Journal of Zhejiang University SCIENCE A
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Research in non-equalization machining method for spatial cam*

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Received Dec. 3, 2007; revision accepted Mar. 24, 2008

Abstract: Many kinds of devices with cam have been widely used in various mechanical equipments. However, non-equalization machining for spatial cam trough remains to be a difficult problem. This paper focuses on the analysis of running conditions and machining processes of spatial cam with oscillating follower. We point out the common errors in the biased distance cutting. By analyzing the motion of oscillating follower of spatial cam, we present a new 3D curve expansion model of spatial cam trough-outline. Based on this model, we have proposed a machining method for trochoidal milling with non-equalization diameter cutter. This new method has led to a creative and effective way for non-equalization diameter machining for spatial cam with oscillating follower.

Key words: Oscillating follower, Spatial cam, 3D curve expansion, Trochoidal milling, Non-equalization diameter
doi:10.1631/jzus.A0720097 **Document code:** A **CLC number:** TH122; TG711

INTRODUCTION

According to the different motion styles of their followers, spatial cams are generally divided into two categories, i.e., cam with translating follower and cam with oscillating follower. Designing and machining for spatial cam with translating follower are relatively easy based on its expanded plane figure, whereas designing and machining for spatial cam with oscillating follower are complicated and difficult. The latter is especially true for non-equalization diameter machining (also called un-equivalent machining, i.e., when the width of cam trough is bigger than that of machining cutter, the trough-outline needs to be machined at single side with small edge cutter). During the rapid and precise manufacturing of cam trough, one has to ensure its exact dimension and surface quality. The common practice in production is first to machine coarsely with a milling cutter whose diameter is smaller than the roller's diameter, then to precisely machine its single side with small milling cutter or grinding wheel to ensure that cam trough can fit in exactly with follower and to improve the motion precision of spatial cam. As the spatial cam figure is a complicated spatial curve which cannot be expanded, the non-equalization diameter machining is a difficult problem and catches the attention of domestic and oversea researchers.

Hsieh (2007) presented a simple yet comprehensive method for the design and machining of a cylindrical cam with a meshing indexing disc. Chen and Wu (2007) showed that the motion path of tapered roller on oscillating bar could be expanded on the cylinder and presented a method for designing and machining of 3D curve expansion of conical cam with oscillating follower. Lee and Lee (2007) proposed an interference-free toolpath generating method for five-axis machining of a spatial cam. Chen and Xin (2007) applied the 3D curve expansion system to the cylindrical cam design and manufacturing. Yin *et al.*(2002) used the web-based remote

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* Project supported by the National Natural Science Foundation of China (No. 50575205), the Hi-Tech Research and Development Program (863) of China (No. 2006AA04Z233), the Natural Science Foundation of Zhejiang Province (No. Y104243), and the Natural Science Foundation of Ningbo (Nos. 2008A610038 and 200703B 1003018), China

design system to formulate spatial cam mechanisms based on mathematical models. Lee (1998) presented a method for finding the admissible tool orientation by considering both local gouging and rear gouging. Ge *et al.*(2006) deduced the contour surface equation of spatial cam based on pure rolling condition of roller on the cam contour surface. Li and Yin (2003) proposed that the contour surface of cylindrical cam could be expanded from linear surface. Grant and Soni (1999) worked out the analytic expression for cam contour surface of conical cam mechanism with oscillating tapered roller follower in 3D space by applying the theory of envelopes to one parameter family of surfaces.

However, all these studies are limited to the question as how to establish the cam figure models. The information for design and machining is still mostly at the stage of theoretical analysis, which has seriously limited its practical application. Moreover, precise machining of spatial cam with oscillating follower is especially hard to achieve.

By focusing on spatial cam mechanism with oscillating follower, this paper presents that the motion can be divided into two categories, i.e., the oscillating motion of oscillating bar and the rotary motion of oscillating bar relative to spatial cam. With the expansion of rotary motion of oscillating bar, we propose a new 3D curve expansion model of spatial cam trough-outline, and based on it, suggest a machining method for trochoidal milling with non-equalization diameter cutter. This new method has led to a creative and effective way for machining of spatial cam with precise oscillating follower.

ANALYSIS OF THE RUNNING COURSE OF THE SPATIAL CAM WITH OSCILLATING FOLLOWER

The model of spatial cam mechanism with oscillating follower is shown in Fig.1. The oscillating axis of oscillating bar intersects with the rotary axis of spatial cam in different spaces. Line *MN* is the common perpendicular line of these two axes, with *N* as the point of intersection between common perpendicular and rotary axis of spatial cam as the origin of coordinate frames. The rotary axis of spatial cam is the *X*-axis and the common perpendicular is the *Y*-axis.

Here is the hypothesis: assume that *S* is the displacement of roller of oscillating bar (mm); ψ the oscillating angle of oscillating bar (°), *l* the length of oscillating bar (mm), φ the rotary angle of spatial cam (°), *a* the distance from the rotary axis of oscillating bar to the rotary axis of spatial cam (i.e., the length of common perpendicular line *MN*) (mm). The trough in the follower of spatial cam is a 3D spatial figure. It surrounds the spatial cam in line with a certain motion rule. Generating cutting, usually adopted in machining of conical cam trough, means that the spatial cam meeting the demand of design is produced in the following way: replace tapered roller, i.e., follower of spatial cam mechanism with cutter with the same figure and dimension, and then simulate the correlative motion relationship under the practical running condition of spatial cam and tapered roller follower in the way of numerical control.

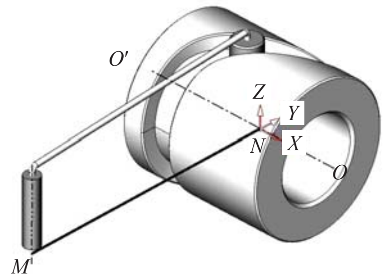

Fig.1 Mechanism of spatial cam with oscillating follower

When the trough of spatial cam with oscillating follower is not wide, there is no strict demand for an exact precision. A milling cutter in the same dimension with oscillating follower roller could be used. It is quite easy to get the trough of spatial cam with oscillating follower in accordance with the above demands through numerical control machining by using the method of generating cutting along trough-center line.

But this machining method has many limitations. When higher precision is demanded for the trough of spatial cam with oscillating follower, in other words, coarsely machining does not work; or when the trough of spatial cam is too wide to find a milling cutter in its dimension, it is hard to guarantee that the width of the obtained trough of spatial cam with oscillating follower is in accordance with the requirement of processing precision. In general, the latter case is called non-equalization diameter machining of spatial cam with oscillating follower.

"Offset" method has been referred in lots of literature (Lin and Tsai, 1996; Tsay and Lin, 1996; Zhang *et al.*, 1997; Xiao *et al.*, 2005). As shown in Fig.2, it is "Offset" machining the return part of trough of spatial cam with oscillating follower. Suppose the trough-width is 16 mm. After rough machining, fine "Offset" machining is practiced with a milling cutter whose diameter is 5 mm.

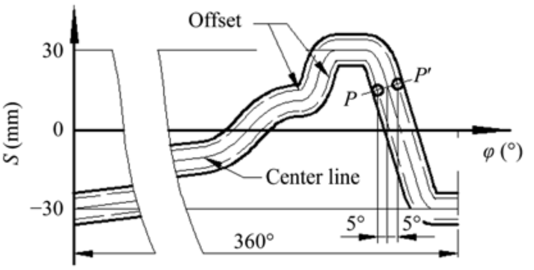

Fig.2 Cutter motion path of "offset" machining

On the trough-width section *P*-*P*′, between the point of "Offset" machining and the point of theoretical figure line (center line) of cam trough, deviation occurs in the rotary angle of cam. It is ±5° as shown in Fig.2, namely, the spatial cam rotary angles corresponding to the "Offset" machining point and theoretical machining point are different. Neither milling cutter center lines of each machining point are parallel to each other, nor are on the same plane. Section *P*-*P*′ can be approximately depicted in Fig.3. When the top of trough-width section meets the dimensional demands, there is a deviation of ±5° between the milling cutter center line of "Offset" machining point and that of the theoretical machining point corresponding to the cam rotary angle. As a result, the bottom of trough-width section is smaller than the dimensional demands. When this happens, the interference between follower and trough would occur (as shown in Fig.3) and lead to the so-called dead-lock phenomenon. Nishioka (2003) evaluated the offset error of cylindrical cam.

To solve this problem, this paper, by using the 3D expansion of spatial cam with oscillating follower, presents a new effective machining method for spatial cam wide-trough with oscillating follower, which can guarantee the precision of processing spatial cam trough.

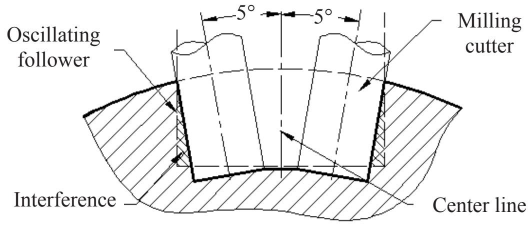

Fig.3 Interference between follower and trough

3D EXPANSION OF SPATIAL CAM WITH OSCILLATING FOLLOWER

Spatial cam is turning around with the axis *OO*′. During the rotary course, the distance between each point of spatial mechanism (including spatial roller, follower and oscillating bar frame) and the common perpendicular line *MN* is invariable, whereas its position is constantly changing with the rotary angle of spatial cam φ. When we apply the kinematical inversion widely used in the design of cam contour surface, the roller follower and oscillating bar frame should reverse every round around point *N* and *YZ*-plane if the spatial cam is kept motionless, as shown in Fig.4.

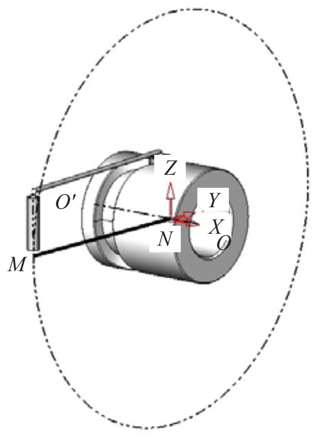

Fig.4 Relative motion path of oscillating bar

As shown in Fig.5, following the double dot line, the rotation of machine frame is expanded to a line and is perpendicular to the surface, on which the oscillating bar is oscillating.

The direction of the line (i.e., φ-direction of spatial cam rotary angle) is parallel to *Z*-axis in the coordinating frame. Arched motion of oscillating follower rotating with spatial cam can then be expanded to a 3D cylinder motion.

Oscillating motion of oscillating bar is driven by the rotation of spatial cam. Based on the demands of motion path of machining, a general relationship can be deduced with spatial cam rotary angle as parameters in the following equation:

$$S=f(\varphi). \tag{1}$$

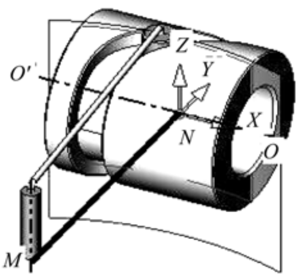

Fig.5 Expansion of spatial curved surface

A displacement curve (subsection curve) is drawn based on Eq.(1). As shown in Fig.6 (if oscillating angle is between ±30°), the motion relationship curve with displacement of oscillating follower and rotary angle of spatial cam can be established.

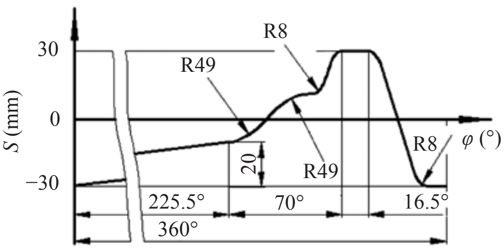

Fig.6 Displacement curve

The (φ, S) value of each point in the displacement curve can determine the correlative point on the cylinder surface, and the value (X, Y, Z) can be established as follows:

$$\begin{cases} X = S, \\ Y = \sqrt{l^2 - S^2} - a, \\ Z = \varphi. \end{cases} \tag{2}$$

The geometrical relationship between oscillating angle ψ of oscillating bar and follower displacement S is shown in Fig.7. As seen from the sketch, through the vertex of oscillating angle ψ, one can make an arc if considering oscillating bar length l as radius, which will intersect with angle border at points A and B, and then make a perpendicular line AC through A, with C as the perpendicular base point. The length of AC is equal to the displacement S of oscillating follower (roller) in cam axis direction (i.e., X-direction), which corresponds to the oscillating angle ψ.

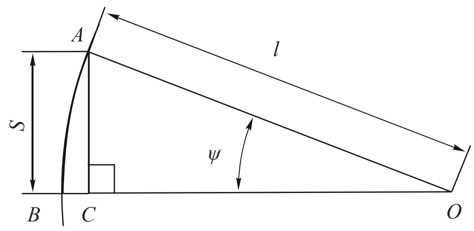

Fig.7 Geometrical relationship between oscillating motion and angle

Mathematical relationship between displacement of oscillating follower and oscillating angle of oscillating bar can be expressed as:

$$S=l\sin\psi. \tag{3}$$

By projecting the displacement curve on the cylinder whose radius is l, the length of oscillating bar, the obtained curve is a 3D expansion of motion path of oscillating follower, as shown in Fig.8.

Accordingly, the value for coordinate-Z of each point in 3D curve is equal to the degree of spatial cam rotary angle; and the values for coordinate-X and coordinate-Y are respectively equal to the coordinate-

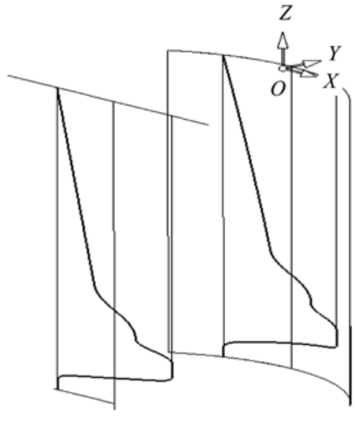

Fig.8 3D expansion of motion path

values of arched motion of oscillating tapered roller follower when spatial cam is rotating to degree φ.

NON-EQUALIZATION DIAMETER "TROCHOIDAL MILLING" MACHINING METHOD OF SPATIAL CAM

Based on the above 3D curve expansion of motion path of oscillating follower, cutting generation can also be adopted in the machining of trough of spatial cam with oscillating follower.

As shown in Fig.9, by moving and rotating motion path of oscillating follower, CAM software can be carried out according to (Lin and Tsai, 1996). This paper will further discuss non-equalization diameter generating cutting of spatial cam.

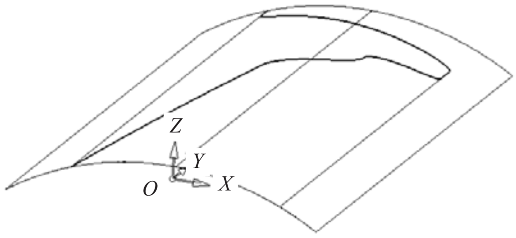

Fig.9 Sketch map of 3D motion path of follower

Cycloidal milling

Cycloidal milling is a new scheme of cutter motion path. The so-called cycloid describes a path formed when a fixed point in the circle is rolling along the curve of the circle. As the cutter is always running along a fixed-curvature curve during the cutting process, the motion of cutter can be kept at an accordant feeding-speed (Wu *et al.*, 2006; Jung and Psang, 2007; Stanislav, 2007).

The cutter motion path of the trochoidal milling, exploited on the basis of cycloidal milling, is extremely similar to that of cycloidal milling. The difference only is that the cutter motion path of trochoidal milling is composed of beelines (i.e., troch step) and circles (as shown in Fig.10). When the value of troch step is tiny, trochoidal milling has an obvious advantage in programming, since it is easier to use a shorter program, and therefore can improve the processing speed of the numerical control system.

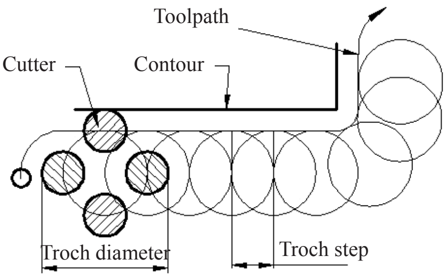

Fig.10 Trochoidal milling

Non-equalization diameter "imitated trochoidal milling" of spatial cam trough

If the bottom of trough-width section is smaller than the dimensional demands when "Offset" method is applied, an interference of oscillating follower and trough would form leading to a dead-lock phenomenon in follower motion.

To avoid this problem, we propose a non-equalization diameter "trochoidal milling". Along the trough center line (i.e., the theoretical line of spatial cam figure), the cutter runs one troch step by one circle, and then another troch step by another circle. The length of each troch step could be different, but the circles are equirotal. The smaller the troch step is, the higher the precision is. As shown in Fig.11, when the cutter finishes one troch step, the cam billet will pause its rotating motion. When the milling cutter is making circling motion in the oscillating plane of oscillating bar, the center of the circle is in the trough center line. The cam billet is immovable during the process of circling motion, as the axes of milling cutter of corresponding machining points is parallel to each other, as shown in *P*-*P*′ of Fig.11. Then the dead-lock phenomenon could be avoided.

In accordance with the 3D expansion of spatial

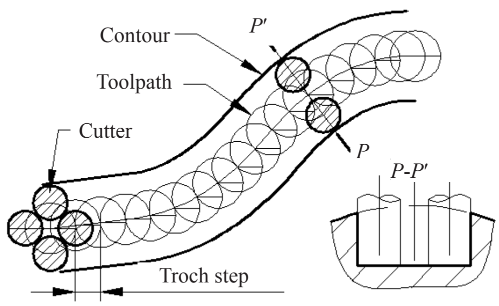

Fig.11 Imitated trochoidal milling

cam trough, the cutter motion path of non-equalization diameter can be established as shown in Fig.12. In order to improve the precision of trough, each troch step should be small enough. The required precision of machining spatial cam trough can be ensured by changing the size of the circle.

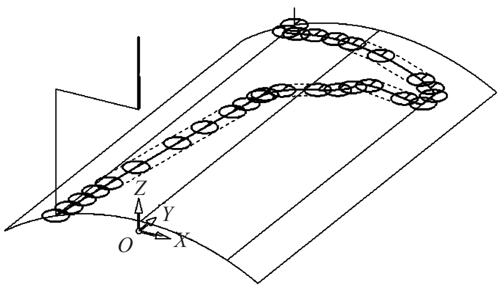

Fig.12 Practical CAM programming

CONCLUSION

By analyzing the errors in the "offset" machining, we have proposed a machining method for trochoidal milling with non-equalization diameter cutter, and the programming and machining with "trochoidal milling" is not only adaptable to spatial cam with oscillating follower, but also adaptable to spatial cam with ordinary follower. In this sense, it is a universal non-equalization diameter programming method of cam. By giving different values to troch diameter, different machining allowances are left in the surface of cam trough. Then coarse machining, semi-fine machining and fine machining can be finished by using a milling cutter with diameter smaller than trough-width, a small-emery-wheel bistrique with handle or a carbide alloy revolution file, etc. It not only solves the existing problems in non-equalization diameter machining of spatial cam, but also meets the different precision demands and reduces the machining cost.

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