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Nonlinear dynamic response of stay cables under axial harmonic excitation*

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Abstract: This paper proposes a new numerical simulation method for analyzing the parametric vibration of stay cables based on the theory of nonlinear dynamic response of structures under the asynchronous support excitation. The effects of important parameters related to parametric vibration of cables, i.e., characteristics of structure, excitation frequency, excitation amplitude, damping effect of the air and the viscous damping coefficient of the cables, were investigated by using the proposed method for the cables with significant length difference as examples. The analysis results show that nonlinear finite element method is a powerful technique in analyzing the parametric vibration of cables, the behavior of parametric vibration of the two cables with different Irvine parameters has similar properties, the amplitudes of parametric vibration of cables are related to the frequency and amplitude of harmonic support excitations and the effect of distributed viscous damping on parametric vibration of the cables is very small.

Key words: Stay cables, Parametric vibration, Nonlinear vibration, Viscous damping
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INTRODUCTION

The vibration of cable in a cable-stayed bridge results in fatigue of the cable anchorage, shortens the service life of cable-stayed bridges, significantly affects the smoothness of the moving vehicles and the safety of the structures. The strong vibration of stay cables and their severe consequences were observed in many bridges (Lilien and Pinto, 1994; Chen and Sun *et al.*, 2003; Yang and Chen, 2005). Therefore, vibration control of stay cables is a very important issue in the design of long span cable-stayed bridges.

The parametric vibration of stay cables is a typical phenomenon for cable-stayed bridges. It happens when the excitation frequency is an integer multiplier of the cable frequencies, and the periodical varying cable tension will induce large amplitude vibrations of the cables.

Parametric vibration of cables was observed long time ago. However, research on this topic started mainly in the 1980s. Takahashi and Konishi (1987a; 1987b) proposed a method for calculating the nonlinear free vibration of cables based on the Galerkin method and Harmonic Wave Equilibrium method, and they also used the eigenvalue method to estimate the range of parameters that may induce potential out-of-plane unstable vibration of cables due to in-plane harmonic loading. Furthermore, Takahashi (1991) analyzed the region of parameters that result in unstable parametric vibration of cables due to periodically varying axial forces. Wu *et al.*(2003) studied the relationship between frequencies of a cable-stayed bridge and those of stay cables, the dynamic responses of cables under simple harmonic axial excitation and the characteristics of parametric vibration. Utilizing the Galerkin method, Uhrig (1993) analyzed and proposed a parameter plane of unstable vibration of the cables due to tower support excitation. Lilien and Pinto (1994) also used the Galerkin method to study the parametric vibration of cables. Georgakis and Taylor (2005a; 2005b) studied the nonlinear dy-

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namic response of cables using finite element method (FEM) and simulated the stay cables by link elements.

The Galerkin method was also widely used by other researchers. Yang and Chen (2005) investigated the parametric vibration problem of Sutong Bridge in China. Kang and Zhong (1998) studied the mechanisms of parametric vibration of cables using a two-degree-of-freedom model, and analyzed the dynamic characteristics of parametric vibration of stay cables using an example of a simple cable-stayed bridge. Chen *et al.*(2002) and Chen and Sun (2003) studied the nonlinear dynamic response of parabolic cable due to axial excitation considering the effects of inclination angle, damping and sag of cables. Sun *et al.*(2003) derived the equations of parametric vibration of stay cables due to cable support excitation, and studied the effects of inclination, initial tension, excitation frequencies and vibration control technique in detail. It is observed that the regularities of parametric vibrations of stay cables are not very similar among the results obtained by different researchers, and the corresponding explanations are different either.

Among the abovementioned researches using the Galerkin method, trigonometric function was employed as the trial function. Compared with the Galerkin method, FEM is more adaptive, since the structural status at different time steps are determined by the equilibrium equations without introducing the assumptions used in the Galerkin method. And the FEM is more feasible in considerations of wind load and the effect of dampers installed on the cable.

This paper proposes a new method for analyzing the parametric vibration of stay cables based on nonlinear vibration theory. The application of this method was demonstrated by two examples of stay cables with different spans, and the effects of distributed viscous damping of the cables, the excitation frequency and the amplitude of support excitation on nonlinear vibrations of cables were thoroughly investigated.

METHOD

Nonlinear vibration equation of flexible cable element

The nonlinear equation of motion for the flexible cables may be written as the following increment form based on updated Lagrange formulation:

$$\boldsymbol{m}\Delta\ddot{\boldsymbol{x}} + \boldsymbol{c}\Delta\dot{\boldsymbol{x}} + \boldsymbol{k}\Delta\boldsymbol{x} = \Delta\boldsymbol{f}, \quad (1)$$

where $\boldsymbol{m}$ is the mass matrix, $\boldsymbol{c}$ the damping matrix, $\boldsymbol{k}$ the tangent stiffness matrix, $\Delta\boldsymbol{x}$ the displacement increment of the cable, $\Delta\boldsymbol{f}$ the load increment; $\Delta\dot{\boldsymbol{x}}$ and $\Delta\ddot{\boldsymbol{x}}$ are respectively the first and second derivatives of $\Delta\boldsymbol{x}$ relative to time.

The mass, damping and tangent stiffness matrices for each element can be derived using isoparametric element method and virtual work principle, and expressed as

$$\begin{cases} \boldsymbol{k}^{\mathrm{e}} = EA\int_S \boldsymbol{B}_L^{\mathrm{T}}\boldsymbol{B}_L\mathrm{d}s + \int_S T\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}\mathrm{d}s, \\ \boldsymbol{m}^{\mathrm{e}} = \overline{m}\int_S \boldsymbol{N}^{\mathrm{T}}\boldsymbol{N}\mathrm{d}s, \\ \boldsymbol{c}^{\mathrm{e}} = \overline{c}\int_S \boldsymbol{N}^{\mathrm{T}}\boldsymbol{N}\mathrm{d}s, \end{cases} \quad (2)$$

where S is the length of the element, T the cable tension, EA the tensile stiffness, $\overline{m}$ and $\overline{c}$ the mass and viscous damping per unit length, respectively. For the element shown in Fig.1, the shape function $\boldsymbol{N}$ and matrices $\boldsymbol{B}_L$ and $\boldsymbol{G}$ can be written as (Xie *et al.*, 2008)

$$\boldsymbol{N} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 \end{bmatrix}, \quad (3)$$

$$\boldsymbol{B}_L = \frac{\partial}{\partial s}\begin{bmatrix} N_1\boldsymbol{e}_{\mathrm{s}}^{\mathrm{T}} & N_2\boldsymbol{e}_{\mathrm{s}}^{\mathrm{T}} & N_3\boldsymbol{e}_{\mathrm{s}}^{\mathrm{T}} \end{bmatrix} - \begin{bmatrix} N_1\boldsymbol{e}_{\mathrm{t}}^{\mathrm{T}} & N_2\boldsymbol{e}_{\mathrm{t}}^{\mathrm{T}} & N_3\boldsymbol{e}_{\mathrm{t}}^{\mathrm{T}} \end{bmatrix}/\mathrm{R}, \quad (4)$$

$$\boldsymbol{G} = \begin{bmatrix} \dfrac{\partial\Delta u_{\mathrm{t}}}{\partial s} + \dfrac{\Delta u_{\mathrm{s}}}{R} \\ \dfrac{\partial\Delta u_{\mathrm{n}}}{\partial s} \\ \dfrac{\partial\Delta u_{\mathrm{s}}}{\partial s} - \dfrac{\Delta u_{\mathrm{t}}}{R} \end{bmatrix}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{N}, \quad (5)$$

and

$$\begin{cases} N_1 = \varsigma(\varsigma-1)/2, \\ N_2 = 1-\varsigma^2, \\ N_3 = \varsigma(\varsigma+1)/2, \end{cases} \quad (6)$$

$$\boldsymbol{e}=[\boldsymbol{e}_\mathrm{t}\quad \boldsymbol{e}_\mathrm{n}\quad \boldsymbol{e}_\mathrm{s}]=\begin{bmatrix} e_{11} & e_{12} & e_{13}\\ e_{21} & e_{22} & e_{23}\\ e_{31} & e_{32} & e_{33}\end{bmatrix}, \tag{7}$$

where ς is the dimensionless local coordinate system, and the coordinates of the two ends of the element are −1 and +1. $\boldsymbol{e}_\mathrm{s}$ is the tangential direction vector, $\boldsymbol{e}_\mathrm{n}$ the direction vector normal to the curvature plane, $\boldsymbol{e}_\mathrm{t}$ the direction vector in curvature plane; $\boldsymbol{e}_{ij}$ is the directional cosine of local coordinate j in global coordinate i; R is the curvature radius of the cable.

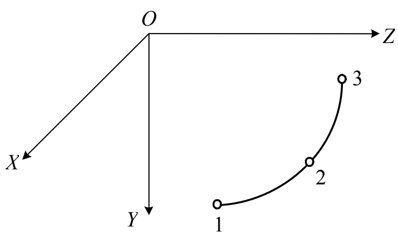

Fig.1 Three-node isoparametric cable element

The drag force F_D acting on the cable introduced by the relative motion of the cable to the air is expressed as

$$F_\mathrm{D}=-\frac{1}{2}\rho C_\mathrm{D}\phi\dot{y}, \tag{8}$$

where ϕ is the outer diameter of the cable, C_D the drag coefficient, which is assumed to be independent of the attack angle, $\dot{y}$ the velocity in y direction, ρ the density of air.

Nonlinear dynamic response of cables under support excitation

Cable vibration due to arbitrary support excitation can be analyzed with the method for the calculation of structural response due to ground motion excitation. As shown in Fig.2, the cable with span L and vertical distance Y between cable ends is excited by asynchronous support movement. The displacement of structure can be expressed as

$$\Delta\boldsymbol{x}=\begin{bmatrix}\Delta\boldsymbol{x}_\mathrm{a}\\ \Delta\boldsymbol{x}_\mathrm{b}\end{bmatrix}, \tag{9}$$

where subscripts "a" and "b" represent displacement of the non-support node and the support node, respectively. Then, Eq.(1) can be rewritten as

$$\begin{bmatrix}\boldsymbol{m}_\mathrm{a} & \boldsymbol{m}_\mathrm{ab}\\ \boldsymbol{m}_\mathrm{ab}^\mathrm{T} & \boldsymbol{m}_\mathrm{b}\end{bmatrix}\begin{bmatrix}\Delta\ddot{\boldsymbol{x}}_\mathrm{a}\\ \Delta\ddot{\boldsymbol{x}}_\mathrm{b}\end{bmatrix}+\begin{bmatrix}\boldsymbol{c}_\mathrm{a} & \boldsymbol{c}_\mathrm{ab}\\ \boldsymbol{c}_\mathrm{ab}^\mathrm{T} & \boldsymbol{c}_\mathrm{b}\end{bmatrix}\begin{bmatrix}\Delta\dot{\boldsymbol{x}}_\mathrm{a}\\ \Delta\dot{\boldsymbol{x}}_\mathrm{b}\end{bmatrix}+\begin{bmatrix}\boldsymbol{k}_\mathrm{a} & \boldsymbol{k}_\mathrm{ab}\\ \boldsymbol{k}_\mathrm{ab}^\mathrm{T} & \boldsymbol{k}_\mathrm{b}\end{bmatrix}\begin{bmatrix}\Delta\boldsymbol{x}_\mathrm{a}\\ \Delta\boldsymbol{x}_\mathrm{b}\end{bmatrix}=\begin{bmatrix}\Delta\boldsymbol{f}_\mathrm{a}(t)\\ \Delta\boldsymbol{f}_\mathrm{b}(t)\end{bmatrix}. \tag{10}$$

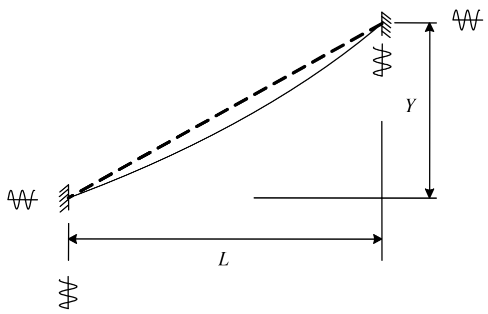

Fig.2 Cable under support excitation

The equation related to dynamic displacement can be derived from Eq.(10) as

$$\boldsymbol{m}_\mathrm{a}\Delta\ddot{\boldsymbol{x}}_\mathrm{a}+\boldsymbol{c}_\mathrm{a}\Delta\dot{\boldsymbol{x}}_\mathrm{a}+\boldsymbol{k}_\mathrm{a}\Delta\boldsymbol{x}_\mathrm{a}=\Delta\boldsymbol{f}_\mathrm{a}(t)-\boldsymbol{m}_\mathrm{ab}\Delta\ddot{\boldsymbol{x}}_\mathrm{b}-\boldsymbol{c}_\mathrm{ab}\Delta\dot{\boldsymbol{x}}_\mathrm{b}-\boldsymbol{k}_\mathrm{ab}\Delta\boldsymbol{x}_\mathrm{b}. \tag{11}$$

If the cable only experiences the end support excitations, then $\boldsymbol{f}_\mathrm{a}(t)$ is calculated according to F_D.

Eq.(11) can be solved by iteration technique using certain convergence criteria. The cable tension T in every step is obtained from the equation of motions and updated automatically.

EXAMPLES

Cable parameters

The parameters of the two cables, which are extracted from a proposed cable-stayed bridge with main span of 1400 m, are shown in Table 1 (Xie *et al.*, 2008), where L is the cable span, which is the horizontal projection length of the cable, Y is the height difference of the cable supports, w the weight per unit length, β the ratio of sag to cable span, and λ the non-dimensional Irvine parameter (Irvine, 1981). The free vibration frequencies of the cables are shown in Table 2.

Table 1 Parameters of the steel cables

Cable	L (m)	Y (m)	EA (kN)	w (kN/m)	β	λ
S-1	660	275.5	3045840	1.3114	0.01750	1.20
S-2	260	195.5	1795360	0.7730	0.00912	2.68

Table 2 Frequencies of the cables (Hz)

Cable	In-plane		Out-of-plane	
	1st	2nd	1st	2nd
S-1	0.1998	0.3260	0.1633	0.3259
S-2	0.3732	0.7189	0.3597	0.7189

Assume that the lower support end is experiencing harmonic excitation in the direction of cable chord, the vertical and horizontal components of the excitation can be written as

$$\begin{cases} u_x = A_0 \sin(2\pi f t)\cos\theta, \\ u_y = -A_0 \sin(2\pi f t)\sin\theta, \end{cases} \tag{12}$$

where A_0 is the amplitude, which is assumed to be 1/5000, 1/10000 and 1/50000 of the cable span, respectively. f is the excitation frequency, and two values are used in this study: $f=f_1$ and $2f_1$, where f_1 is the first in-plane vibration frequency. θ is the angle between the horizontal axis and cable chord. Y axis is positive if downward.

The damping $\bar{c}$ of the cable is assumed to be zero if there is no explicit indication. The drag coefficient C_D=0.7.

Nonlinear dynamic characteristics of the long cable S-1

Fig.3 shows the vertical displacement response and corresponding frequency spectra at the middle point of the longer cable S-1 due to excitation of frequency f_1. The vibration lasts for 2500 s. Because of the effect of drag force, the amplitude of dynamic displacement experiences a periodic variation and becomes smooth and stationary.

Pattern of cable vibration due to support excitations with different amplitudes is identical, which is initially nonlinear forced vibration with periodic varying amplitude and ultimately parametric vibration with steady amplitude. The amplitude of parametric vibration is proportional to the support excitation amplitude, since the parametric vibration of the cable is affected by cable tension, and the support excitation with a big amplitude will induce significant changes of cable tension, thus result in a remarkable variation of the cable parameters. On the contrary, it is hard for small amplitude excitation to induce para-

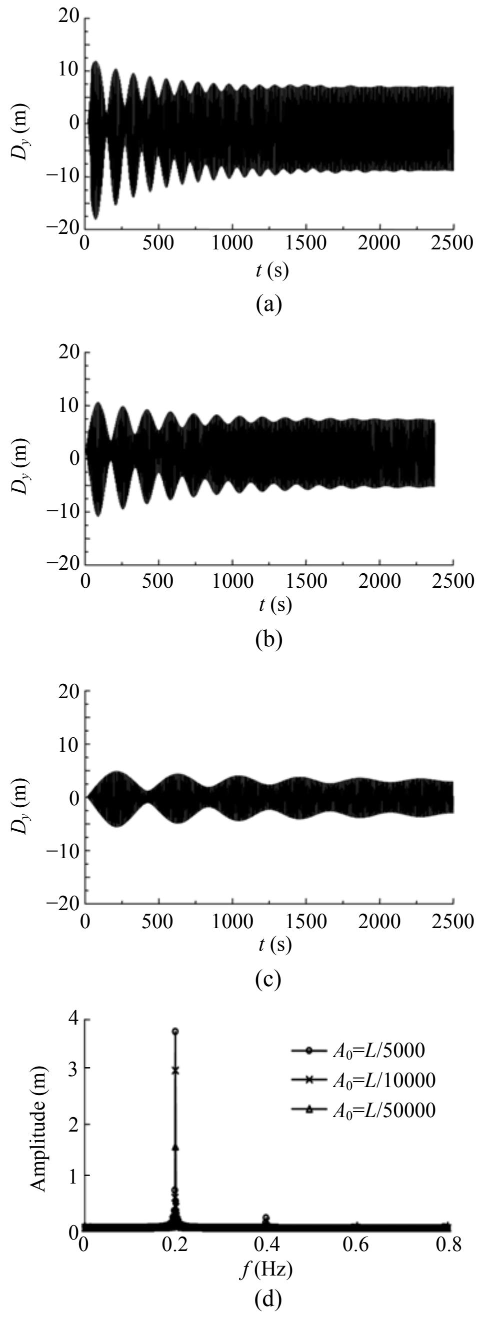

Fig.3 Vertical displacement response and spectrum at the middle point of the cable S-1 due to excitation of frequency $f=f_1$ without damping. (a) A_0=L/5000; (b) A_0=L/10000; (c) A_0=L/50000; (d) Spectrum of vertical displacement

metric vibration of cable. The vibration with periodic varying amplitudes is the "beat" phenomenon.

Fig.3d shows the corresponding frequency spectra of the displacement vs time histories in Figs.3a~3c. The results demonstrate that the characteristics of the dynamic responses are identical due to the three different support excitations. Two modes with frequencies f_1 and $2f_1$ are included in cable vibration and the dominant one is the mode with f_1.

Fig.4 depicts the vertical displacement response and corresponding frequency spectra at the middle point of cable S-1 due to excitation of frequency $2f_1$. When the amplitude of the excitation is 1/50000 of the cable span, the vibration amplitude is smaller than 0.2 m and parametric vibration with big amplitude does not occur, thus corresponding results are not presented.

According to the results, the pattern of cable vibrations due to support excitations of frequency $2f_1$ is similar to that due to support excitations of frequency f_1. The beat pattern occurs in the early stage and is followed by parametric vibrations with steady amplitudes. The excitation frequency is not the natural frequency of the cable, so the initial vibration amplitude and vibration of cable tension are relatively small. As the excitation continues, the variation of cable tension induces self-motivated vibration, which is called parametric vibration.

Fig.4c shows the corresponding spectra of the dynamic responses. It is observed that the mode with frequency f_1 is still the dominant mode of the vibration although the excitation frequency is $2f_1$.

The characteristics of cable vibration due to support excitation obtained in this study are similar to that in Wu (2002)'s research. The vibration of the cable varies from nonlinear vibration due to support excitation to parametric vibration with steady amplitude induced by periodic variation of the cable tension. The amplitude of parametric vibration is related to the amplitude A_0 of support excitation. When A_0 is relatively big, the beat vibration will last for a short time, and the steady amplitude of parametric vibration is much bigger.

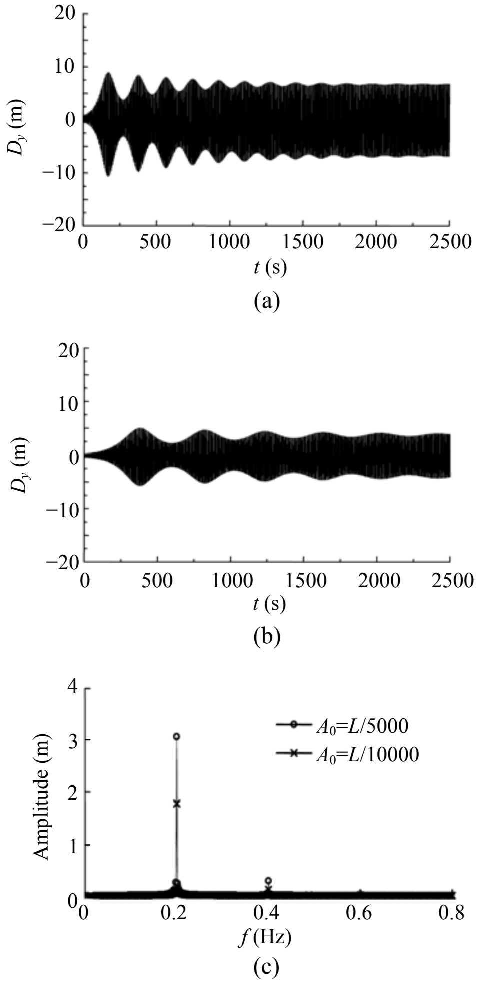

Fig.4 Vertical displacement response and spectrum at the middle point of the cable S-1 due to excitation of frequency f=2f_1 without damping. (a) A_0=L/5000; (b) A_0= L/10000; (c) Spectrum of vertical displacement

Nonlinear dynamic characteristics of the short cable S-2

Fig.5 shows the vertical displacement response at the middle point of cable S-2. When the amplitude of the excitation is 1/50000 of the cable span and the excitation frequency is $2f_1$, the parametric vibration with a big amplitude has not happened with the excitation duration of 2500 s, therefore the corresponding results are not presented in this paper.

It is observed from Fig.5 that the vibration of cable S-2 is similar to that of cable S-1. However, because the structural properties of the two cables are different, the amplitudes and the period for beat vibration between the two cables are different.

According to the results, although the Irvine parameters between the cables S-1 and S-2 are different, of which the characteristics of parametric vibration are identical.

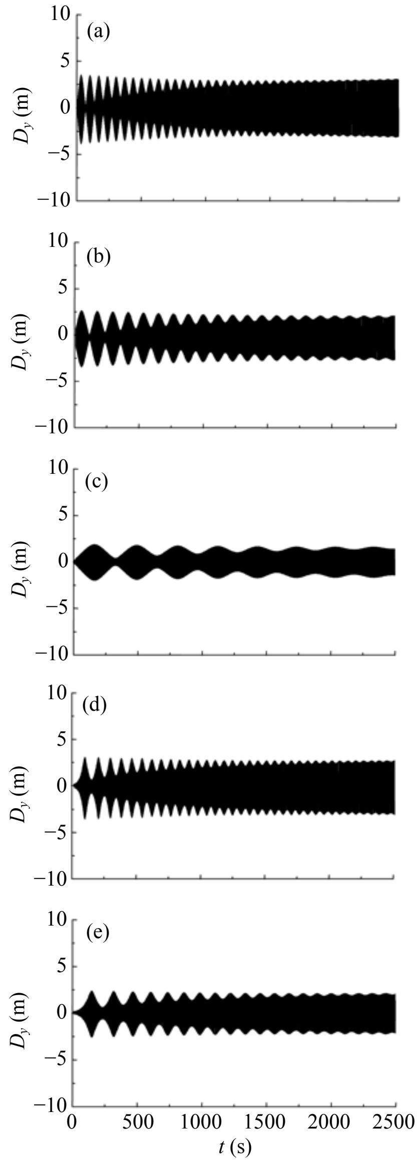

Fig.5 Vertical displacement response at the middle point of the cable S-2 due to excitation of frequencies $f=f_1$ and $2f_1$ without damping. (a) $A_0=L/5000$, $f=f_1$; (b) $A_0=L/10000$, $f=f_1$; (c) $A_0=L/50000$, $f=f_1$; (d) $A_0=L/5000$, $f=2f_1$; (e) $A_0=L/10000$, $f=2f_1$

Effects of viscous damping

In order to discuss the damping effect of distributed damping on parametric vibration of cables, nonlinear dynamic responses are obtained for different distributed damping coefficients.

Assume that the uniformly distributed damping is of Rayleigh mass-proportional type (Clough and Penzien 1995),

$$\boldsymbol{c} = \frac{\bar{c}}{\bar{m}} \boldsymbol{m}, \tag{13}$$

the s mode damping ratio is therefore

$$\xi_s = \bar{c} / (4\pi \bar{m} f_s), \tag{14}$$

where f_s is the sth modal frequency. Then the logarithmic decrement δ_s is

$$\delta_s = 2\pi\xi_s \Big/ \sqrt{1-\xi_s^2}\,. \tag{15}$$

Fig.6 shows the relationship between δ and frequency with $\bar{c}$ =0.0002 kN·s/m^2. The logarithmic decrement of steel cable is 0.5% as usual.

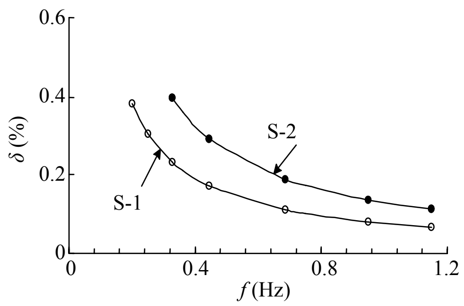

Fig.6 Curves for the logarithmic decrement δ with c=0.0002 kN·S/m^2

Since the characteristics of S-1 and S-2 are similar, only S-1 is studied to investigate the influence of damping on parametric vibration of cables. Fig.7 shows the vertical displacement time history at the middle point of cable S-1 when $\bar{c}$ =0.0002 kN·s/m^2. The frequencies of support excitations are f_1 and $2f_1$; the amplitudes of support excitations are 1/5000 and 1/10000 of the cable spans, respectively. Comparing the results with those in Figs.3 and 4, we can conclude that the influence of distributed viscous damping on parametric vibration of cable is not significant.

CONCLUSION

This study propose a new method for analyzing parametric vibration of stay cables due to support excitation based on nonlinear dynamic FEM. Applying this method to two examples of stay cables

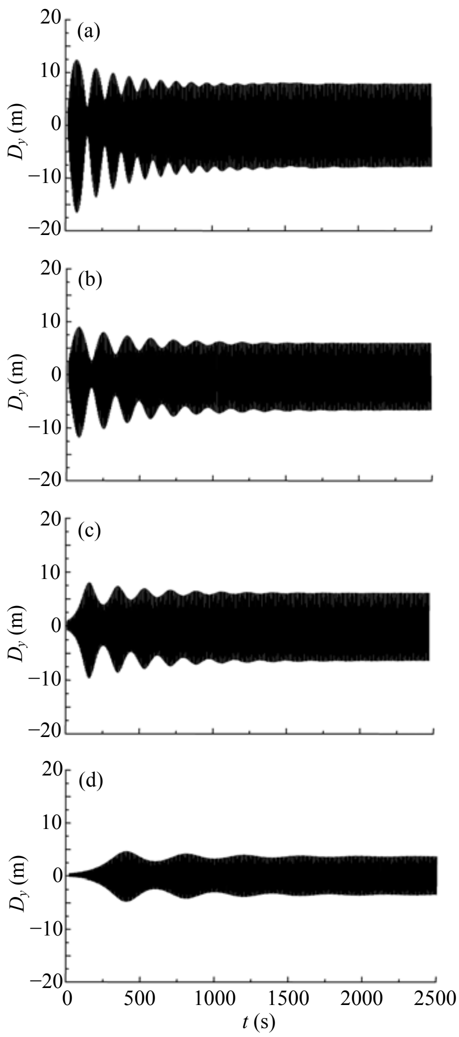

Fig.7 Vertical displacement response at the middle point of the cable S-1 considering damping effects. (a) $A_0=L/5000$, $f=f_1$; (b) $A_0=L/10000$, $f=f_1$; (c) $A_0=L/5000$, $f=2f_1$; (d) $A_0=L/10000, f=2f_1$

with different spans, we analyzed the characteristics of parametric vibration of stay cables, and achieved the following conclusions:

(1) Nonlinear FEM is a powerful and adaptive technique in analyzing the parametric vibration of cables. It can be used to analyze the vibration of cables acted by arbitrary complicated load, and dampers could be easily taken into consideration in the analysis.

(2) The parametric vibrations of stay cables with different Irvine parameters follow the same pattern under support excitation. However, because of the different structural properties of the two cables, the amplitudes and the period of beat pattern between the two cables are different.

(3) When the frequency of support excitation is f_1, the beat vibration with a large amplitude occurs because of the resonance, and the periodic variation of cable tension will induce parametric vibration of cables very soon. When the frequency of support excitation is $2f_1$, the large amplitude parametric vibration only occurs after a certain duration of excitation, and is related to the excitation amplitude. The duration of necessary excitation will increase as the support excitation amplitude decreases.

(4) Regardless of the excitation frequency, the dominant frequencies of parametric vibration are the first in-plane vibration frequency and twice of the first in-plane vibration frequency of the cable.

(5) The amplitude of parametric vibration is proportional to that of excitation support, while the period of beat pattern is inversely proportional to the amplitude of support excitation.

(6) The influence of distributed viscous damping on parametric vibration of cable is insignificant.

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