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Bayesian networks modeling for thermal error of numerical control machine tools*

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Abstract: The interaction between the heat source location, its intensity, thermal expansion coefficient, the machine system configuration and the running environment creates complex thermal behavior of a machine tool, and also makes thermal error prediction difficult. To address this issue, a novel prediction method for machine tool thermal error based on Bayesian networks (BNs) was presented. The method described causal relationships of factors inducing thermal deformation by graph theory and estimated the thermal error by Bayesian statistical techniques. Due to the effective combination of domain knowledge and sampled data, the BN method could adapt to the change of running state of machine, and obtain satisfactory prediction accuracy. Experiments on spindle thermal deformation were conducted to evaluate the modeling performance. Experimental results indicate that the BN method performs far better than the least squares (LS) analysis in terms of modeling estimation accuracy.

Key words: Bayesian networks (BNs), Thermal error model, Numerical control (NC) machine tool
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INTRODUCTION

Thermal error is one of the most significant factors affecting the precision of machine tools. It is believed that 40%~70% of the error in precision parts arises from thermal error (Pahk and Lee, 2002). A great deal of work has been carried out over the last decade in the estimation and compensation of temperature dependent errors (Ramesh *et al*., 2000). Researchers have employed various techniques such as least squares (LS) fitting technique to model the thermal behavior.

However, some difficulties have all along remained in modeling for the influence of properties of thermal error. Yang and Ni (2005) indicated that the complex thermal behaviors of a machine are created by interactions among the heat source location, heat source intensity, thermal expansion coefficient, the machine system configuration and the running environment. This situation leads to two problems: one is that the thermally induced error becomes a time-varying nonlinear and non-stationary process (Huang and Liang, 2005), and the other is that the various interactions make it hard to understand the relationships among various factors. These problems affect the model accuracy directly, and limit the use of traditional fitting methods in modeling thermal error. Therefore, new modeling approaches are needed to overcome these difficulties.

With the development and progress of uncertain theory and artificial intelligence, some novel methods for error compensation have been proposed. Fu and Chen (2004) presented an artificial neural network method which was combined with fuzzy logic to predict thermal error. A new method of principal component analysis was presented to identify the structure and parameters of the fuzzy model. Then, the artificial neural network method was used to

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overcome the problem which the fuzzy model could not resolve, because the effective data number was smaller than the consequence parameter number. Lin *et al*.(2008) proposed a modeling method based on on-line LS support vector machine (OLS-SVM) to implement thermal error compensation. The data collected by temperature sensors and laser position sensors were trained to construct the thermal error model based on OLS-SVM. The thermal error could be predicted by this model, and at the same time the model was modified recursively according to new input data. However, a common shortcoming with these models was the "black-box" nature of themselves. That is to say, these models that predicted thermal error only depended on the temperature data collected on measuring points, but without taking the relationships among various factors into account. Thus, much experiential knowledge cannot be used by these models to improve their calculation precision and speed.

A BN is a probabilistic representation for uncertain relationships and is useful for modeling real-world problems such as diagnosis, forecasting, manufacturing control, etc., wherein there exist multiple cause and effect dependency relations (Heckerman and Breese, 1996). It is adept in reasoning and decision-making by well combining experiential knowledge and sampled data. This paper seeks to present a thermal error model for numerical control (NC) machine tool based on BNs and is organized as follows: in the next section, a concept of BNs is introduced; In Section 3, the modeling method based on BNs is proposed and how to use it to analyze thermal error is then presented; To test the validity of the method, an experiment is shown in Section 4 and the result is analyzed; Finally, some conclusions are presented in Section 5.

BAYESIAN NETWORKS

A BN is a graphical model consisting of nodes which represent causes and effects in real-world situations and a set of edges each of which connects two nodes. If there exists a causal relationship between any two of the nodes, the edge would be directional leading from the cause variable to the effect variable. For such directed edges, the edge is said to be from parent to child. A BN is also called a 'directed acyclic graph' for this reason as all the edges of the graph point in a particular direction and there is no way to start from one node, travel along a set of directed edges in the proper direction and arrive at the same node.

Mathematical representations of Bayesian networks

A BN for a set of variables $\boldsymbol{X}=\{X_1, ..., X_n\}$ consists of a network S that encodes a set of conditional independence assertions about variables in $\boldsymbol{X}$ and a set of local probability distributions associated with each variable (Heckerman and Breese, 1996). Together, these components define the joint probability distribution for $\boldsymbol{X}$.

For a joint probability distribution, from the chain rule of probability, we have

$$P(X_1,\cdots,X_n)=\prod_{i=1}^{n}P(X_i\mid X_1,X_2,\cdots,X_{i-1}). \quad (1)$$

For every X_i there will be some subset $\pi(X_i) \subseteq\{X_1, X_2, ..., X_{i-1}\}$ such that X_i and $\{X_1, ..., X_{i-1}\}\backslash\pi(X_i)$ are conditionally independent. That is, for any X_i,

$$P(X_i\mid X_1,\ ...,\ X_{i-1})=P(X_i\mid\pi(X_i)), \quad (2)$$

where X_i denotes the variable and $\pi(X_i)$ represents the parents of node X_i in S. Combining Eqs.(1) and (2),

$$P(X_1,\cdots,X_n)=\prod_{i=1}^{n}P(X_i\mid\pi(X_i)). \quad (3)$$

Now, we assume that the number of nodes in a BN is n, the average number of parent nodes is m, and each variable owns k values on average. Then, the computation complexity of Eq.(3) is nk^m, while it is k^n-1 to Eq.(1). Obviously, the time cost will decline by using Eq.(3) when m is far smaller than n.

Parameter learning for Bayesian networks

The process to refine the local probability distributions of a BN using given data is called 'parameter learning'. Given net structure S^h and background knowledge ξ, we denote

$$\theta_{ijk}=P(X_i^k\mid\pi(X_i)^j,S^h,\xi), \quad (4)$$

where θ_{ijk} is the parameter of a BN, representing the probability of X_i whose value is k and the value of its parents is j. For convenience, we define the vector of parameters

$$\theta_{ij}=\bigcup_{k=1}^{r_i}\{\theta_{ijk}\},\ \ \theta_i=\bigcup_{j=1}^{q_i}\{\theta_{ij}\},\ \ \theta_s=\bigcup_{i=1}^{n}\{\theta_i\},$$

where r_i and q_i denote the numbers of possible values of X_i and their parents, respectively.

Given this class of local distribution functions, the posterior distribution can be calculated efficiently and in a closed form under three assumptions (Heckerman, 1997): (1) There are no missing data in Sample D, that is Sample D is complete; (2) The parameter vectors θ_{ij} are mutually independent; (3) The distribution of θ_{ij} is a Dirichlet distribution.

Then, the posterior distribution can be obtained as follows:

$$\begin{aligned}P(\theta_s\mid D,S^h,\xi)&=\prod_{i=1}^{n}\prod_{j=1}^{q_i}P(\theta_{ij}\mid D,S^h,\xi)\\&=\prod_{i=1}^{n}\prod_{j=1}^{q_i}\mathrm{dir}(N'_{ij1}+N_{ij1},\ldots,N'_{ijr_i}+N_{ijr_i})\\&=\prod_{i=1}^{n}\prod_{j=1}^{q_i}\frac{\Gamma\left(\sum_{k=1}^{r_i}(N'_{ijk}+N_{ijk})\right)}{\prod_{k=1}^{r_i}\Gamma(N'_{ijk}+N_{ijk})}\prod_{k=1}^{r_i}\theta_{ijk}^{N'_{ijk-1}}.\end{aligned}\quad(5)$$

where N'_{ijk} is the coefficient of Dirichlet distribution indicating the prior distribution of variables. N_{ijk} is the number of cases in D in which $X_i=x_i^k$ and the value of its parents is j.

Probabilistic inference for Bayesian networks

In general, the computation of a probability of a constructed model is known as probabilistic inference. In this section, probabilistic inference in BNs is described briefly.

To a BN whose structure has been fixed, if there are N cases in sampled data, the inference can be calculated as follows (Heckerman, 1997):

$$\begin{aligned}P(x_{N+1}\mid D,S^h)&=\int P(x_{N+1}\mid\theta_s,D,S^h)P(\theta_s\mid D,S^h)\mathrm{d}\theta_s\\&=\prod_{i=1}^{n}\prod_{j=1}^{q_i}\frac{N'_{ijk}+N_{ijk}}{N'_{ijk}+N_{ijk}}.\end{aligned}\quad(6)$$

Especially, if there is only one variable X_i to be calculated, Eq.(6) can be simplified as

$$\hat{X}_i=x_i^{k_0},\quad(7)$$

where $\hat{X}_i$ is the prediction value of X_i, and k_0 should satisfy

$$\theta_{ij_0k_0}=\max_k\{\theta_{ijk}\}=\max_k\{P(x_i^k\mid\pi(X_i)^j,S^h)\}.$$

THERMAL ERROR MODELING WITH BAYESIAN NETWORKS

Complex behaviors of a machine tool (as described in INTRODUCTION) need to be addressed by the thermal error model in arriving at an accurate prediction under varying operation conditions. In order to account for these effects, the BN—especially dealing with uncertain relationships—is therefore very essential.

Modeling process

With the help of Bayesian statistics, prior knowledge of a domain can be combined well with sampled data. Accordingly, modeling of thermal errors in machine tools will be implemented by two steps: (1) Construct network from prior knowledge. Here we should identify the factors that are of critical importance in inducing thermal errors, analyze the interrelation among them, then describe all this information by building a directed acyclic graph, and finally assess prior probability of each variable in graph. (2) Conduct probabilities learning and inference in the network based on sampled data. The parameters of network will be refined, and the result of modeling can be calculated by probabilistic inference.

The entire modeling flowchart is described as Fig.1 in detail.

Network design

Thermal errors in machine tools are caused by several factors, such as machining time, cooling liquid and working environment. A BN involving more factors will have higher precision but increased complexity (Lo *et al.*, 1999). Based on an overall consideration of various factors, a network with 16

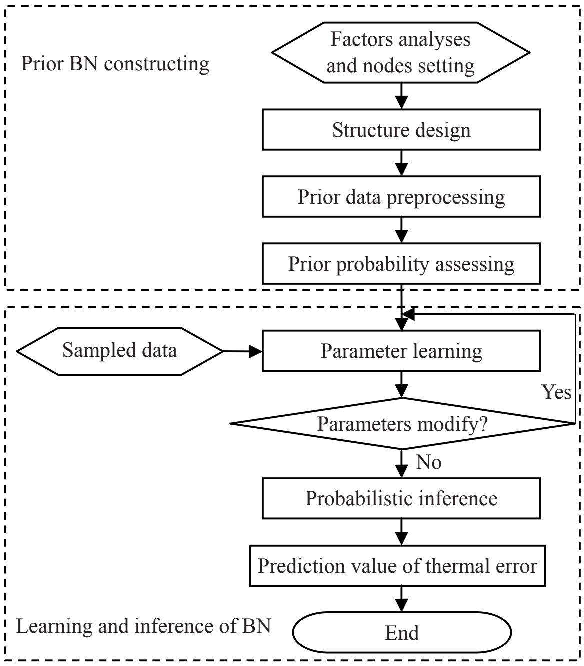

Fig.1 Modeling flowchart of BN method

factors—comprising 13 thermal variables, rotate speed of spindle, feed rate and cooling liquid—has been constructed in experiments. However, to express clearly and compactly, a network with 4 nodes is taken as an example herein. Three temperature variables which consist of ambient temperature T_0, front bearing temperature T_1 and electric machine temperature T_2, together with thermal deformation in axial direction D_0 and in radial direction D_1, form a network. Here variables T_0, T_1 and T_2 represent the alterative values of temperature. According to causal relationships among variables, the network can be constructed as Fig.2. Obviously, any variable X_i in this network and $\{X_1, ..., X_{i-1}\}\backslash\pi(X_i)$ are conditionally independent.

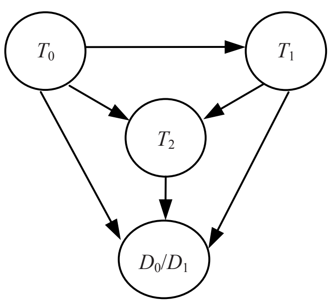

Fig.2 Network for thermal errors prediction

Data preprocess

The variables of BNs should be discrete, so it is essential to discretize variables of temperature and thermal deformation. Suppose that the range of a variable X_i is $V_i=[low_i, up_i]$, then it should be divided into a finite number of non-overlapping intervals with equal interval width w, that is

$$V_i=\{[C_{i0}, C_{i1}]\cup[C_{i1}, C_{i2}]\cup\ldots\cup[C_{i,k-1}, C_{ik}]\},$$

and

$$w=C_{ij}-C_{i,j-1},\ \forall j=1, 2, \ldots, k,$$

where $low_i=C_{i0}<C_{i1}<C_{i2}<\ldots<C_{i,k-1}<C_{ik}=up_i$. After this transition, it can be seen that the variable X_i owns k discrete values.

The issue about how to determine the value of w will be discussed here. In principle, the interval width should satisfy two requirements: (1) Precision requirement. As BNs estimate the value of a variable according to the probability of each interval, interval size limits the precision of modeling. Furthermore, the middle value of the interval which has maximal probability will be used in this paper as the result of the prediction, interval width therefore should have the same magnitude as precision requirement. (2) Performance requirement. As mentioned in Section 2, the computation complexity of BN is nk^m. When the structure of a network is fixed, the number of nodes n and the average number of parent nodes m are also fixed. Thus the computation complexity will increase swiftly when the number of intervals k grows. Obviously, setting large size will be beneficial to reduce the number of intervals and reduce the computation complexity.

To sum up, the size of intervals is relative to modeling precision and computation complexity, and its value should depend on certain applications. To some applications with high-precision requirements, using hierarchical means to truncate the range of variables gradually will be an ideal method.

MODEL VALIDATION

A series of experiments on an NC machining center have been conducted to verify the method based on BNs. With smart temperature sensors and laser position sensors (KEYENCE LK-150 H), we have collected the temperature on different positions of the machine tool and thermal deformation of spindle, respectively (Fig.3).

Fig.3 Thermal error measurement

Modeling test

The machine runs continuously for 2.5 h in an experiment, and the data is collected once every minute. This experiment is repeated several times under similar conditions and thirty sets of data are collected. In order to explain how to process the data, the thermal error at the moment t=20 min is taken as an example. Table 1 cites a portion of data.

Table 1 Portion of experimental data when *t*=20 min

No.	Thermally induced error				
	T_0 (°C)	T_1 (°C)	T_2 (°C)	D_0 (μm)	D_1 (μm)
1	0.003	1.687	1.525	10.7	6.1
2	0.001	1.776	1.437	9.3	5.8
3	0.001	2.062	1.737	12.9	7.1
4	0.002	2.001	1.666	12	6.5
5	0.003	2.211	1.718	13.2	7
6	0	1.512	1.436	9.7	6.2
7	0.001	1.818	1.512	9.3	6.1
8	0	1.438	1.399	10.1	6.3
9	0.001	1.7	1.411	8.7	5.6
10	0.001	1.937	1.598	11.4	6.6
⋮	⋮	⋮	⋮	⋮	⋮
30	0.002	1.926	1.587	11.7	6.5

Some characteristic values of statistics are shown in Table 2. Based on it, together with the precision requirements, the intervals of each variable are set as Table 3. Then the number of sampled data in every interval can be obtained, and the probability distribution for variables D_0 and D_1 can be calculated according to Eq.(7). The parameter N'_{ijk} in this formula indicates the priori distribution which should be offered by experts in general. Here N'_{ijk} is set as 1. The result of the calculation can be seen in Table 4, where $P_k=\theta_{ijk}$ (k=1, 2, …, 6), i.e., the probability of each interval of the thermal deformation. To axial thermal deformation D_0, for the interval [11.0~12.0) has the maximal probability, the middle value of it (i.e., 11.5 μm) is the predictive value of thermal deformation. Similarly, it can be estimated that the value of D_1 is 6.4 μm.

Experimental result analysis

To further confirm the validity of the BN method, the modeling performance of it is compared with the LS method in terms of modeling accuracy. The modeling results for axial thermal deformation from two methods and the actual value of machine are shown in Fig.4, and the results for radial thermal deformation are shown in Fig.5. Furthermore, the

Table 2 Values of statistical characteristic

	Variance	Maximum	Minimum	Mean
T_0 (°C)	1.05×10^{-6}	0.003	0	0.001
T_1 (°C)	0.047	2.492	1.432	1.885
T_2 (°C)	0.013	1.803	1.357	1.582
D_0 (μm)	2.180	13.600	8.100	11.300
D_1 (μm)	1.710	7.600	5.600	6.300

Table 3 Intervals of variables

Variable	Number of interval	Interval
T_0 (°C)	3	[0~0.001)/[0.001~0.002)/[0.002~0.003)
T_1 (°C)	4	[1.350~1.400)/[1.400~1.550)/[1.550~1.700)/[1.700~1.850)
T_2 (°C)	4	[1.400~1.700)/[1.700~2.000)/[2.000~2.300)/[2.300~2.600)
D_0 (μm)	6	[8.0~9.0)/[9.0~10.0)/[10.0~11.0)/[11.0~12.0)/[12.0~13.0)/[13.0~14.0)
D_1 (μm)	6	[5.4~5.8)/[5.8~6.2)/[6.2~6.6)/[6.6~7.0)/[7.0~7.4)/[7.4~7.8)

Table 4 Predictive result of thermal deformation

	Probability of thermal error	
	D_0 (μm)	D_1 (μm)
P_1	0.033	0.1
P_2	0.133	0.133
P_3	0.133	0.567
P_4	0.533	0.067
P_5	0.1	0.1
P_6	0.067	0.033
Interval	[11.0~12.0)	[6.2~6.6)
Predictive value	11.5	6.4

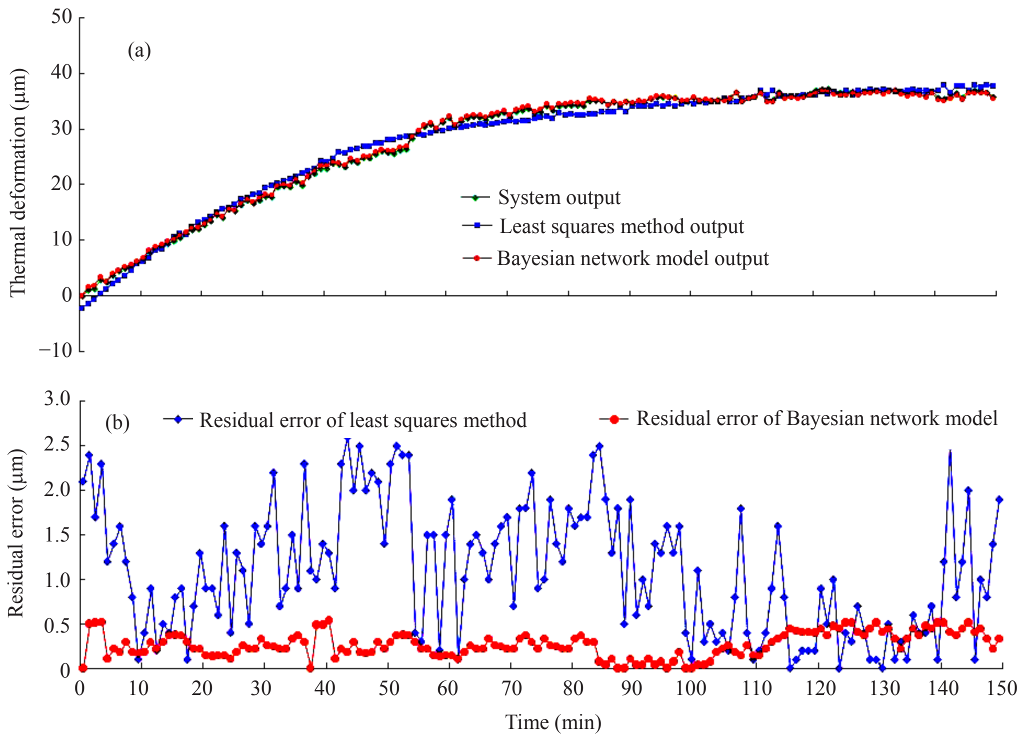

Fig.4 Modeling validation results in axial direction. (a) Thermal deformation; (b) Residual error

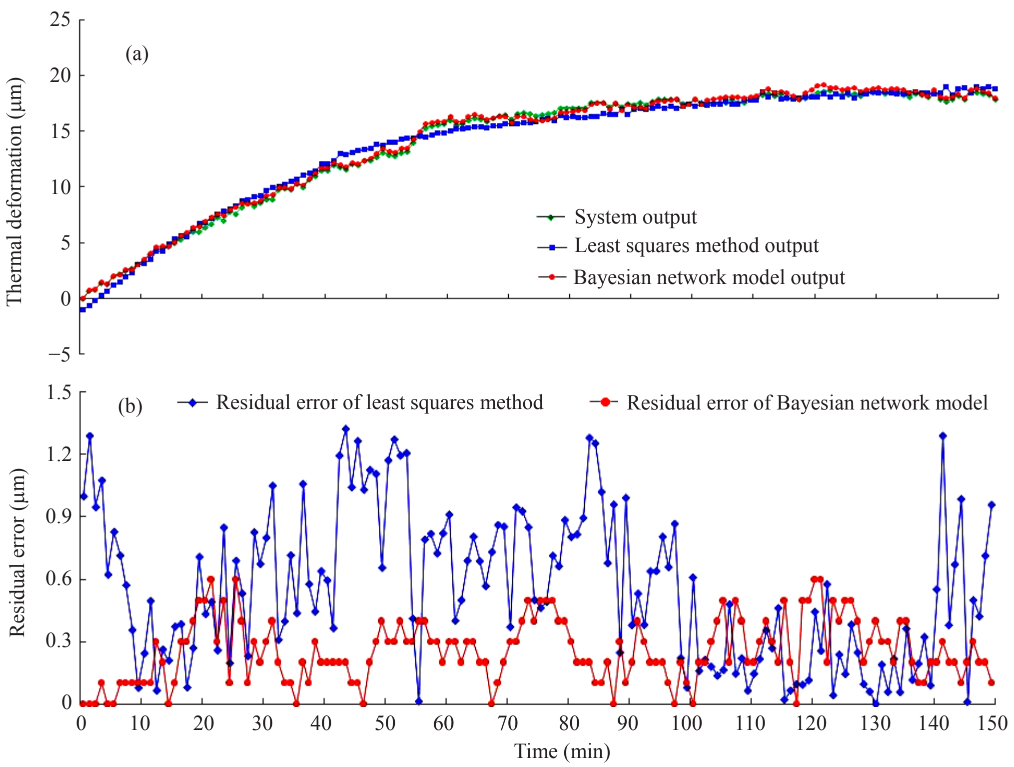

Fig.5 Modeling validation results in radial direction. (a) Thermal deformation; (b) Residual error

modeling results are compared based on the mean absolute percentage error (MAPE) as defined below:

$$MAPE = 100 \frac{\sum_{i=1}^{n} \left| (d_i - \hat{d}_i) / \hat{d}_i \right|}{n}, \quad n = 150, \tag{8}$$

where d_i is the modeling value, while $\hat{d}_i$ the actual value.

With regard to MAPE shown in Table 5, the BN model performs far better than the LS method as expected, improving the accuracy by approximately 80%.

Table 5 Comparison of MAPE between the BN and the LS methods

Method	*MAPE* (%)	
	Axial direction	Radial direction
BN	1.33	1.58
LS	7.26	7.75

CONCLUSION

In this paper, a novel method based on the BN model is developed to estimate the thermal errors of NC machine. Different from traditional fitting methods, it has the following characteristics:

(1) The method describes causal relationships of factors inducing thermal error by graph theory. It helps us to gain understanding about a problem domain readily, and the causal semantics also reduces the complexity of probability calculation.

(2) BNs in conjunction with Bayesian statistical techniques facilitate the combination of domain knowledge and sampled data. Therefore, the parameters of the model can adapt to the change of running state of machine, and the results of the model have satisfactory prediction accuracy.

The experiments show that BNs can model the thermal errors effectively. However, when the network is quite complex and the sampling frequency is fairly high, the calculation of the model will become slow. Fortunately, dynamic BNs (Bilmes, 2000) proposed recently, also the main emphasis of our future research, is an effective tool to solve this problem.

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