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Improved concept models for straight thin-walled columns with box cross section

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Abstract: This paper focuses on developing improved concept models for straight thin-walled box sectional columns which can better predict the peak crushing force that occurs during crashworthiness analyses. We develop a nonlinear translational spring based on previous research and apply such a spring element to build the enhanced concept models. The work presented in this article is developed on the basis of the publication of the author (Liu and Day, 2006b) and has been applied in a crashworthiness design issue, which is presented by the author in another paper (Liu, 2008).

Key words: Concept model, Peak crushing force, Box section, Nonlinear translational springs, Crashworthiness analysis, Thin-walled column
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INTRODUCTION

Thin-walled box section columns have been increasingly used in architecture, automotive engineering, shipbuilding and other applications because of their high strength-weight ratio, low cost, and excellent energy absorption capability during crashworthiness analysis. Therefore it is very important for analysts and engineers to create finite element (FE) models for such columns in order to simulate and evaluate their design and performance on a computer.

Recently, a new modeling technique, concept modeling, has been developed to model the thin-walled structures, especially beam-like components. The concept modeling method applies the elements with simpler formulations to create concept models. Compared to the traditional detailed models, the use of concept models allows a quicker and more efficient computer simulation and also enables designers to easily switch several alternative designs and evaluate them. Prater *et al.*(2002; 2005) and Kim *et al.*(1997) demonstrated theories behind the concept modeling and developed concept vehicle models that were used for design optimization, noise, vibration, harshness (NVH) analysis and crashworthiness analysis. Drazetic *et al.*(1993) focused on generating concept models for thin-walled box section columns. Based on the above research, Liu and Day (2006a; 2006b) developed a complete methodology of concept modeling for regular thin-walled structures creating very promising concept models for common thin-walled columns, including straight thin-walled columns with box cross section.

The basic idea of Liu and Day (2006a; 2006b)'s design comes from the collapse mechanisms of the straight thin-walled columns which were thoroughly studied by Wierzbicki and Abramowicz (1983). This means that during the crash the crumpling process of a box section column is progressive, with each new fold being formed after the previous one is completed. This phenomenon is also verified by computer simulation. Fig.1 displays the configurations of the box section column at different time during the crash, clearly reflecting the progressive crushing process of the model. The configurations were captured at a specified time from the animation file presented in the post-processor of LS-DYNA. As illustrated in Fig.1, the buckling of the thin-walled box section column model during the crash is realized through the generation of the consecutive plastic folds, which have a "spring-like" characteristic. Thus it is possible to have several spring elements simulate the column's axial collapse mechanisms in the concept model. Following this idea Liu and Day (2006b) developed their concept thin-walled box section column models.

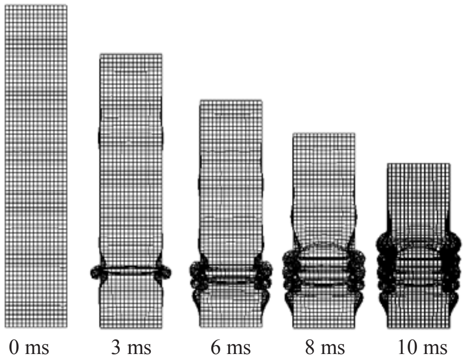


Fig.1 Configurations of the straight box section column model at a series of time

Liu and Day's concept models were composed of beam elements and linear translational spring elements, which were defined based on Wierzbicki and Abramowicz (1983; 1989)'s theory. In Liu and Day's concept models it is the spring elements that simulated the buckling behavior of the column and absorbed most impact energy during the crashworthiness analyses. These linear spring elements are defined based on the prediction of the mean crushing force occurring on the box section column during crashworthiness analysis (as shown in Eqs.(1)~(2) and Fig.2), therefore they can correctly reflect the mean crushing force during the crash processes but may not evaluate the peak crushing force accurately. Unfortunately, during the crashworthiness analysis, the maximum crushing force is one of the most important criteria that influence people's safety in many real applications. The energy absorption is also dependent on the structure's load and plastic deformation capacity. Thus an eligible concept model should be capable of reflecting the peak crushing force correctly. The major objective of this paper is to employ current collapse theories to re-develop the spring elements that can accurately predict the maximum crushing force occurring in the crashes. These spring elements, together with the beam elements, will then be used to generate improved concept models for the straight thin-walled box section columns. Besides Liu and Day's model, Nikravesh *et al.*(1983) and Sousa *et al.*(2008) also developed crash models following the plastic hinge approach and implemented them in commercial codes.

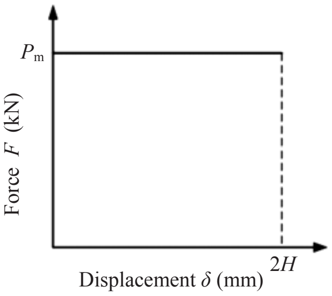


Fig.2 *F*-*δ* curve of linear spring element used in current concept models

The force-deflection relationship of the old spring elements is determined by the following equations (Wierzbicki and Abramowicz, 1983) and shown in Fig.1.

$$H = 0.98\sqrt[3]{tC^2}, \tag{1}$$

$$P_m = 9.56\sigma_0 t^{5/3} C^{1/3}, \tag{2}$$

where t is the wall thickness; C is the average edge length of the cross section's depth a and width b, $C=(a+b)/2$; H is the length of the half plastic fold that was generated during the axial collapse (Fig.3); σ_0 is the material's energy equivalent flow stress which can be approximated based on the material's ultimate stress σ_u as $\sigma_0=0.92\sigma_u$; and P_m is the calculated mean crushing force.

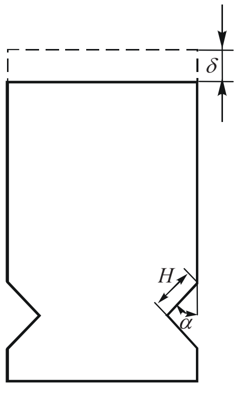


Fig.3 Length of half plastic fold, *H*

DEVELOPMENT OF NONLINEAR SPRING ELEMENT

In order to correctly predict the maximum crushing force, the characteristic of the spring elements has to be determined based on the relationship between the axial deflection and the instantaneous crushing force P during the crash, rather than the mean crushing force P_m. Wierzbicki *et al.*(1994) concluded that the instantaneous force P depended on the instantaneous folding angle α that appeared during the formation of one plastic fold (Fig.3), and could be evaluated through the mean crushing force P_m. Eq.(3) is a simple closed form that shows how to calculate P from P_m and α. Thus, given a straight thin-walled box section column with known dimensions and material properties, the instantaneous crushing force that may occur during an impact can be evaluated from Eqs.(2) and (3) as Eq.(4):

$$P = P_m(0.6 + 0.512/\alpha),\ 0\le\alpha\le\pi/2, \tag{3}$$

$$P = 9.56\sigma_0 t^{5/3} C^{1/3}(0.6 + 0.512/\alpha). \tag{4}$$

Eq.(4) shows that in the relationship between P and α, in order to determine the force-deflection relationship for the new spring element, the axial deflection δ has to be correlated with α. As shown in Fig.3, δ can be calculated based on H and α as

$$\delta = 2H(1 - \cos\alpha). \tag{5}$$

Substituting Eq.(1) into Eq.(5) and yields

$$\delta = 1.96 t^{1/3} C^{2/3}(1 - \cos\alpha). \tag{6}$$

The force-deflection relationship of the spring element can then be defined through Eqs.(4) and (6) where the relationship between the F and δ is correlated through the folding angle α. Obviously from Fig.3 it can be seen that during a crash process α varies from 0° to 90° and δ therefore varies from 0 to $2H$ (Eq.(5)).

Eq.(4) can be used to predict the instantaneous force during the impact process with varying α. However, from that equation it is inferred that P tends towards infinity as α approaches zero, and δ also approaches zero according to Eq.(5). Therefore another equation is required to describe P when α approaches zero. Kecman (1983) discussed this problem. In this paper P under very small α is assumed as linearly increasing from zero to its “full plastic force” (Eq.(7)) and then decreasing to follow the relationship defined by Eq.(4). Thus the entire F-δ relationship is composed of two parts: the linear relationship where P increases from 0 to its full plastic force and the relationship defined by Eq.(4). Fig.4 shows how to evaluate the full plastic force, with the energy equivalent flow stress σ_0 evenly distributed along its cross section area. The developed F-δ relationship of the new spring element is plotted in Fig.5 and such a spring is a nonlinear translational spring and will be used for creating the improved concept models for the thin-walled box section columns.

$$P_f = \sigma_0[2a + 2(b - 2t)]t = 4\sigma_0(C - t)t. \tag{7}$$

VALIDATION OF CONCEPT MODELS

By using the developed nonlinear translational spring elements, we employed Liu and Day (2006b)’s

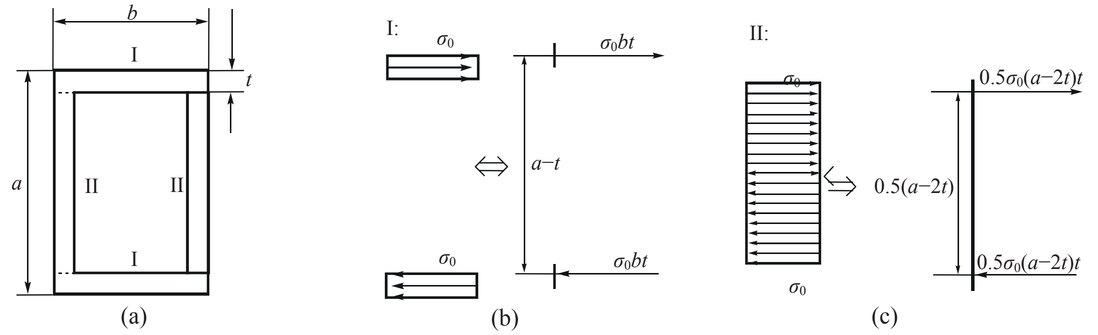


Fig.4 Flow stress distribution along the cross-section

(a) A typical box cross section; (b) Flow stress distribution along top and bottom webs; (c) Flow stress distribution along left and right flanges

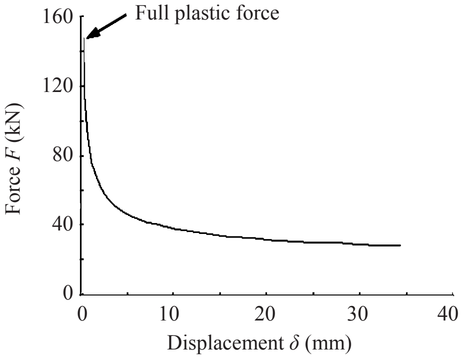


Fig.5 F-δ curve of developed nonlinear spring element for the column listed in Table 1

concept modeling method to develop concept models for the straight thin-walled box section columns, which are expected to correctly capture the peak crushing force during the crashworthiness analyses. The developed spring elements are validated through the concept models for two typical box section columns: one is the box column with square cross section while the other is the one with rectangular cross section. The results of the analyses obtained from these concept models are compared to the results from the detailed models and the concept models using the old linear spring elements, to verify the accuracy and advantages of the developed nonlinear spring elements.

The following steps are taken in creating the concept models. Step 1: Apply Eqs.(1) and (2) to determine the mean crushing force P_m and the length of one plastic fold $2H$ for the given column. Step 2: Based on these results, define the force-deflection characteristic of the nonlinear spring element using Eqs.(4), (6) and (7), as shown in Fig.5. Step 3: Divide the entire column into a certain number of segments of equal length $2H$. Step 4: Use beam element whose correct cross-sectional information is determined by the given box-section beam, to model those segments; Then use the defined spring element to connect these segments with the remaining part to finish the concept model.

Box columns with square section

Fig.6a shows a detailed model for a straight thin-walled column with a square cross section, which is modeled using a shell element. Two concept models are then developed based on the detailed model composed only of the beam and the developed nonlinear spring elements (defined from Eqs.(4), (6) and (7)) or the previous linear spring elements (defined from Eqs.(1) and (2)), respectively (Fig.6b). Within the concept models the cross-sectional type and dimensions are assigned to the beam elements; the number and arrangement of the spring elements are determined following Liu and Day (2006b)'s method. After modeling the same crash analysis is performed on these models and the results are compared to show the correlation between the three models. The dimensions, material properties, and the analysis conditions of the square column are listed in Table 1. All the results and comparisons are displayed in Table 2 and plotted through Fig.7a~7c.

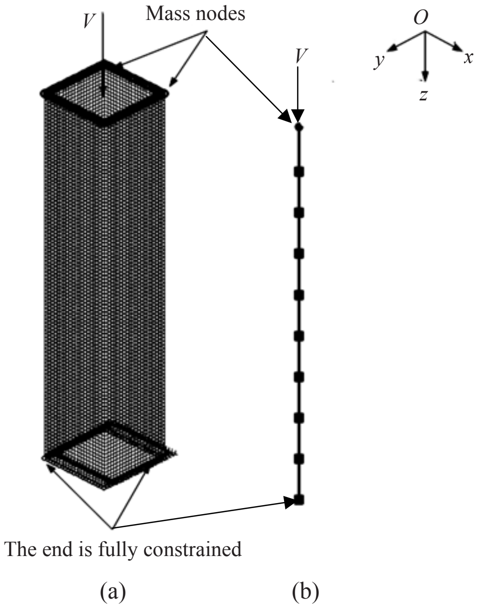


Fig.6 Detailed model (a) and concept model (b) for straight thin-walled column with square section

Box columns with rectangular section

The improved concept model for a thin-walled column with a rectangular section is also validated in this study. Fig.8 shows the detailed model for this straight thin-walled column with rectangular cross section. Similarly, two concept models are also developed for this rectangular section column, which are modeled with beam elements and apply different spring elements to simulate the column's axial collapse behavior. These three FE models are then used for the same crash analysis and the results are displayed in Table 3 and plotted through Fig.9a~9c. The geometries, material properties and the analysis

Table 1 Properties and impact conditions

Parameter		Value
Material properties	Young's modulus (GPa)	207
	Density (kg/m^3)	7830
	Yield stress (MPa)	200
	Ultimate stress (MPa)	448
	Hardening modulus (MPa)	630
	Poisson's ratio	0.3
Geometries	Total length (mm)	300
	Cross section (mm×mm)	60×60
	Wall thickness (mm)	1.5
Impact conditions	Added mass (kg)	480
	Initial velocity (m/s)	15
	Crash time (s)	0.01

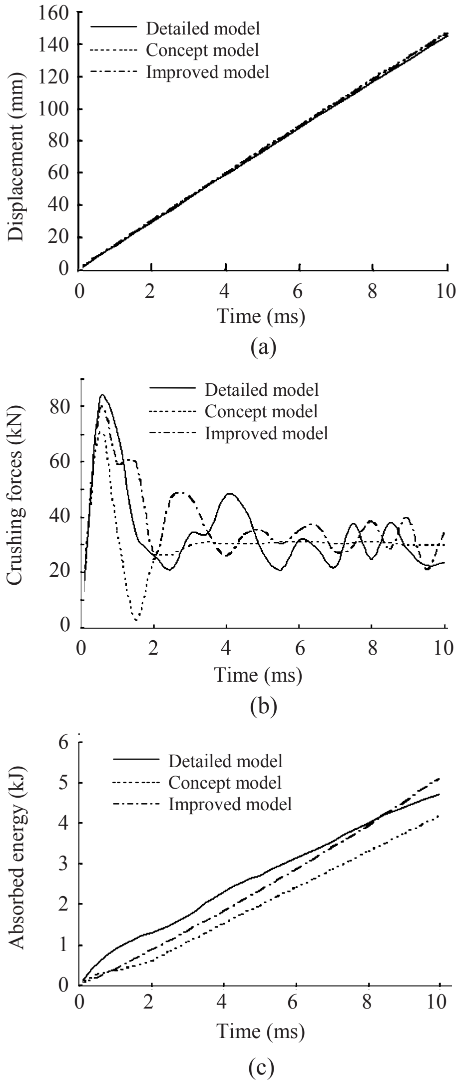


Fig.7 Displacements at moving end (a), crushing forces (b) and absorbed energies (c) of square section column models

Table 2 Comparisons of crash results from detailed model and two concept models for the thin-walled box column with square section

Model	Peak crushing force (kN)	Global displacement (mm)	Absorbed energy (kJ)
DM	82.6	145.4	4.74
CM	70.4	147.4	4.16
ICM	77.5	146.6	5.10
$d_{\mathrm{DM\&CM}}$ (%)	−14.8	1.4	−12.24
$d_{\mathrm{DM\&ICM}}$ (%)	−6.2	0.8	7.59

DM: detailed model; CM: concept model; ICM: improved concept model; $d_{\mathrm{DM\&CM}}$: the difference between DM and CM; $d_{\mathrm{DM\&ICM}}$: the difference between DM and ICM

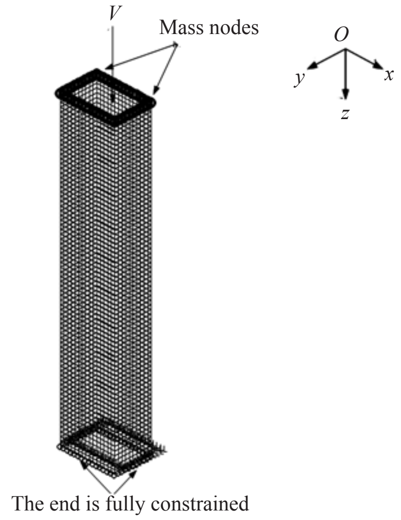


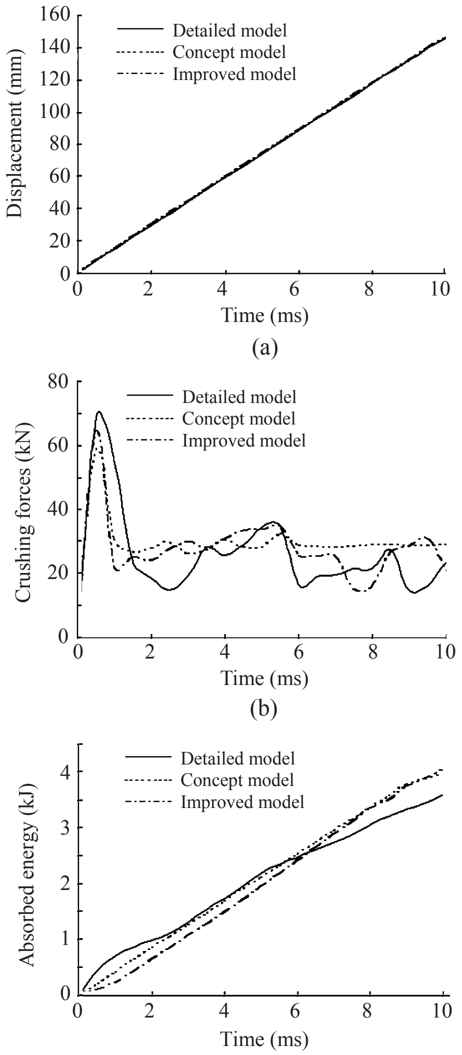
Fig.8 Detailed model for straight thin-walled column with rectangular section

conditions are the same as listed in Table 1, except that the size of the rectangular cross section is 40 mm×60 mm.

DISCUSSIONS

As shown in Figs.7 and 9 and Tables 2 and 3, both concept models can generate good results while saving more computer resources and modeling effort because they require less elements which take simpler formulation. Furthermore, the concept models allow an easy transition between different cross sectional designs, which is extremely significant and useful in their early design stage and product optimization. Tables 2 and 3 show that errors of all the important crash results are within 15%.

On comparing the results yielded from the two concept models, it can be observed that the concept models using the developed nonlinear spring elements are better in capturing the peak crushing force, as well as exactly simulating the entire crushing force history, than the concept models using the linear spring elements. The errors of the peak crushing force caused by the improved concept models are −14.8% for both the square section column and the rectangular column models, while the errors of the previous concept models are −6.2% and −6.7%. Also, as shown in Fig.7b and Fig.9b, the improved concept models properly reflect the whole crushing force history while the previous concept models almost yield constant forces immediately after the first peak force. The major reason for the difference is that the linear spring elements used in the previous concept models were defined as being based on a prediction of the mean crushing force, while in the improved concept models the nonlinear spring elements were developed based on capturing the instantaneous force.



(a)

(b)

(c)

Fig.9 Displacements at moving end (a), crushing forces (b) and absorbed energies (c) of rectangular section column models

Table 3 Comparisons of crash results from detailed model and two concept models for the thin-walled box column with rectangular section

Model	Peak crushing force (kN)	Global displacement (mm)	Absorbed energy (kJ)
DM	68.8	147.1	3.60
CM	58.6	147.0	3.98
ICM	64.2	147.0	4.05
$d_{\mathrm{DM\&CM}}$ (%)	−14.8	−0.1	10.56
$d_{\mathrm{DM\&ICM}}$ (%)	−6.7	−0.1	12.50

DM: detailed model; CM: concept model; ICM: improved concept model; $d_{\mathrm{DM\&CM}}$: the difference between DM and CM; $d_{\mathrm{DM\&ICM}}$: the difference between DM and ICM

CONCLUSION

This paper improves the current concept models for straight thin-walled box section columns so that they can correctly predict the maximum crushing force as well as properly reflecting the whole crushing force history during the crashworthiness analyses. The improved concept models employ the nonlinear translational spring element, which is developed in this paper. The force-deflection characteristic of this spring element is determined based on the equations that illustrate the relations between the instantaneous crushing force F and the instant deflection δ of the box section columns. Compared to the previous concept models and the corresponding detailed models (Liu and Day, 2006b), the concept models presented in this study can better predict the peak crushing force while still accurately reflecting the global displacement and energy absorption. This conclusion is verified through modeling and validating two typical box section columns: the column with square cross section and the one with rectangular cross section. In real product designs and evaluations the maximum crushing force may be a significant criterion because it does influence human safety in most applications. Thus the improved concept models are recommended for further crashworthiness analyses because of their excellent capability to predict the peak crushing force and their simple configurations.

This paper focuses on creating new concept models for straight thin-walled columns with box sections. Similarly, this idea can be extended and applied in creating concept models for straight thin-walled columns with other types of cross sections, such as hexagonal sections, octagonal sections, circular sections, and so on. Moreover, since the thin-walled box section column is an important energy absorber in modern vehicles, the concept column models developed in such a way can be applied in constructing the whole concept vehicle model for further assessments and analyses. Geometric CAD models for the box section columns are created using the implicit FE code ANSYS (2005) and all the FE analysis modeling and computer simulation involved in this study are performed using the explicit code LS-DYNA (Hallquist, 1993).

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