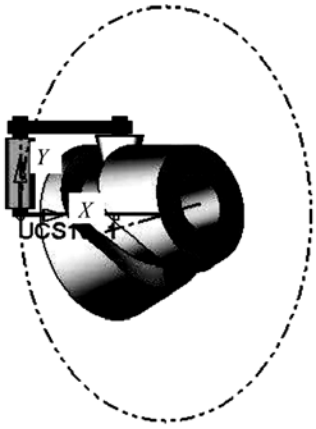

Fig.5 Relative motion path of oscillating bar

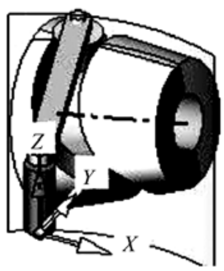

Fig.6 Cylinder expansion

Oscillating angle curve (subsection curve) figure is drawn based on Eq.(1). Supposing that the start angle of oscillating bar rotary angle is the middle of two ultimate positions, and is also paralleled to *Y*-direction as shown in Fig.7 (if oscillating angle is between ±20°), motion relationship curve with rotary angle of conical cam and oscillating angle of oscillating bar can be established.

The (φ, ψ) value of each point in the oscillating angle curve can find a correlative point on the conical surface, and value (*X*, *Y*, *Z*) can be established as follows:

$$\begin{cases} X = l \times \sin\psi, \\ Y = l \times \cos\psi, \\ Z = \varphi. \end{cases} \tag{2}$$

Besides, all the correlative points with the (φ, ψ) values in the above curve could be drawn on the cylinder surface by applying CAD, namely to draw a curve (3D curve). Therefore we can obtain a figure of "3D curve expansion".

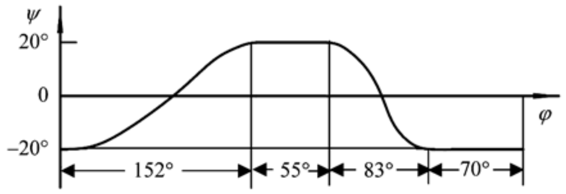

Fig.7 Oscillating-angle curve

Using curve pasting method of CAD, we can paste oscillating angle curve as shown in Fig.7 on the conical surface, magnify ψ-axis at proportion of $2\pi l/360$, then the distance between each curve point and φ-axis is equal to the arc length of tapered roller oscillating relative to ψ-value of the point on the oscillating bar. We can also change Fig.7 into plane curve for ψ-value expanded based on arc length, and paste the new curve on the cylinder whose radius is equal to l. φ-axis is along the direction of cylinder generatrix and is kept as an invariable. As shown in Fig.8, the original planar curve can now be changed into the desired 3D curve (e.g. 3D curve expanded from conical cam surface). Accordingly, value for coordinate-Z of each point in 3D curve is equal to the degree of conical cam rotary angle; and values for coordinate-X and coordinate-Y are respectively equal to the coordinate-values of arched motion of oscillating tapered roller follower along X- and Y-directions.

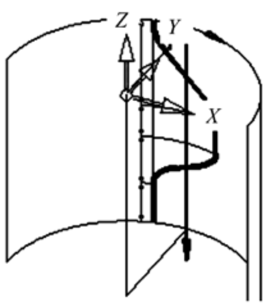

Fig.8 3D curve expansion

MACHINING OF 3D CURVE EXPANSION

Generating cutting is also adopted in machining of trough of conical cam with oscillating follower. The practical method for generating cutting using 3D curve expansion method has been thoroughly analyzed as follows.

Uniform tri-axial milling tool path is adopted in programming, first by moving and rotating the obtained 3D curve. As shown in Fig.9, we will move the 3D curve for a (assuming that a is the distance between rotary axis of oscillating bar and rotary axis of conical cam), then rotate the expanding direction (direction of Z-axis in designed coordinate frame) to the direction of Y-axis in coordinate frame during machining.

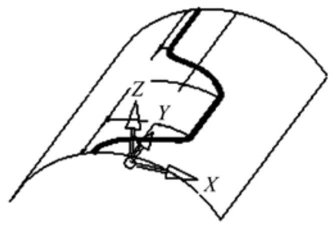

Fig.9 Practical CAM programming

When applying CAM software, numerical control programming along 3D curve can be carried out, namely producing conical cam trough that meets the above demands through using generating cutting. However, during the course of programming, convex mills with tiny diameter, even ∅0 milling cutters should be selected. One should not use end mills according to the practical machining, since they may lead to deviation from the cutter route. As shown in Fig.10, if ∅20 end mill according to the practical machining was selected in programming, a deviation from the cutter route would happen and the actual cutter route should not be superimposed to the 3D curve.

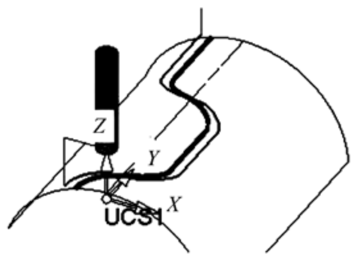

Fig.10 Deviation in tool path

After bringing out tool path by applying CAM software and carrying out post-process, one should replace Y in the program with A, Z in the program with Y, and edit manually for additional tool feed and withdrawal. After these steps, a 4-axes numerical control machining program for conical cam trough is obtained. If the diameter of tool is shorter than that of conical roller, method of trochoid milling for programming should be used. Details of this method can be found in Details of this method can be found in (Wu *et al*., 2006).

CONCLUSION

Many literature (Yan and Chen, 1994; Lee and Lee, 2007) have presented the problems concerning design and machining of trough of conical cam with oscillating follower. While they carried out some specific analyses and might even give formula for calculating coordinates, a practical method for design and machining remains to be established. Based on the analysis and research of practical running condition and machining process of conical cam with oscillating follower, this paper presented a new design and machining method for trough of conical cam with oscillating follower—the 3D curve expansion method. This new method has practical value and will improve our design and machining using oscillating follower conical CAM.

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