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Research on cubic polynomial acceleration and deceleration control model for high speed NC machining*

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Abstract: To satisfy the need of high speed NC (numerical control) machining, an acceleration and deceleration (acc/dec) control model is proposed, and the speed curve is also constructed by the cubic polynomial. The proposed control model provides continuity of acceleration, which avoids the intense vibration in high speed NC machining. Based on the discrete characteristic of the data sampling interpolation, the acc/dec control discrete mathematical model is also set up and the discrete expression of the theoretical deceleration length is obtained furthermore. Aiming at the question of hardly predetermining the deceleration point in acc/dec control before interpolation, the adaptive acc/dec control algorithm is deduced from the expressions of the theoretical deceleration length. The experimental result proves that the acc/dec control model has the characteristic of easy implementation, stable movement and low impact. The model has been applied in multi-axes high speed micro fabrication machining successfully.

Key words: High speed NC machining, Acceleration and deceleration (acc/dec) control model, Cubic speed curve, Discrete mathematical model, Adaptive acceleration and deceleration control algorithm
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INTRODUCTION

NC (numerical control) machining is now developing towards high speed and high efficiency. In high speed machining, each motion axis must accelerate into moving state and realize precise stop in few seconds. So, researching on an efficient acceleration and deceleration (acc/dec) control method to meet the demand of high speed machining is one of the critical problems in modern high performance NC system.

The commonly used methods in most domestic economical CNC (computer numerical control) system are linear acc/dec mode and exponential acc/dec mode. But vibration is easily caused by discontinuity of acceleration, which affects machining quality and equipment life (Hu *et al.*, 1999; Zhang, 2002). To decrease the vibration, the *s*-curve acc/dec motion planning method is adopted in advanced CNC system. The acceleration (or deceleration) stage in the *s*-curve acc/dec motion planning method is composed of increasing acceleration phase, constant acceleration phase and decreasing acceleration phase (or increasing deceleration phase, constant deceleration phase and decreasing deceleration phase). Through graded control of acceleration in each stage, machining feedrate can be changed smoothly. However, the algorithm is too complex (Kaan and Yusuf, 2001; Nam and Yang, 2004). The trigonometric function acc/dec method is more flexible, but the algorithm is also computation extensive and more complex, which is relatively difficult to satisfy real-time requirement (Guo and Li, 2003). The method selecting polynomial functions can generate so many kinds of acc/dec characteristics and, furthermore, can make the characteristics of deceleration be independent from those of acceleration. In order to achieve high-performance motion control, the motion profiles must be matched

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to the system limits such as the maximum acceleration and the maximum velocity. If position trajectories of which the velocity profiles are smooth are generated by the method selecting polynomial functions, it requires a lot of computations (Inaba and Sakakibara, 1985; Park, 1996). The digital convolution method is much more efficient than the method selecting polynomial functions and is easily implemented by hardware. But, in the velocity profiles generated by the method, the acceleration interval is always the same as the deceleration interval and the characteristics of the deceleration are dependent on those of the acceleration (Khalsa and Mahoney, 1990; Chen and Lee, 1998). A simple coefficients stored method for generating velocity profiles is proposed. According to the desired characteristics of acc/dec, each set of coefficients is calculated and stored. Given a moving distance and acc/dec intervals, a velocity profile having the desired characteristics of acc/dec can be efficiently generated by using these coefficients. But for long and short distances, the same acc/dec intervals are selected; the efficient velocity profile cannot be calculated for the varied distance movements (Jeon and Ha, 2000).

This paper is organized as follows. Section 2 proposes a cubic polynomial acc/dec control model for high speed NC machining. Based on the discrete characteristic of the data sampling interpolation, the cubic polynomial acc/dec control discrete mathematical model is set up in Section 3. The adaptive acc/dec control algorithm for predetermining the deceleration point of arbitrary route segment is deduced in Section 4. Experimental results are presented in Section 5, and conclusions are summarized in Section 6.

CUBIC POLYNOMIAL ACC/DEC CONTROL MODEL

To meet the need of high speed machining, the feedrate must be changed smoothly and the acceleration must be continuous. The boundary conditions are: (1) the displacement at the beginning time is 0; (2) both the beginning speed and the end speed are the same as required; (3) the accelerations both at the beginning time and the end time are 0.

The acc/dec feedrate curve function is constructed by the cubic polynomial,

$$V(u)=(a_1+2a_2u+3a_3u^2+4a_4u^3)/t_{\rm m}. \tag{1}$$

It is assumed that $t_{\rm m}$ is the accelerating or decelerating duration time, which is taken as synchronized motion axes to accelerate or decelerate from the beginning speed to the end speed, $u=t/t_{\rm m}$, $t\in[0,t_{\rm m}]$.

The other kinetic characteristic curves of acceleration and jerk can be obtained by differentiating the feedrate curve,

$$\begin{cases} A(u)=(2a_2+6a_3u+12a_4u^2)/t_{\rm m}^2, \\ J(u)=(6a_3+24a_4u)/t_{\rm m}^3. \end{cases} \tag{2}$$

Again, integrating Eq.(1) with respect to time yields the displacement curve function as,

$$S(u)=a_0+a_1u+a_2u^2+a_3u^3+a_4u^4. \tag{3}$$

The boundary conditions are,

$$S(0)=0,\ V(0)=V_{\rm s},\ V(1)=V_{\rm e},\ A(0)=A(1)=0,$$

where $V_{\rm s}$ and $V_{\rm e}$ stand for the beginning speed in acc stage (or dec stage) and the end speed. Then the curvilinear functions are derived as,

$$\begin{cases} J(u)=6(V_{\rm e}-V_{\rm s})(1-2u)/t_{\rm m}^2, \\ A(u)=6(V_{\rm e}-V_{\rm s})(u-u^2)/t_{\rm m}, \\ V(u)=V_{\rm s}+3(V_{\rm e}-V_{\rm s})u^2+2(V_{\rm s}-V_{\rm e})u^3, \\ S(u)=t_{\rm m}V_{\rm s}u+(V_{\rm e}-V_{\rm s})t_{\rm m}u^3+0.5(V_{\rm s}-V_{\rm e})t_{\rm m}u^4. \end{cases} \tag{4}$$

From Eq.(4), let u=0.5, the acceleration $A(u)$ is equal to the maximum acceleration $A_{\max}$. Then $t_{\rm m}$ can be deduced as,

$$t_{\rm m}=3|V_{\rm e}-V_{\rm s}|/(2A_{\max})=nT. \tag{5}$$

n is a real number, denoting the theoretical times of the theoretical running time $t_{\rm m}$ to interpolation period T when decelerating (or accelerating) from $V_{\rm s}$ to $V_{\rm e}$.

When $t=t_{\rm m}$, the theoretical acceleration (or deceleration) length S_1 is obtained from Eq.(4) as,

$$S_1=(V_s+V_e)t_m/2=3|V_s^2-V_e^2|/(4A_{max}).$$

The relation of speed (V), acceleration (A) and jerk (J) is expressed as shown in Fig.1. From Fig.1, in the cubic polynomial acc/dec control model, the acceleration is consecutive, which avoids the occurrence of intense vibration in high speed machining. The calculations of jerk, acceleration, speed and displacement in varying speed process are simple and easy to realize, because of few four fundamental operations.

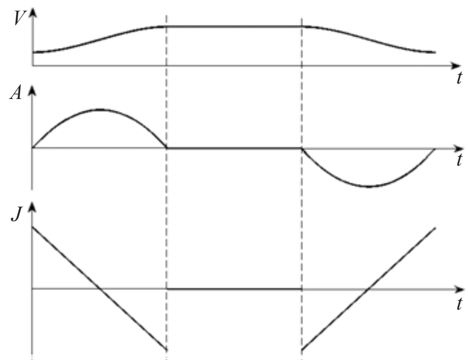

Fig.1 Speed (*V*), acceleration (*A*) and jerk (*J*) of the cubic polynomial acc/dec control model

CUBIC POLYNOMIAL ACC/DEC CONTROL DISCRETE MODEL

Characteristics of data sampling interpolation

The controlling with data sampling interpolation is a type of discrete controlling mode. The data sampling interpolation is based on approximating a curve with straight-line segments, whose lengths are proportional to the local axial planned velocities. The acc/dec control algorithm is performed on the desired feedrate command V' firstly. Then the feedrate command V (after acc/dec control algorithm) is sent to the interpolator to compute the traveling distance component for each axis in the Cartesian coordinates (ΔX, ΔY and ΔZ). The traveling distance components ΔX, ΔY and ΔZ along the X, Y and Z axes are transmitted to the motion control routine as position commands to the position control loop for the desired machining finally.

When data sampling interpolation is used in position control, it is the condition to make sure that every synchronized motion axis reaches the destination simultaneously and their movement is continuous so that each axis running time t is just integral times that of the interpolation period T, i.e. time partitioning rule. This condition can be realized by adjusting the command feedrate of each synchronized motion axis (i.e. accelerating or decelerating) (Guo *et al.*, 2003).

Through the above analyses, n should be an integer. Set N as the minimum integer not smaller than n,

$$N=ceil(n),\quad N\geq n.$$

Replacing the real number n with the integer N in Eq.(5) yields,

$$t_m=3|V_e-V_s|/(2A_{max})=NT. \tag{6}$$

Construction of discrete model

Given any curve given by the function $y=f(t)$, the interval $[t_0, t_n]$ is divided into N divisions. Picking the left endpoints to determine the heights is shown in Fig.2a; the approximate area under $y=f(t)$ on the interval $[t_0,t_n]$ is,

$$\int_{t_0}^{t_N} f(t)\mathrm{d}t\approx\sum_{i=1}^{N} f\left(t_0+(i-1)\frac{t_N-t_0}{N}\right)\frac{t_n-t_0}{N}.$$

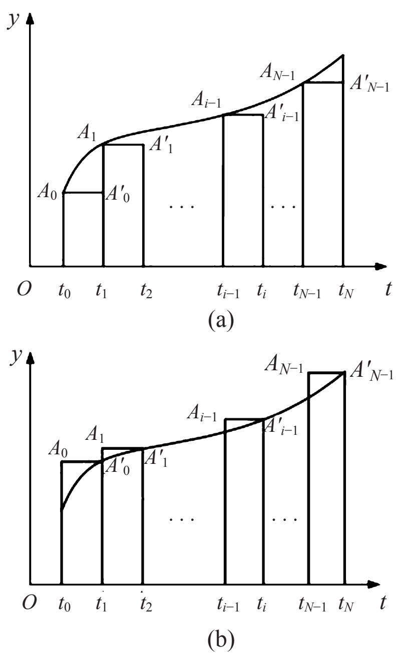

Fig.2 Implement curve of *f*(*t*). (a) Left endpoints used for the heights of rectangles; (b) Right endpoints used for the heights of rectangles

This way of approximating the area under the curve is called the left sum method.

Choosing the right endpoints to determine the heights is shown in Fig.2b; the approximate area under $y=f(t)$ on the interval $[t_0,t_n]$ is,

$$\int_{t_0}^{t_N} f(t)\mathrm{d}t \approx \sum_{i=1}^{N} f\left(t_0 + i\frac{t_N - t_0}{N}\right)\frac{t_n - t_0}{N}.$$

This way of approximating the area under the curve is called the right sum method.

Suppose that the slope of jerk is K_J and initial values of jerk, acceleration, speed and displacement are J_0, A_0, V_0 and S_0, respectively. The discrete model of jerk curve, implemented by left sum method, is,

$$J(iT) = J_0 + K_J(i-1)T, \quad 1 \le i \le N. \tag{7}$$

The discrete models of acceleration, speed and displacement curve, realized by right sum method, are,

$$\begin{cases} A(iT) = A_0 + \sum_{j=1}^{i} J(jT)T, & 1 \le i \le N+1, \\ V(iT) = V_0 + \sum_{j=1}^{i} A(jT)T, & 1 \le i \le N, \\ S(iT) = S_0 + \sum_{j=1}^{i} V(jT)T, & 1 \le i \le N. \end{cases} \tag{8}$$

To satisfy that the acceleration at the end time is 0 and considering characteristics of implementing the acceleration curve by right sum method (the height of corresponding rectangle area at time t_i is $f(t_i+1)$), the boundary conditions of the acceleration curve are,

$$A_0 = 0, \quad A((N+1)T) = 0.$$

The boundary conditions of speed and displacement curve are,

$$V_0 = V_\mathrm{s}, \quad V(NT) = V_\mathrm{e}, \quad S_0 = 0.$$

So the discrete model of the jerk curve, implemented by left sum method, is,

$$J(iT) = \frac{6(V_\mathrm{e} - V_\mathrm{s})}{(N+1)(N+2)T^2} - \frac{12(V_\mathrm{e} - V_\mathrm{s})}{N(N+1)(N+2)T^2}i, \quad 1 \le i \le N.$$

The discrete models of acceleration, speed and displacement curve, implemented by right sum method, are,

$$\begin{cases} A(iT) = \frac{6(V_\mathrm{e} - V_\mathrm{s})}{N(N+2)T}i - \frac{6(V_\mathrm{e} - V_\mathrm{s})}{N(N+1)(N+2)T}i^2, \\ \qquad 1 \le i \le N+1; \\ V(iT) = V_\mathrm{s} + \frac{(V_\mathrm{e} - V_\mathrm{s})(3N+2)}{N(N+1)(N+2)}i + \frac{3(V_\mathrm{e} - V_\mathrm{s})}{(N+1)(N+2)}i^2 \\ \qquad - \frac{2(V_\mathrm{e} - V_\mathrm{s})}{N(N+1)(N+2)}i^3, \qquad 1 \le i \le N; \\ S(iT) = \frac{(N^3 + 3N^2 - 1)V_\mathrm{s} + (2N+1)V_\mathrm{e}}{N(N+1)(N+2)}Ti \\ \qquad + \frac{(V_\mathrm{e} - V_\mathrm{s})(6N+1)}{2N(N+1)(N+2)}Ti^2 + \frac{(V_\mathrm{e} - V_\mathrm{s})(N-1)}{N(N+1)(N+2)}Ti^3 \\ \qquad - \frac{V_\mathrm{e} - V_\mathrm{s}}{2N(N+1)(N+2)}Ti^4, \qquad 1 \le i \le N. \end{cases} \tag{9}$$

When $t_\mathrm{m}=NT$, the discrete expression of the theoretical acceleration (or deceleration) length S_1 is,

$$S_1 = \frac{(N-1)V_\mathrm{s}T}{2} + \frac{(N+1)V_\mathrm{e}T}{2}; \tag{10}$$

For the acceleration curve, when $V_\mathrm{e}=F_\mathrm{max}$, the discrete expression of the theoretical acceleration length S_a is,

$$S_\mathrm{a} = \frac{(M-1)V_\mathrm{s}T}{2} + \frac{(M+1)F_\mathrm{max}T}{2};$$

For the deceleration curve, when $V_\mathrm{s}=F_\mathrm{max}$, the discrete expression of the theoretical deceleration length S_d is,

$$S_\mathrm{d} = \frac{(N-1)F_\mathrm{max}T}{2} + \frac{(N+1)V_\mathrm{e}T}{2},$$

where F_max is the maximum allowed feedrate, M is total interpolation periods in acceleration process, N is total interpolation periods in deceleration process.

When both acceleration and deceleration stages exist, the minimum length S_2 is computed as,

$$S_2 = S_a + S_d = (M+N)F_{max}T/2 + (M-1)V_sT/2 + (N+1)V_eT/2. \quad (11)$$

Through the above deductions, the cubic polynomial acc/dec control discrete model is set up and the discrete expression of theoretical deceleration length is also obtained.

ADAPTIVE ACC/DEC CONTROL ALGORITHM

Interpolation preceded by acc/dec or acc/dec preceded by interpolation is available in the interpolation routine. While the former is called the acc/dec control before interpolation, the later is called the acc/dec control after interpolation. In the acc/dec control before interpolation, the interpolation process involves performing acc/dec by applying acc/dec to the feedrate of the interpolator. The acc/dec control after interpolation causes respective delays in controlling the axes of motion. The delays in controlling the respective axes resulting from acc/dec degrade machining accuracy (Jeon, 1996). So the former mode is adopted in this paper. Aiming at the question of hardly predetermining the deceleration point in acc/dec control before interpolation, the adaptive acc/dec control algorithm is deduced.

In general, the controlling process includes three stages: the acceleration or deceleration stage, acc/dec stages, and acceleration, constant-speed and deceleration stages.

Acc (or dec) stage

If $V_s>V_e$, the route segment is the deceleration stage. From Eq.(10), the discrete expression of the maximum deceleration length S_1 can be computed. If $S_1>L$, the end feedrate V_e cannot be reached at the end point, where L is the length of the route segment. For the deceleration stage, the theoretical deceleration point should be calculated beforehand, otherwise overshoot would result. Let $S_1=L$,

$$\begin{cases} S_1 = (N-1)V_sT/2 + (N+1)V_eT/2 = L, \\ t_m = 3(V_s - V_e)/(2A_{max}) = NT. \end{cases}$$

Then, the value of N can be computed, and the beginning speed V_s is modified as,

$$V_s = \frac{2L-(N+1)V_eT}{(N-1)T}.$$

The discrete model of the deceleration curve is expressed as,

$$V(iT) = V_s + \frac{(V_e - V_s)(3N+2)}{N(N+1)(N+2)}i + \frac{3(V_e - V_s)}{(N+1)(N+2)}i^2 - \frac{2(V_e - V_s)}{N(N+1)(N+2)}i^3, \ 1 \le i \le N.$$

If $V_s<V_e$, the route segment is the acceleration stage. From Eq.(10), the discrete expression of the maximum acceleration length S_1 can be calculated. If $S_1>L$, the end speed V_e also cannot be achieved. The value of M can be calculated and the end speed V_e is modified as described above,

$$V_e = \frac{2L-(M-1)V_sT}{(M+1)T}.$$

The discrete model of the acceleration curve is expressed as,

$$V(iT) = V_s + \frac{(V_e - V_s)(3M+2)}{M(M+1)(M+2)}i + \frac{3(V_e - V_s)}{(M+1)(M+2)}i^2 - \frac{2(V_e - V_s)}{M(M+1)(M+2)}i^3, \ 1 \le i \le M.$$

Both acc/dec stages exist

When both acc/dec stages exist, the minimum length S_2 is obtained from Eq.(11). Suppose that V_{max} is the actual maximum speed of the route segment.

Combination of Eqs.(6) and (11) yields

$$\begin{cases} \dfrac{3(V_{max} - V_s)}{2A_{max}} = MT, \\ \dfrac{3(V_{max} - V_e)}{2A_{max}} = NT, \\ \dfrac{1}{2}(M+N)V_{max}T + \dfrac{1}{2}(M-1)V_sT + \dfrac{1}{2}(N+1)V_eT = L. \end{cases}$$

Then, values of M, N, V_{max} can be computed. The discrete model of the acceleration curve is expressed as,

$$V(iT) = V_s + \frac{(V_{max} - V_s)(3M+2)}{M(M+1)(M+2)} i + \frac{3(V_{max} - V_s)}{(M+1)(M+2)} i^2 - \frac{2(V_{max} - V_s)}{M(M+1)(M+2)} i^3, \quad 1 \le i \le M.$$

The discrete model of the deceleration curve is expressed as,

$$V(iT) = V_{max} + \frac{(V_e - V_{max})(3N+2)}{N(N+1)(N+2)} i + \frac{3(V_e - V_{max})}{(N+1)(N+2)} i^2 - \frac{2(V_e - V_{max})}{N(N+1)(N+2)} i^3, \quad 1 \le i \le N.$$

Acc, constant-speed and dec stages all exist

If $S_2<L$, acceleration, constant-speed and deceleration stages all exist. The actual maximum speed of the route segment is the maximum allowed feedrate F_{max}. The discrete model of the acceleration curve is expressed as,

$$V(iT) = V_s + \frac{(F_{max} - V_s)(3M+2)}{M(M+1)(M+2)} i + \frac{3(F_{max} - V_s)}{(M+1)(M+2)} i^2 - \frac{2(F_{max} - V_s)}{M(M+1)(M+2)} i^3, \quad 1 \le i \le M.$$

The discrete model of the deceleration curve is expressed as,

$$V(iT) = F_{max} + \frac{(V_e - F_{max})(3N+2)}{N(N+1)(N+2)} i + \frac{3(V_e - F_{max})}{(N+1)(N+2)} i^2 - \frac{2(V_e - F_{max})}{N(N+1)(N+2)} i^3, \quad 1 \le i \le N.$$

The discrete model of the constant-speed curve is expressed as,

$$V(iT) = F_{max}.$$

Thus the adaptive acc/dec control algorithm is deduced. To guarantee the interpolation precision and speed in high speed NC machining, the parametric interpolation algorithm is used in this paper.

EXPERIMENTAL RESULTS

Based on the above-mentioned model, the data sampling parametric interpolator is adopted in the software to implement in the CNC experimental system, whose control system is based on the step motor system, which is a framework of the PC-NC system and similar to the synchronization control of 3-axis motion system (Fig.3).

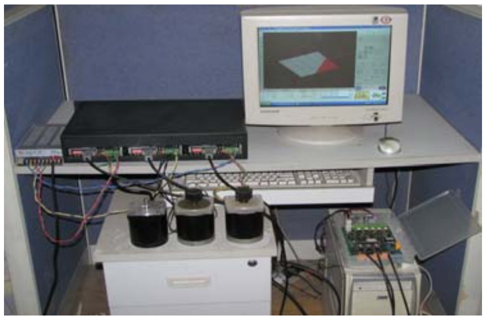

Fig.3 CNC experimental system based on step motor

In order to evaluate the effectiveness of the proposed acc/dec control model, an arbitrary 3D tool path composed of 11 broken line segments is adopted in Fig.4, which is a part of the shoe mold surface. The parameters used in experiment are as follows: The maximum jerk is set at 8000 mm/s^3, the acceleration limits are set at 600 mm/s^2 respectively, the maximum allowed feedrate is given as 50 mm/s, the interpolation period is set at 0.004 s, the impulse equivalent is given as 0.5 μm, the initial velocity and the final velocity are 2 mm/s. The experimental results are listed in Table 1. It is shown that the beginning speed and

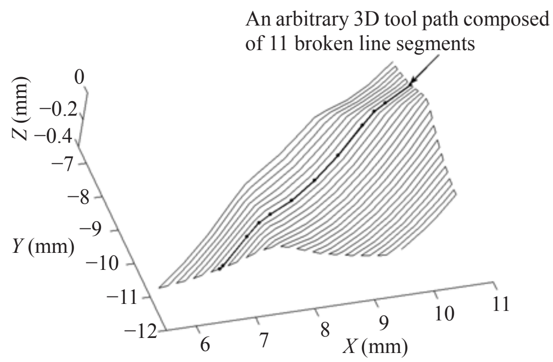

Fig.4 Shoe mold surface

Table 1 Experimental data of speed

No.	Length L (mm)	Without using adaptive acc/dec control algorithm		Using adaptive acc/dec control algorithm		Actual maximum speed V_{max} (mm/s)
		V_s (mm/s)	V_e (mm/s)	V_s (mm/s)	V_e (mm/s)	
1	0.852	2.000	50.000	2.000	28.725	28.725
2	0.375	50.000	23.115	28.725	23.115	28.725
3	0.376	23.115	50.000	23.115	23.726	23.726
4	0.752	50.000	46.310	23.726	21.800	24.469
5	0.751	46.310	48.679	21.800	30.557	30.557
6	0.750	48.679	40.145	30.557	23.984	30.557
7	0.752	40.145	50.000	23.984	23.128	23.984
8	0.376	50.000	23.724	23.128	23.724	23.724
9	0.375	23.724	50.000	23.724	23.286	23.724
10	0.752	50.000	25.107	23.286	9.554	26.679
11	0.100	25.107	2.000	9.554	2.000	9.554

the end speed can be modified adaptively according to the length of each broken line segment, and then the actual maximum speed is obtained and the deceleration point can be predetermined precisely by adopting the proposed adaptive acc/dec control algorithm. The experiment curves of displacement, speed and acceleration are shown in Fig.5. It can be seen that the cubic polynomial acc/dec control model provides continuity of acceleration, which avoided the intense vibration in high speed NC machining. The calculations of jerk, acceleration, speed and displacement in varying speed process are efficient and easy to realize, because of few four arithmetic operations.

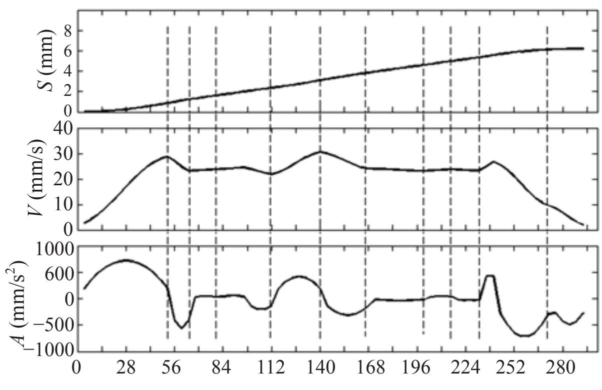

Fig.5 Results of displacement, speed and acceleration curve

CONCLUSION

In this paper, a cubic polynomial acc/dec control model for high speed NC machining is proposed. Aiming at the question of hardly predetermining the deceleration point in acc/dec control before interpolation, the adaptive acc/dec control algorithm is deduced furthermore. The experimental results prove that the proposed acc/dec control model provides continuity of acceleration, which avoids the occurrence of intense vibration in high speed machining and the deceleration point can be predicted precisely by the deduced algorithm. In the real-time interpolation process, the calculations of jerk, acceleration, speed and displacement are efficient and easy to realize, which can satisfy real-time demand. At present, the control model has been applied in multi-axes high speed micro fabrication machining successfully.

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