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Delay-dependent robust control for uncertain discrete singular systems with time-varying delay*

Hui-jiao WANG[†1,2], Xiao-dong ZHAO[2], An-ke XUE[2], Ren-quan LU[2]

([1]*Institute of Automation, Zhejiang Sci-Tech University, Hangzhou 310018, China*)

([2]*Institute of Information and Control, Hangzhou Dianzi University, Hangzhou 310018, China*)

[†]E-mail: hjwang@hdu.edu.cn

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Abstract: The design problem of delay-dependent robust control for uncertain discrete singular systems with time-varying delay is addressed in this paper. The uncertainty is assumed to be norm-bounded. By establishing a finite sum inequality based on quadratic terms, a new delay-dependent robust stability condition is derived and expressed in terms of linear matrix inequalities (LMIs). A suitable robust state feedback control law is presented, which guarantees that the resultant closed-loop system is regular, causal and stable for all admissible uncertainties. Numerical examples are given to demonstrate the applicability of the proposed method.

Key words: Uncertain discrete singular system, Time-varying delay, Robust control, Linear matrix inequality (LMI)
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INTRODUCTION

Time delays are frequently encountered in many fields of science and engineering, including communication network, manufacturing systems, biology, economy and other areas (Gu *et al*., 2003). During the last two decades, the problem of stability analysis and control of time-delay systems has been the subject of considerable research efforts. Many significant results have been reported in the literature, see e.g. (Gao *et al*., 2004; Han, 2002a; 2002b; 2004a; 2004b; 2005a; 2005b; 2005c; Han *et al*., 2004; Jiang and Han, 2005; 2006; Yue and Han, 2005b; Chen *et al*., 2007) and references therein. On the other hand, singular systems, which are known as descriptor systems, implicit systems, generalized state-space systems or semi-state systems, have received much attention since the singular model can preserve the structure of practical systems and can better describe a large class of physical systems than state-space ones (Lewis, 1986; Dai, 1989). Therefore, the study of robust stability and stabilizability for uncertain singular time-delay systems is of theoretical and practical importance.

It should be pointed out that when the robust stability problem for singular systems is investigated, the regularity and absence of impulse (for continuous systems) and causality (for discrete systems) must be considered simultaneously (Xu *et al*., 2002; Zhou and Lam, 2003; Chen and Lin, 2004; Wang *et al*., 2007; 2008). Hence, the robust stability problem for singular time-delay systems is much more complicated than that for state-space ones. Some delay-dependent stability criteria for singular time-delay systems were presented in (Fridman, 2001; Boukas and Liu, 2003), where it was required to assume that the considered system is regular and impulse free. In (Yue and Han, 2005a), a delay-dependent robust H_∞ controller is designed for uncertain descriptor systems with time-varying discrete and distributed delays, but the given results are based on a set of nonconvex matrix inequalities, not on strict linear matrix inequalities

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(LMIs). In the discrete setting, Wang and Liu (2000) used the spectral radius inequality of the modulus matrix to deal with the stability problem of discrete-time singular systems with multiple time delays, subjected to a class of highly structured uncertainties. However, the robustness of regularity and causality were not discussed in (Wang and Liu, 2000). For discrete singular time-delay systems, some sufficient conditions were obtained for the problem of robust stability (Chen and Chou, 2003; Xu and Lam, 2004; Ji *et al*., 2006). However, these conditions were established under the assumption that the delay was constant; when the delay is time-varying, they are inapplicable. To the best of our knowledge, the class of uncertain discrete singular systems with time-varying delay has not yet been fully investigated. In particular, delay-dependent sufficient conditions of robust stability are few, not even existing in the literature.

In this paper, the problem of robust control is considered for a class of discrete singular systems with time-varying delay and norm-bounded uncertainties. With the introduction of a new finite sum inequality, which avoids using both model transformation and bounding technique for cross terms (Zhang and Han, 2006), a strict LMI sufficient criterion for discrete singular time-varying delay systems is obtained. The robust control problem is also solved and an explicit expression of the desired state feedback control law is given, which can be obtained by solving the feasibility problem of a strict LMI.

Notations: Throughout this paper, the superscripts "T" and "−1" stand for the transpose and inverse of a matrix, respectively; $\mathbb{R}^n$ denotes n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all real matrices with n rows and m columns; $\boldsymbol{P}>\boldsymbol{0}$ means that $\boldsymbol{P}$ is positive definite; $\boldsymbol{I}$ is the identity matrix with appropriate dimensions; $\lambda_{\max}(\boldsymbol{P})$ and $\lambda_{\min}(\boldsymbol{P})$ denote the maximal and minimal eigenvalues of the matrix $\boldsymbol{P}$, respectively; $\|\boldsymbol{x}\|$ refers to the Euclidean norm of the vector $\boldsymbol{x}$, that is $\|\boldsymbol{x}\|=\sqrt{\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}}$; for a symmetric matrix, * denotes the matrix entries implied by symmetry.

PROBLEM FORMULATION

Consider a class of uncertain discrete singular systems with time-varying delay described by

$$\begin{cases}\boldsymbol{E}\boldsymbol{x}(k+1)=(\boldsymbol{A}+\Delta\boldsymbol{A})\boldsymbol{x}(k)+(\boldsymbol{A}_d+\Delta\boldsymbol{A}_d)\\ \qquad\cdot\boldsymbol{x}(k-d(k))+(\boldsymbol{B}+\Delta\boldsymbol{B})\boldsymbol{u}(k),\\ \boldsymbol{x}(k)=\boldsymbol{\phi}(k),\quad k=-\bar{d},-\bar{d}+1,\cdots,0,\end{cases}\tag{1}$$

where $\boldsymbol{x}(t)\in\mathbb{R}^n$ is the state vector, $\boldsymbol{u}(t)\in\mathbb{R}^m$ is the control input vector, $\boldsymbol{\phi}(k)$ $(k=-\bar{d},-\bar{d}+1,\cdots,0)$ is a known given initial condition sequence, and $d(k)$ is a positive integer time-varying delay satisfying

$$0<\underline{d}\le d(k)\le\bar{d}<\infty,\quad k=1,2,\cdots\tag{2}$$

$\boldsymbol{E}$, $\boldsymbol{A}$, $\boldsymbol{A}_d$, and $\boldsymbol{B}$ are known constant matrices of appropriate dimensions, where $\boldsymbol{E}$ may be singular and we assume rank($\boldsymbol{E}$)=$r\le n$. $\Delta\boldsymbol{A}$, $\Delta\boldsymbol{A}_d$ and $\Delta\boldsymbol{B}$ are unknown and possibly time-varying parameter uncertainties representing norm-bounded parameter uncertainties and are assumed to be of the following form:

$$\begin{bmatrix}\Delta\boldsymbol{A} & \Delta\boldsymbol{A}_d & \Delta\boldsymbol{B}\end{bmatrix}=\boldsymbol{M}\boldsymbol{F}(k)\begin{bmatrix}\boldsymbol{N}_a & \boldsymbol{N}_d & \boldsymbol{N}_b\end{bmatrix},\tag{3}$$

where $\boldsymbol{M}$, $\boldsymbol{N}_a$, $\boldsymbol{N}_d$ and $\boldsymbol{N}_b$ are known matrices of appropriate dimensions, and $\boldsymbol{F}(k)$ is an unknown matrix function bounded by $\boldsymbol{F}^{\mathrm{T}}(k)\boldsymbol{F}(k)\le\boldsymbol{I}$. Clearly, $\bar{d}-\underline{d}=0$ means that the time-delay $d(k)$ is time-invariant.

Before moving on, we give some definitions and lemmas concerning the following nominal unforced counterpart of the system Eq.(1):

$$\boldsymbol{E}\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{A}_d\boldsymbol{x}(k-d(k)).\tag{4}$$

Definition 1 (Lewis, 1986; Dai, 1989) (1) The pair ($\boldsymbol{E}$, $\boldsymbol{A}$) is said to be regular if det($z\boldsymbol{E}-\boldsymbol{A}$) is not identically zero; (2) The pair ($\boldsymbol{E}$, $\boldsymbol{A}$) is said to be causal if deg[det($z\boldsymbol{E}-\boldsymbol{A}$)]=rank($\boldsymbol{E}$).

Lemma 1 Suppose the pair ($\boldsymbol{E}$, $\boldsymbol{A}$) to be regular and causal, then the solution to the discrete singular time-varying delay system Eq.(4) exists and is unique and causal.

Proof Noting the regularity and causality of the pair ($\boldsymbol{E}$, $\boldsymbol{A}$) and the decomposition method as in (Dai, 1989), the desired result follows immediately.

Definition 2 (1) The discrete singular time-varying delay system Eq.(4) is said to be regular and causal, if the pair ($\boldsymbol{E}$, $\boldsymbol{A}$) is regular and causal; (2) The discrete singular time-varying delay system Eq.(4) is said to be stable if for any $\varepsilon>0$ there exists a scalar $\delta(\varepsilon)>0$ such that, for any compatible initial conditions $\sup_{-\bar{d}\le k\le 0}\|\boldsymbol{\phi}(k)\|\le\delta(\varepsilon)$, the solution $\boldsymbol{x}(k)$ of the system Eq.(4) satisfies $\|\boldsymbol{x}(k)\|\le\varepsilon$. Furthermore, $\boldsymbol{x}(k)\to\boldsymbol{0}$ as $k\to\infty$.

Definition 3 The uncertain discrete singular time-varying delay system Eq.(1) is said to be robustly stable if the system Eq.(1) with $\boldsymbol{u}(k)\equiv\boldsymbol{0}$ is regular, causal and stable for all admissible uncertainties $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{A}_d$.

Definition 4 The uncertain discrete singular time-varying delay system Eq.(1) is said to be robustly stabilizable if there exists a linear state feedback control law $\boldsymbol{u}(k)=\boldsymbol{Kx}(k)$, $\boldsymbol{K}\in\mathbb{R}^{m\times n}$ such that its closed-loop system is robustly stable in the sense of Definition 3. In this case, $\boldsymbol{u}(k)=\boldsymbol{Kx}(k)$ is called a "robust state feedback controller".

The problem to be addressed in this paper is to obtain a feedback gain $\boldsymbol{K}$ for designing a robust state feedback controller $\boldsymbol{u}(k)=\boldsymbol{Kx}(k)$.

We conclude this section by introducing the following lemmas that will be used in the proof of our main results.

Lemma 2 (Petersen, 1987) Given matrices $\boldsymbol{\Gamma}$, $\boldsymbol{\Lambda}$ and symmetric matrix $\boldsymbol{\Omega}$, we have $\boldsymbol{\Omega}+\boldsymbol{\Gamma F\Lambda}+\boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{\Gamma}^{\mathrm{T}}<\boldsymbol{0}$ for any $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}\le\boldsymbol{I}$, if and only if there exists a scalar $\varepsilon>0$ such that $\boldsymbol{\Omega}+\varepsilon^{-1}\boldsymbol{\Gamma}\boldsymbol{\Gamma}^{\mathrm{T}}+\varepsilon\boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{\Lambda}<\boldsymbol{0}$.

Lemma 3 (Ji *et al.*, 2006) Given a series $\alpha_i\ge 0$ (i=0, 1, 2, …), if $\lim_{n\to\infty}\sum_{i=0}^{n}\alpha_i=\alpha<\infty$, then $\lim_{i\to\infty}\alpha_i=0$.

MAIN RESULTS

In this section, we give a solution to the problem of robust control for the singular system Eq.(1) by using the LMI approach.

Delay-dependent stability analysis for nominal singular system

We first consider the stability problem of the nominal unforced discrete singular time-varying delay system Eq.(4).

For this nominal system Eq.(4), we introduce two vectors: $\boldsymbol{\xi}(k)=[\boldsymbol{x}^{\mathrm{T}}(k)\ \ \boldsymbol{x}^{\mathrm{T}}(k-d(k))]^{\mathrm{T}}$ and $\boldsymbol{y}(l)=\boldsymbol{x}(l+1)-\boldsymbol{x}(l)$. Then

$$\begin{cases}\boldsymbol{Ex}(k+1)=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{A}_d\end{bmatrix}\boldsymbol{\xi}(k),\\ \boldsymbol{Ey}(k)=\begin{bmatrix}\boldsymbol{A}-\boldsymbol{E} & \boldsymbol{A}_d\end{bmatrix}\boldsymbol{\xi}(k).\end{cases}\tag{5}$$

The following lemma gives the relationship between the vectors $\boldsymbol{\xi}(k)$ and $\boldsymbol{Ey}(k)$, which will play a key role in the delay-dependent stability analysis.

Lemma 4 (Zhang and Han, 2006) For any constant matrices $\boldsymbol{N}_1$, $\boldsymbol{N}_2\in\mathbb{R}^{n\times n}$, a positive definite symmetric matrix $\boldsymbol{Z}\in\mathbb{R}^{n\times n}$ and a positive integer time-varying $d(k)$,

$$-\sum_{l=k-d(k)}^{k-1}\boldsymbol{y}^{\mathrm{T}}(l)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{ZEy}(l)\le\boldsymbol{\xi}^{\mathrm{T}}(k)[\boldsymbol{\Pi}+d(k)\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Z}^{-1}\boldsymbol{Y}]\boldsymbol{\xi}(k),\tag{6}$$

where

$$\boldsymbol{\Pi}=\begin{bmatrix}\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_1 & \boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_2-\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E}\\ * & -\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_2\end{bmatrix},\ \boldsymbol{Y}=\begin{bmatrix}\boldsymbol{N}_1 & \boldsymbol{N}_2\end{bmatrix}.\tag{7}$$

Based on Lemma 4, the following theorem presents a stability condition of the nominal system Eq.(4).

Theorem 1 The nominal discrete singular system Eq.(4) with time-varying delay is regular, causal and stable, if there exist positive definite symmetric matrices $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{Z}$ and matrices $\boldsymbol{S}$, $\boldsymbol{S}_d$, $\boldsymbol{N}_1$, $\boldsymbol{N}_2$ of appropriate dimensions such that

$$\boldsymbol{\Xi}=\begin{bmatrix}\boldsymbol{\Xi}_{11} & \boldsymbol{\Xi}_{12} & \bar{d}\boldsymbol{N}_1^{\mathrm{T}} & \bar{d}(\boldsymbol{A}-\boldsymbol{E})^{\mathrm{T}}\boldsymbol{Z} & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\\ * & \boldsymbol{\Xi}_{22} & \bar{d}\boldsymbol{N}_2^{\mathrm{T}} & \bar{d}\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{Z} & \boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{P}\\ * & * & -\bar{d}\boldsymbol{Z} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & * & -\bar{d}\boldsymbol{Z} & \boldsymbol{0}\\ * & * & * & * & -\boldsymbol{P}\end{bmatrix}<\boldsymbol{0},\tag{8}$$

where

$$\boldsymbol{\Xi}_{11}=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{RS}^{\mathrm{T}}+\boldsymbol{SR}^{\mathrm{T}}\boldsymbol{A}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{PE}+\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_1+(\bar{d}-\underline{d}+1)\boldsymbol{Q},$$
$$\boldsymbol{\Xi}_{12}=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{RS}_d^{\mathrm{T}}+\boldsymbol{SR}^{\mathrm{T}}\boldsymbol{A}_d+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_2-\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E},$$
$$\boldsymbol{\Xi}_{22}=\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{RS}_d^{\mathrm{T}}+\boldsymbol{S}_d\boldsymbol{R}^{\mathrm{T}}\boldsymbol{A}_d-\boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{E}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_2-\boldsymbol{Q},$$

and $\boldsymbol{R}\in\mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $\boldsymbol{ER}=\boldsymbol{0}$.

Proof Since rank($\boldsymbol{E}$)=$r\le n$, there must exist two invertible matrices $\boldsymbol{G}$, $\boldsymbol{H}\in\mathbb{R}^{n\times n}$ such that

$$\bar{\boldsymbol{E}}=\boldsymbol{GEH}=\begin{bmatrix}\boldsymbol{I}_r & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}. \tag{9}$$

Then $\boldsymbol{R}$ can be parameterized as $\boldsymbol{R}=\boldsymbol{G}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{0}\\ \bar{\boldsymbol{\Phi}}\end{bmatrix}$, where $\bar{\boldsymbol{\Phi}}\in\mathbb{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix.

Similar to Eq.(9), we define

$$\bar{\boldsymbol{A}}=\boldsymbol{GAH}=\begin{bmatrix}\bar{\boldsymbol{A}}_{11} & \bar{\boldsymbol{A}}_{12}\\ \bar{\boldsymbol{A}}_{21} & \bar{\boldsymbol{A}}_{22}\end{bmatrix},\ \bar{\boldsymbol{P}}=\boldsymbol{G}^{-\mathrm{T}}\boldsymbol{P}\boldsymbol{G}^{-1}=\begin{bmatrix}\bar{\boldsymbol{P}}_{11} & \bar{\boldsymbol{P}}_{12}\\ \bar{\boldsymbol{P}}_{21} & \bar{\boldsymbol{P}}_{22}\end{bmatrix},$$

$$\bar{\boldsymbol{N}}_1=\boldsymbol{G}^{-\mathrm{T}}\boldsymbol{N}_1\boldsymbol{H}=\begin{bmatrix}\bar{\boldsymbol{N}}_{1,11} & \bar{\boldsymbol{N}}_{1,12}\\ \bar{\boldsymbol{N}}_{1,21} & \bar{\boldsymbol{N}}_{1,22}\end{bmatrix},$$

$$\bar{\boldsymbol{S}}=\boldsymbol{H}^{\mathrm{T}}\boldsymbol{S}=\begin{bmatrix}\bar{\boldsymbol{S}}_{11}\\ \bar{\boldsymbol{S}}_{21}\end{bmatrix},\ \bar{\boldsymbol{R}}=\boldsymbol{G}^{-\mathrm{T}}\boldsymbol{R}=\begin{bmatrix}\boldsymbol{0}\\ \bar{\boldsymbol{\Phi}}\end{bmatrix}.$$

Since $\boldsymbol{\Xi}_{11}<\boldsymbol{0}$ and $(\bar{d}-\underline{d}+1)\boldsymbol{Q}>\boldsymbol{0}$, we can formulate the following inequality easily:

$$\boldsymbol{\Psi}=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{S}^{\mathrm{T}}+\boldsymbol{S}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{A}-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}+\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{N}_1<\boldsymbol{0}.$$

Pre- and post-multiplying $\boldsymbol{\Psi}<\boldsymbol{0}$ by $\boldsymbol{H}^{\mathrm{T}}$ and $\boldsymbol{H}$, respectively, yield

$$\begin{aligned}\bar{\boldsymbol{\Psi}}&=\boldsymbol{H}^{\mathrm{T}}\boldsymbol{\Psi}\boldsymbol{H}\\ &=\bar{\boldsymbol{A}}^{\mathrm{T}}\bar{\boldsymbol{R}}\bar{\boldsymbol{S}}^{\mathrm{T}}+\bar{\boldsymbol{S}}\bar{\boldsymbol{R}}^{\mathrm{T}}\bar{\boldsymbol{A}}-\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{P}}\bar{\boldsymbol{E}}+\bar{\boldsymbol{N}}_1^{\mathrm{T}}\bar{\boldsymbol{E}}+\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{N}}_1\\ &=\begin{bmatrix}\bar{\boldsymbol{\Psi}}_{11} & \bar{\boldsymbol{\Psi}}_{12}\\ * & \bar{\boldsymbol{A}}_{22}^{\mathrm{T}}\bar{\boldsymbol{\Phi}}\bar{\boldsymbol{S}}_{21}^{\mathrm{T}}+\bar{\boldsymbol{S}}_{21}\bar{\boldsymbol{\Phi}}^{\mathrm{T}}\bar{\boldsymbol{A}}_{22}\end{bmatrix}<\boldsymbol{0}.\end{aligned} \tag{10}$$

Since $\bar{\boldsymbol{\Psi}}_{11}$ and $\bar{\boldsymbol{\Psi}}_{12}$ are irrelevant to the results of the following discussion, the real expression of these two variables are omitted here. From Eq.(10), it is easy to see that

$$\bar{\boldsymbol{A}}_{22}^{\mathrm{T}}\bar{\boldsymbol{\Phi}}\bar{\boldsymbol{S}}_{21}^{\mathrm{T}}+\bar{\boldsymbol{S}}_{21}\bar{\boldsymbol{\Phi}}^{\mathrm{T}}\bar{\boldsymbol{A}}_{22}<\boldsymbol{0}, \tag{11}$$

and thus $\bar{\boldsymbol{A}}_{22}$ is nonsingular. Otherwise, supposing $\bar{\boldsymbol{A}}_{22}$ is singular, there must exist a non-zero vector $\boldsymbol{\varsigma}\in\mathbb{R}^{n-r}$, which ensures $\bar{\boldsymbol{A}}_{22}\boldsymbol{\varsigma}=\boldsymbol{0}$. And then we can conclude that $\boldsymbol{\varsigma}^{\mathrm{T}}(\bar{\boldsymbol{A}}_{22}^{\mathrm{T}}\bar{\boldsymbol{\Phi}}\bar{\boldsymbol{S}}_{21}^{\mathrm{T}}+\bar{\boldsymbol{S}}_{21}\bar{\boldsymbol{\Phi}}^{\mathrm{T}}\bar{\boldsymbol{A}}_{22})\boldsymbol{\varsigma}=\boldsymbol{0}$. This contradicts inequality (11). So $\bar{\boldsymbol{A}}_{22}$ is nonsingular.

Then, it can be shown that

$$\begin{aligned}&\det(s\boldsymbol{E}-\boldsymbol{A})=\det(\boldsymbol{G}^{-1})\det(z\bar{\boldsymbol{E}}-\bar{\boldsymbol{A}})\det(\boldsymbol{H}^{-1})\\ &=\det(\boldsymbol{G}^{-1})\det(-\bar{\boldsymbol{A}}_{22})\det(z\boldsymbol{I}_r-(\bar{\boldsymbol{A}}_{11}-\bar{\boldsymbol{A}}_{12}\bar{\boldsymbol{A}}_{22}^{-1}\bar{\boldsymbol{A}}_{21}))\det(\boldsymbol{H}^{-1}),\end{aligned}$$

which implies that $\det(z\boldsymbol{E}-\boldsymbol{A})$ is not identically zero and $\deg[\det(z\boldsymbol{E}-\boldsymbol{A})]=r=\mathrm{rank}(\boldsymbol{E})$. Then the pair of $(\boldsymbol{E}, \boldsymbol{A})$ is regular and causal, which implies that the system Eq.(4) is regular and causal. We will prove that the system Eq.(4) is also stable as the following.

We define the function

$$V(k)=V_1(k)+V_2(k)+V_3(k)+V_4(k), \tag{12}$$

where

$$\begin{aligned}V_1(k)&=\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}\boldsymbol{x}(k),\\ V_2(k)&=\sum_{\theta=-\bar{d}+1}^{0}\sum_{l=k-1+\theta}^{k-1}\boldsymbol{y}^{\mathrm{T}}(l)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{E}\boldsymbol{y}(l),\\ V_3(k)&=\sum_{l=k-d(k)}^{k-1}\boldsymbol{x}^{\mathrm{T}}(l)\boldsymbol{Q}\boldsymbol{x}(l),\\ V_4(k)&=\sum_{\theta=-\bar{d}+2}^{-\underline{d}+1}\sum_{l=k-1+\theta}^{k-1}\boldsymbol{x}^{\mathrm{T}}(l)\boldsymbol{Q}\boldsymbol{x}(l).\end{aligned}$$

Then, the forward difference of $V(k)$, $\Delta V(k)=V(k+1)-V(k)$, along the trajectory of the system Eq.(4) satisfies the following relation:

$$\begin{aligned}\Delta V_1(k)&=\boldsymbol{x}^{\mathrm{T}}(k+1)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}\boldsymbol{x}(k+1)-\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}\boldsymbol{x}(k)\\ &=\boldsymbol{\xi}^{\mathrm{T}}(k)[\boldsymbol{A}\ \ \boldsymbol{A}_d]^{\mathrm{T}}\boldsymbol{P}[\boldsymbol{A}\ \ \boldsymbol{A}_d]\boldsymbol{\xi}(k)-\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}\boldsymbol{x}(k),\\ \Delta V_2(k)&=\bar{d}\boldsymbol{y}^{\mathrm{T}}(k)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{E}\boldsymbol{y}(k)-\sum_{l=k-\bar{d}}^{k-1}\boldsymbol{y}^{\mathrm{T}}(l)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{E}\boldsymbol{y}(l)\\ &\le\bar{d}\boldsymbol{y}^{\mathrm{T}}(k)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{E}\boldsymbol{y}(k)-\sum_{l=k-d(k)}^{k-1}\boldsymbol{y}^{\mathrm{T}}(l)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{E}\boldsymbol{y}(l),\\ \Delta V_3(k)&=\sum_{l=k+1-d(k+1)}^{k}\boldsymbol{x}^{\mathrm{T}}(l)\boldsymbol{Q}\boldsymbol{x}(l)-\sum_{l=k-d(k)}^{k-1}\boldsymbol{x}^{\mathrm{T}}(l)\boldsymbol{Q}\boldsymbol{x}(l)\\ &\le\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{Q}\boldsymbol{x}(k)-\boldsymbol{x}^{\mathrm{T}}(k-d(k))\boldsymbol{Q}\boldsymbol{x}(k-d(k))\\ &\quad+\sum_{l=k+1-\bar{d}}^{k-\underline{d}}\boldsymbol{x}^{\mathrm{T}}(l)\boldsymbol{Q}\boldsymbol{x}(l),\\ \Delta V_4(k)&=(\bar{d}-\underline{d})\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{Q}\boldsymbol{x}(k)-\sum_{l=k+1-\bar{d}}^{k-\underline{d}}\boldsymbol{x}^{\mathrm{T}}(l)\boldsymbol{Q}\boldsymbol{x}(l).\end{aligned} \tag{13}$$

Furthermore, noting $\boldsymbol{E}^{\mathrm{T}}\boldsymbol{R}=\boldsymbol{0}$, we can deduce

$$\boldsymbol{0}=2\boldsymbol{x}^{\mathrm{T}}(k+1)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{R}[\boldsymbol{S}^{\mathrm{T}}\boldsymbol{x}(k)+\boldsymbol{S}_d^{\mathrm{T}}\boldsymbol{x}(k-d(k))]. \tag{14}$$

Then it follows from Eqs.(13) and (14) and Lemma 4 that

$$\Delta \boldsymbol{V}(k) \le \boldsymbol{\xi}^{\mathrm{T}}(k)\boldsymbol{\varXi}\boldsymbol{\xi}(k).$$

It is easy to see that inequality (8) guarantees $\Delta\boldsymbol{V}(k)<0$ and thus

$$\lambda_1\|\boldsymbol{x}(k+1)\|^2 - \boldsymbol{V}(0) \le \boldsymbol{x}^{\mathrm{T}}(k+1)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{PEx}(k+1) - \boldsymbol{V}(0)$$
$$\le \boldsymbol{V}(k+1) - \boldsymbol{V}(0) = \sum_{i=0}^{k}\Delta\boldsymbol{V}(i) \le -\lambda_2\sum_{i=0}^{k}\|\boldsymbol{x}(i)\|^2, \quad (15)$$

where

$$\lambda_1 = \lambda_{\min}(\boldsymbol{E}^{\mathrm{T}}\boldsymbol{PE}) > 0,\ \ \lambda_2 = -\lambda_{\max}(\boldsymbol{\varXi}) > 0.$$

Obviously, inequality (15) implies that

$$0 \le \sum_{i=0}^{k}\|\boldsymbol{x}(i)\|^2 \le \boldsymbol{V}(0)/\lambda_2. \quad (16)$$

From inequality (16), it is easy to see that $\sum_{i=0}^{k}\|\boldsymbol{x}(i)\|^2$ is bounded for k=0, 1, 2, …, n, …,∞. By Lemma 3, it follows that $\lim_{i\to\infty}\|\boldsymbol{x}(i)\|=0$ and thus $\lim_{i\to\infty}\boldsymbol{x}(i)=\boldsymbol{0}$, which implies that the system Eq.(4) is stable for the arbitrary time-varying delay $d(k)$ that satisfies Eq.(2). This completes the proof.

Remark 1 When $\boldsymbol{E}=\boldsymbol{I}$, it follows from $\boldsymbol{E}^{\mathrm{T}}\boldsymbol{R}=\boldsymbol{0}$ that $\boldsymbol{R}=\boldsymbol{0}$. Therefore, it is easy to see that Theorem 1 coincides with Proposition 1 in (Zhang and Han, 2006) not considering the H_∞ performance by slight modifications.

Applying inequality (8) to the uncertain singular time-varying delay system Eq.(1) with $\boldsymbol{u}(k)=\boldsymbol{0}$, we can easily obtain the following theorem:

Theorem 2 The uncertain discrete singular time-varying system Eq.(1) with $\boldsymbol{u}(k)=\boldsymbol{0}$ is robustly stable for all admissible uncertainties satisfying Eqs.(2) and (3), if there exist positive definite symmetric matrices $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{Z}$, matrices $\boldsymbol{S}$, $\boldsymbol{S}_d$, $\boldsymbol{N}_1$, $\boldsymbol{N}_2$ with appropriate dimensions and a scalar $\varepsilon>0$ such that

$$\begin{bmatrix} \boldsymbol{\varXi}_{11} & \boldsymbol{\varXi}_{12} & \bar{d}\boldsymbol{N}_1^{\mathrm{T}} & \boldsymbol{\varXi}_{14} & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} & \boldsymbol{SR}^{\mathrm{T}}\boldsymbol{M} & \varepsilon\boldsymbol{N}_a^{\mathrm{T}} \\ * & \boldsymbol{\varXi}_{22} & \bar{d}\boldsymbol{N}_2^{\mathrm{T}} & \bar{d}\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{Z} & \boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{P} & \boldsymbol{S}_d\boldsymbol{R}^{\mathrm{T}}\boldsymbol{M} & \varepsilon\boldsymbol{N}_d^{\mathrm{T}} \\ * & * & -\bar{d}\boldsymbol{Z} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & -\bar{d}\boldsymbol{Z} & \boldsymbol{0} & \bar{d}\boldsymbol{ZM} & \boldsymbol{0} \\ * & * & * & * & -\boldsymbol{P} & \boldsymbol{PM} & \boldsymbol{0} \\ * & * & * & * & * & -\varepsilon\boldsymbol{I} & \boldsymbol{0} \\ * & * & * & * & * & * & -\varepsilon\boldsymbol{I} \end{bmatrix} < \boldsymbol{0}, \quad (17)$$

where $\boldsymbol{\varXi}_{14} = \bar{d}(\boldsymbol{A}-\boldsymbol{E})^{\mathrm{T}}\boldsymbol{Z}$, and $\boldsymbol{R}$, $\boldsymbol{\varXi}_{11}$, $\boldsymbol{\varXi}_{12}$ and $\boldsymbol{\varXi}_{22}$ share the same definitions as those in inequality (8).

Proof Replacing $\boldsymbol{A}$ by $\boldsymbol{A}+\boldsymbol{MF}(k)\boldsymbol{N}_a$ and $\boldsymbol{A}_d$ by $\boldsymbol{A}_d+\boldsymbol{MF}(k)\boldsymbol{N}_d$ in inequality (8), we have

$$\boldsymbol{\varXi} + \boldsymbol{\varGamma}\boldsymbol{F}(k)\boldsymbol{\varPhi} + \boldsymbol{\varPhi}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(k)\boldsymbol{\varGamma}^{\mathrm{T}} < \boldsymbol{0}, \quad (18)$$

where

$$\boldsymbol{\varGamma} = \begin{bmatrix}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{RS}^{\mathrm{T}} & \boldsymbol{M}^{\mathrm{T}}\boldsymbol{RS}_d^{\mathrm{T}} & \boldsymbol{0} & \bar{d}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{Z} & \boldsymbol{M}^{\mathrm{T}}\boldsymbol{P}\end{bmatrix}^{\mathrm{T}},$$
$$\boldsymbol{\varPhi} = \begin{bmatrix}\boldsymbol{N}_a & \boldsymbol{N}_d & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}.$$

By Lemma 2, it follows that inequality (18) holds for any $\boldsymbol{F}^{\mathrm{T}}(k)\boldsymbol{F}(k)\le\boldsymbol{I}$ if there exists a scalar $\varepsilon>0$ such that

$$\boldsymbol{\varXi} + \varepsilon^{-1}\boldsymbol{\varGamma}\boldsymbol{\varGamma}^{\mathrm{T}} + \varepsilon\boldsymbol{\varPhi}^{\mathrm{T}}\boldsymbol{\varPhi} < \boldsymbol{0}, \quad (19)$$

which is equal to inequality (17) in the sense of the Schur complement.

Robust state feedback controller design

In the sequel, we give a strict LMI design algorithm for the singular system Eq.(1). For notation simplicity, we first consider the system Eq.(1) with $\Delta\boldsymbol{A}=\Delta\boldsymbol{A}_d=\Delta\boldsymbol{B}=\boldsymbol{0}$. The controller $\boldsymbol{u}(k)=\boldsymbol{Kx}(k)$ results in the following closed-loop system:

$$\boldsymbol{Ex}(k+1) = (\boldsymbol{A}+\boldsymbol{BK})\boldsymbol{x}(k) + \boldsymbol{A}_d\boldsymbol{x}(k-d(k)). \quad (20)$$

For this system, we have the following theorem:

Theorem 3 The discrete singular system Eq.(20) with time-varying delay is robustly stabilizable if there exist positive definite symmetric matrices $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{Z}$ and matrices $\boldsymbol{S}$, $\boldsymbol{N}_1$, $\boldsymbol{N}_2$, $\boldsymbol{X}$, $\boldsymbol{L}$ of appropriate dimensions such that

$$\begin{bmatrix} \boldsymbol{\varUpsilon}_{11} & \boldsymbol{\varUpsilon}_{12} & \boldsymbol{\varUpsilon}_{13} & \bar{d}\boldsymbol{N}_1^{\mathrm{T}} & \boldsymbol{0} \\ * & \boldsymbol{\varUpsilon}_{22} & \boldsymbol{X}^{\mathrm{T}}\boldsymbol{A}_d^{\mathrm{T}} & \boldsymbol{0} & \bar{d}\boldsymbol{Z} \\ * & * & \boldsymbol{\varUpsilon}_{33} & \bar{d}\boldsymbol{N}_2^{\mathrm{T}} & \boldsymbol{0} \\ * & * & * & -\bar{d}\boldsymbol{Z} & \boldsymbol{0} \\ * & * & * & * & -\bar{d}\boldsymbol{Z} \end{bmatrix} < \boldsymbol{0}, \quad (21)$$

where

$$\boldsymbol{\varUpsilon}_{11} = (\boldsymbol{A}-\boldsymbol{E})\boldsymbol{X} + \boldsymbol{X}^{\mathrm{T}}(\boldsymbol{A}-\boldsymbol{E})^{\mathrm{T}} + \boldsymbol{BL} + \boldsymbol{L}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} + \boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{E}^{\mathrm{T}} + \boldsymbol{EN}_1 + (\bar{d}-\underline{d}+1)\boldsymbol{Q},$$
$$\boldsymbol{\varUpsilon}_{12} = \boldsymbol{EP} + \boldsymbol{SR}^{\mathrm{T}} - \boldsymbol{X}^{\mathrm{T}} + (\boldsymbol{A}-\boldsymbol{E})\boldsymbol{X} + \boldsymbol{BL},$$

$\Upsilon_{13}=X^{\mathrm{T}}A_d^{\mathrm{T}}+EN_2-N_1^{\mathrm{T}}E^{\mathrm{T}}$, $\Upsilon_{22}=-X-X^{\mathrm{T}}+P$,
$\Upsilon_{33}=-Q-EN_2-N_2^{\mathrm{T}}E^{\mathrm{T}}$,

and $R\in\mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $ER=0$. Furthermore, a suitable state feedback controller is given by $u(k)=LX^{-1}x(k)$.

Proof Following the same philosophy as that in (Fridman and Shaked, 2002), we represent the singular system Eq.(20) as the following form:

$$\bar{E}\bar{x}(k+1)=\bar{A}\bar{x}(k)+\bar{A}_d\bar{x}(k-d(k)), \tag{22}$$

where

$$\bar{E}=\begin{bmatrix}E & 0\\ 0 & 0\end{bmatrix},\ \bar{x}(k)=\begin{bmatrix}x(k)\\ Ey(k)\end{bmatrix},$$
$$\bar{A}_d=\begin{bmatrix}0 & 0\\ A_d & 0\end{bmatrix},\ \bar{A}=\begin{bmatrix}E & I\\ A+BK-E & -I\end{bmatrix}.$$

Then, by Theorem 1, we have that system Eq.(22) is robustly stable if inequality (8) holds, where E, A, A_d, P, Q, Z, R, S, S_d, N_1, N_2 are replaced by their respective transformation. Especially, we select

$$\bar{P}=\begin{bmatrix}P & 0\\ 0 & \beta I\end{bmatrix},\ \bar{Q}=\begin{bmatrix}Q & 0\\ 0 & \beta I\end{bmatrix},$$
$$\bar{R}=\begin{bmatrix}R & 0\\ 0 & X\end{bmatrix},\ \bar{S}=\begin{bmatrix}S & I\\ 0 & I\end{bmatrix},\ \bar{Z}=\begin{bmatrix}Z & 0\\ 0 & \beta I\end{bmatrix},$$
$$\bar{N}_1=\begin{bmatrix}N_1 & 0\\ 0 & \beta I\end{bmatrix},\ \bar{N}_2=\begin{bmatrix}N_2 & 0\\ 0 & \beta I\end{bmatrix},\ \bar{S}_d=0,$$

where P, Q, $Z\in\mathbb{R}^{n\times n}$ are positive definite symmetric matrices; $R\in\mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^{\mathrm{T}}R=0$; $X\in\mathbb{R}^{n\times n}$ is any nonsingular matrix; $S\in\mathbb{R}^{n\times(n-r)}$, N_1, $N_2\in\mathbb{R}^{n\times n}$ are any matrices. It is easy to see that $\bar{R}$ is with full column rank and satisfies $\bar{E}^{\mathrm{T}}\bar{R}=0$. Then the following condition can be obtained by using the Shur complement and letting $\beta\to 0$,

$$\begin{bmatrix}\Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \bar{d}N_1^{\mathrm{T}} & 0\\ * & \Lambda_{22} & X^{\mathrm{T}}A_d & 0 & \bar{d}Z\\ * & * & \Upsilon_{33} & \bar{d}N_2^{\mathrm{T}} & 0\\ * & * & * & -\bar{d}Z & 0\\ * & * & * & * & -\bar{d}Z\end{bmatrix}<0, \tag{23}$$

where

$$\Lambda_{11}=(A+BK-E)^{\mathrm{T}}X+X^{\mathrm{T}}(A+BK-E)+N_1^{\mathrm{T}}E+E^{\mathrm{T}}N_1+(\bar{d}-\underline{d}+1)Q,$$
$$\Lambda_{12}=E^{\mathrm{T}}P+SR^{\mathrm{T}}-X^{\mathrm{T}}+(A+BK-E)^{\mathrm{T}}X,$$
$$\Lambda_{13}=X^{\mathrm{T}}A_d+E^{\mathrm{T}}N_2-N_1^{\mathrm{T}}E,\ \Lambda_{22}=-X-X^{\mathrm{T}}+P,$$
$$\Upsilon_{33}=-Q-EN_2-N_2^{\mathrm{T}}E^{\mathrm{T}}.$$

Now, consider the following discrete singular time-varying delay system:

$$E^{\mathrm{T}}\varsigma(k+1)=(A+BK)^{\mathrm{T}}\varsigma(k)+A_d^{\mathrm{T}}\varsigma(k-d(k)), \tag{24}$$

where $\varsigma(k)\in\mathbb{R}^n$ is the state vector.

Note that $\det(zE-A)=\det(zE^{\mathrm{T}}-A^{\mathrm{T}})$, then the pair (E, A) is regular and causal if and only if the pair $(E^{\mathrm{T}}, A^{\mathrm{T}})$ is regular and causal and thus, system Eq.(20) is regular and causal if and only if system Eq.(24) is regular and causal. Since the solutions of $\det[zE-(A+BK)-A_dz^{-d(k)}]=0$ are the same as those of $\det[zE^{\mathrm{T}}-(A+BK)^{\mathrm{T}}-A_d^{\mathrm{T}}z^{-d(k)}]=0$, system Eq.(20) is stable if and only if system Eq.(24) is stable.

Therefore, as long as the regularity, causality and stability are concerned, we can consider system Eq.(24) instead of system Eq.(20). Then, LMI (21) can be obtained by replacing E, $(A+BK)$, A_d in inequality (23) by E^{T}, $(A+BK)^{\mathrm{T}}$, A_d^{T} respectively and introducing a matrix $L=KX$. According to Definition 4, the discrete singular time-varying delay system Eq.(20) is robustly stabilizable.

The robust stabilizability result is presented in the following theorem:

Theorem 4 Consider the uncertain discrete singular system Eq.(1) with time-varying delay, if there exist positive definite symmetric matrices P, Q, Z, matrices S, N_1, N_2, X, L of appropriate dimensions and scalars ε_1, $\varepsilon_2>0$, such that

$$\begin{bmatrix}\Theta_{11} & \Upsilon_{12} & \Upsilon_{13} & \bar{d}N_1^{\mathrm{T}} & 0 & \Theta_{16} & X^{\mathrm{T}}N_d^{\mathrm{T}}\\ * & \Upsilon_{22} & X^{\mathrm{T}}A_d^{\mathrm{T}} & 0 & \bar{d}Z & \Theta_{16} & X^{\mathrm{T}}N_d^{\mathrm{T}}\\ * & * & \Theta_{33} & -\bar{d}N_2^{\mathrm{T}} & 0 & 0 & 0\\ * & * & * & -\bar{d}Z & 0 & 0 & 0\\ * & * & * & * & -\bar{d}Z & 0 & 0\\ * & * & * & * & * & -\varepsilon_1 I & 0\\ * & * & * & * & * & * & -\varepsilon_2 I\end{bmatrix}<0, \tag{25}$$

then we can construct a robust state feedback control law

$$\boldsymbol{u}(k)=\boldsymbol{L}\boldsymbol{X}^{-1}\boldsymbol{x}(k),$$

such that the resultant closed-loop system is robustly stable for all admissible uncertainties, where $\boldsymbol{R}\in\mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $\boldsymbol{ER}=\boldsymbol{0}$, $\boldsymbol{\Theta}_{11}=\boldsymbol{\Upsilon}_{11}+\varepsilon_1\boldsymbol{M}\boldsymbol{M}^{\mathrm{T}}$, $\boldsymbol{\Theta}_{16}=(\boldsymbol{N}_a\boldsymbol{X}+\boldsymbol{N}_b\boldsymbol{L})^{\mathrm{T}}$, $\boldsymbol{\Theta}_{33}=\boldsymbol{\Upsilon}_{33}+\varepsilon_2\boldsymbol{M}\boldsymbol{M}^{\mathrm{T}}$ and $\boldsymbol{\Upsilon}_{11},\boldsymbol{\Upsilon}_{12},\boldsymbol{\Upsilon}_{13},\boldsymbol{\Upsilon}_{22},\boldsymbol{\Upsilon}_{33}$ follow the same definitions as those in inequality (21).

Proof Replacing $\boldsymbol{A}$, $\boldsymbol{A}_d$ and $\boldsymbol{B}$ in inequality (21) by $\boldsymbol{A}+\boldsymbol{MF}(k)\boldsymbol{N}_a$, $\boldsymbol{A}_d+\boldsymbol{MF}(k)\boldsymbol{N}_d$ and $\boldsymbol{B}+\boldsymbol{MF}(k)\boldsymbol{N}_b$ respectively and following the same line as that in the proof of Theorem 2, LMI (25) follows immediately.

Remark 2 It is easy to see that LMI (25) implies $\boldsymbol{\Upsilon}_{22}=-\boldsymbol{X}-\boldsymbol{X}^{\mathrm{T}}+\boldsymbol{P}<\boldsymbol{0}$ and thus $\boldsymbol{X}$ is nonsingular. Then, $\boldsymbol{K}$ is well-defined if LMI (25) is feasible. It is worth pointing out that the given robust controller design method has not been reported in the literature.

Remark 3 A delay-dependent condition, which is used to design a suitable robust state feedback controller for uncertain discrete singular systems with time-varying delay, is provided by Theorem 4. This condition depends on the upper bound as well as the lower bound of the time-varying delay. So, when these bounds are available, Theorem 4 can achieve the expected results. Moreover, if the lower bound is not exactly known, we can replace it with zero; if the lower bound is equal to the upper bound, then the delay is time-invariant. In this case, Theorem 4 can handle the question of robust control for a large class of discrete singular systems.

Remark 4 On the other hand, Gao and Chen (2007) provided a less conservative solution to delay-dependent stability for discrete-time systems with time-varying state delay by using new Lyapunov functions and novel techniques, which maybe supplies a further research topic for discrete singular systems with time-varying delay.

NUMERICAL SIMULATION

In this section, we give three examples to demonstrate the effectiveness of the proposed method.

Example 1 (Stability analysis) Consider the nominal discrete singular system Eq.(4) with parameters as follows:

$$\boldsymbol{E}=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ \boldsymbol{A}=\begin{bmatrix}0.6&0.25\\0.8&1\end{bmatrix},\ \boldsymbol{A}_d=\begin{bmatrix}0.7&0.3\\0.25&-0.1\end{bmatrix}.$$

And the time-varying delay $d(k)$ satisfies Eq.(2).

Since the delay is time-varying, the results in (Xu and Lam, 2004; Ji *et al*., 2006) fail to make any decision for this example. We calculate the maximum value of $d(k)$ when $\underline{d}=0$ for which robust stability is guaranteed. According to Theorem 1, by letting $\boldsymbol{R}=[0\ 1]^{\mathrm{T}}$ and solving the feasibility problem of LMI (8), we have that this system is robustly stable for the maximum $\overline{d}=9$.

Example 2 Consider the singular system Eq.(1) with the following parameters:

$$\boldsymbol{E}=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ \boldsymbol{A}=\begin{bmatrix}0.6&0.25\\0.8&1\end{bmatrix},\ \boldsymbol{A}_d=\begin{bmatrix}0.7&0.3\\0.25&-0.1\end{bmatrix},$$

$$\boldsymbol{B}=\begin{bmatrix}1\\2\end{bmatrix},\ \boldsymbol{M}=\begin{bmatrix}0.2\\0.2\end{bmatrix},\ \boldsymbol{N}_a=\boldsymbol{N}_d=\begin{bmatrix}0.2&0.2\end{bmatrix},\ \boldsymbol{N}_b=1.$$

Supposing $\underline{d}=0$, according to Theorem 4, by letting $\boldsymbol{R}=[0\ 1]^{\mathrm{T}}$ and solving the feasibility problem of LMI (25), we have that the maximum $\overline{d}=8$ and a suitable robust state feedback controller

$$u(k)=\begin{bmatrix}-9.1371&-5.8610\end{bmatrix}\boldsymbol{x}(k),$$

which guarantees the resultant closed-loop system is robustly stable.

Example 3 Consider the following discrete-time system with a time-varying state delay (Gao and Chen, 2007):

$$\boldsymbol{x}(k+1)=\begin{bmatrix}0.8&0\\0.05&0.9\end{bmatrix}\boldsymbol{x}(k)+\begin{bmatrix}-0.1&0\\-0.2&-0.1\end{bmatrix}\boldsymbol{x}(k-d(k)).$$

Here, $d(k)$ represents a time-varying state delay. Now assume $\underline{d}=2$. We are interested in $\overline{d}$ below which the above system is stable for all $\underline{d}\le d(k)\le\overline{d}$. By using Theorem 1 in (Gao *et al*., 2004), it is found that $\bar{d}=7$. However, by applying Theorem 1 and letting $\boldsymbol{E}$=$\boldsymbol{I}$, $\boldsymbol{R}$=$\mathbf{0}$, we obtain $\bar{d}=10$, but $\bar{d}=13$ by using the method in (Gao and Chen, 2007). A more detailed comparison is given in Table 1, from which we can see that the stability conditions presented in this paper are less conservative than those in (Gao *et al*., 2004), but more conservative than those in (Gao and Chen, 2007), which further demonstrates Remark 4. The method in (Gao and Chen, 2007) might be a further research topic for discrete singular systems with time-varying delay.

Table 1 Maximum allowed time delay for different cases

$\underline{d}$	$\bar{d}$ by Gao *et al*.(2004)	$\bar{d}$ by this paper	$\bar{d}$ by Gao and Chen (2007)
2	7	10	13
4	8	11	13
6	9	11	14
10	12	13	15
12	13	14	17

CONCLUSION

The delay-dependent robust control for uncertain discrete singular systems with time-varying delay is studied in this paper. By establishing a finite sum inequality based on quadratic terms, a new delay-dependent robust stability condition is derived and expressed in terms of LMIs. Meanwhile, a control law design algorithm is also given, which guarantees that the resultant closed-loop system is regular, causal and stable for all admissible uncertainties and time-varying delay. Finally, three numerical examples are given to show the effectiveness of the proposed approach.

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