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Science Letters:

Lattice type transmission line of negative refractive index*

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Abstract: In this letter, we introduce a novel passive transmission line of negative refractive index (i.e., left-handedness) based on identical symmetrical lattice type structures [thus called "lattice type transmission line" (LT-TL)]. The dispersion characteristic and the transmission response of the proposed LT-TL are analyzed. While all the other left-handed passive transmission lines are of high pass, the present passive left-handed transmission line is of low pass. Compared with a conventional transmission line, the LT-TL has a phase shift of 180° in the entire wide pass-band.

Key words: Negative refractive index (NRI), Transmission line (TL), Symmetrical lattice, Low pass
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INTRODUCTION

Electric permittivity and magnetic permeability are two fundamental parameters for describing the electromagnetic responses of a continuous media. An artificial dielectric medium that exhibits simultaneously negative permittivity and permeability was studied theoretically by Veselago (1968). This medium is also known as a left-handed (LH) material with a negative refractive index (NRI). By periodically loading a conventional transmission line (C-TL) with lumped-elements of series capacitors and shunt inductors, a planar NRI TL was later realized (Iyer and Eleftheriades, 2002; Eleftheriades *et al*., 2002). To obtain an equivalent material with these negative parameters, the authors replaced the distributed series inductance and parallel capacitance in a C-TL with distributed series capacitance and parallel inductance and proposed a high pass NRI TL (Iyer and Eleftheriades, 2002). So far, all the existing LH TLs are of high pass, except recent work by Hu and He (2007), in which the authors demonstrated an active NRI TL of low pass with active components.

In this paper we propose a new model for an NRI TL without active components. In the design, we utilize a structure of identical symmetrical lattice type, and call it "lattice type transmission line" (LT-TL). Due to its balanced configuration, the model can be realized using coplanar stripline (CPS), which is useful in applications such as printed dipole antenna feeding, rectennas, and uniplanar mixers.

THEORY

In the proposed TL, we make use of the symmetric lattice section (Scott and Essigman, 1960) as shown in Fig.1a. The lattice section is a two-terminal-pair (i.e., four-terminal) network of balanced symmetry. We first calculate the impedance between terminals 1 and 2 when terminals 3 and 4 are open (denoted as open circuit impedance Z_{oc}) and when terminals 3 and 4 are shorted (denoted as short circuit impedance Z_{sc}). One can show easily that this lattice section is equivalent to the T-section of Fig.1b, as in each case the open/short circuit impedances have the

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have the same values:

$$Z_{\rm oc}=\frac{Z_{\rm A}+Z_{\rm B}}{2},\quad Z_{\rm sc}=\frac{2Z_{\rm A}Z_{\rm B}}{Z_{\rm A}+Z_{\rm B}}. \tag{1}$$

In this way a lattice network can be transformed into its equivalent unbalanced circuit of T-section network.

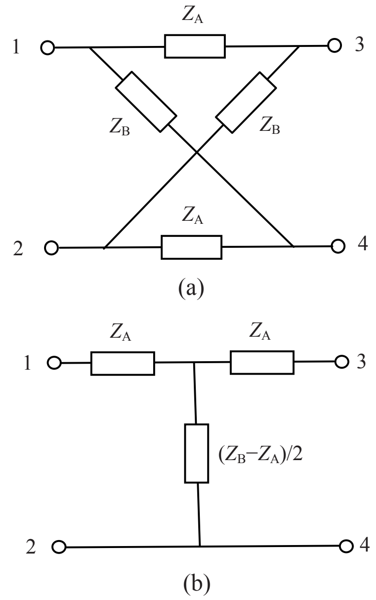

Fig.1 Symmetrical lattice section (a) and its equivalent T-section (b)

When $Z_{\rm A}$ is a capacitance [i.e., $Z_{\rm A}=1/({\rm j}\omega C)$] and $Z_{\rm B}$ a wire (i.e., $Z_{\rm B}=0$), we obtain the lattice structure as shown in Fig.2a (which will be utilized in our proposed NRI TL) and

$$(Z_{\rm B}-Z_{\rm A})/2=1/[{\rm j}\omega(-2C)]. \tag{2}$$

Thus, we obtain its equivalent T-section circuit as shown in Fig.2b.

Fig.3a shows the unit cell of the proposed LT-TL consisting of a lattice of capacitors and wires (as shown in Fig.2a) and 4 series inductances. Its equivalent unbalanced circuit of T-section network can be presented in Fig.3b, where the series impedance $Z_{\rm ser}$ and parallel admittance $Y_{\rm par}$ are given by

$$Z_{\rm ser}={\rm j}\omega\left(2L-\frac{1}{\omega^2 C}\right),\quad Y_{\rm par}=-2{\rm j}\omega C. \tag{3}$$

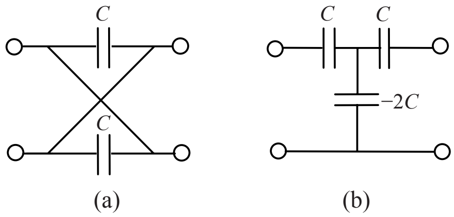

Fig.2 The lattice structure used in our design (a) and its equivalent circuit (b)

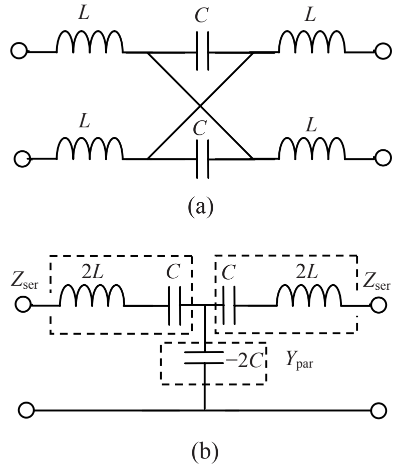

Fig.3 The unit cell of the proposed lattice type transmission line (LT-TL) (a) and its equivalent circuit (b)

From the equivalent circuit we can derive the following transmission matrix of the unit cell:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} 1+Z_{\rm ser}Y_{\rm par} & Z_{\rm ser}(2+Z_{\rm ser}Y_{\rm par}) \\ Y_{\rm par} & 1+Z_{\rm ser}Y_{\rm par} \end{pmatrix}. \tag{4}$$

Thus, the dispersion relationship for such a periodic structure is

$$\cosh(rd)=\frac{A+D}{2}=1+Z_{\rm ser}Y_{\rm par}=\omega^2(2L)(2C)-1, \tag{5}$$

where $r=\alpha+{\rm j}\beta$ is the propagation constant for the periodic structure.

For comparison, the unit cell of a C-TL with balanced symmetry or four terminals [e.g., CPS] and its equivalent circuit are shown in Figs.4a and 4b, respectively.

The dispersion relation of the C-TL can be written as:

$$\cosh(rd)=1+Z_{\rm ser}Y_{\rm par}=1-\omega^2(2L)(2C), \tag{6}$$

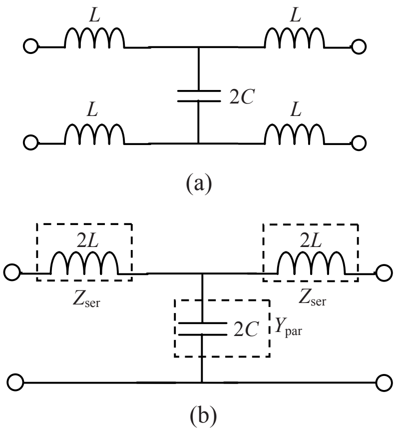

Fig.4 The unit cell of a conventional transmission line (C-TL) (a) and its equivalent circuit (b)

The dispersion relations (5) and (6) are plotted in Fig.5 with L=1 nH and C=0.2 pF. In Fig.5 we can see that the dispersion curve of LT-TL (solid curve) shows the same magnitude response, i.e., a low-pass band between 0 and cut-off frequency ω_c, as the C-TL (dashed curve). In the pass-band for LT-TL, the phase velocity (i.e., the slope of the line from the origin to a point on the dispersion curve) has a sign opposite to the energy velocity (i.e., group velocity, the slope of the tangent to the dispersion curve). This indicates that it is a left-handed (or backward wave) pass-band. Furthermore, the phase shift difference between the LT-TL and C-TL keeps a constant of 180° over the entire pass-band [this is caused by the different sign of Eqs.(5) and (6)].

To get a perfect match to the LC network of the proposed structure, we should also consider the ratio of voltages and currents in the network (with the unit cell of Fig.3a). If the network is periodic, the ratio, which is called the Bloch impedance Z_B or characteristic impedance, has a constant value at any nth point of the structure. We can derive the following expression for Z_B:

$$\begin{aligned} Z_B &= \frac{B}{\sqrt{A^2-1}} = \sqrt{(Z_{ser}Y_{par})^2 + 2Z_{ser}Y_{par}} \Big/ Y_{par} \\ &= \sqrt{\frac{L}{C}\left[1-(4\omega^2 LC-1)\right]} = \sqrt{\frac{L}{C}\left[1-\cosh(rd)\right]}. \end{aligned} \quad (7)$$

Since the characteristic impedance Z_B is frequency-dependent, broadband matching is complicated. However, if the frequency of interest lies in the range where cosh(rd)<<1, the characteristic impedance can be simplified as $Z_B \approx \sqrt{L/C}$, which is almost a constant.

To verify our analysis, we make a comparison in Fig.6 between the transmission responses of a C-TL and the proposed LT-TL consisting of 5 unit cells as shown in Fig.3a. In our calculation, the values of C

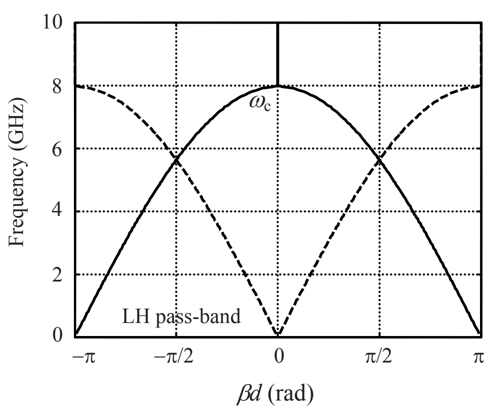

Fig.5 Dispersion relations of the proposed lattice type transmission line (LT-TL) (solid curve) and conventional transmission line (C-TL) (dashed curve)

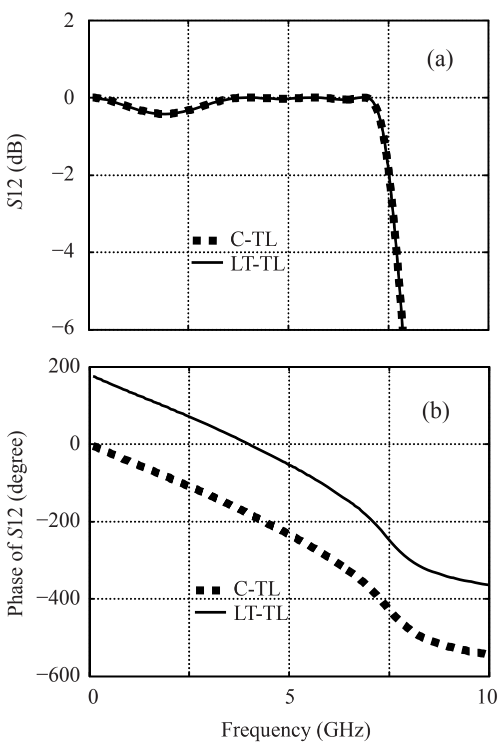

Fig.6 Amplitude responses (a) and phase responses (b) of the proposed lattice type transmission line (LT-TL) and conventional transmission line (C-TL)

and L in the unit cells are 0.2 pF and 1 nH, respectively, and the terminal impedance is 70.7 Ω. Fig.6 also shows that the LT-TL has the same low pass-band as a C-TL, but a 180° phase difference, which is consistent with Fig.5.

CONCLUSION

We have demonstrated a novel realization of a left-handed transmission line by utilizing a lattice circuit in the unit cell. Its dispersion relation and characteristic impedance have been analyzed. The transmission response of the LT-TL has also been given. While all the other left-handed passive transmission lines are of high pass, the present passive left-handed transmission line is of low pass. The proposed LT-TL has been shown to have the same low-pass property as a C-TL, but with a 180° phase shift from a C-TL over the whole pass-band. This can lead to many interesting applications such as hybrid rings and filters. The present theory can be generalized to high dimensions.

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