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New numerical solution for self-acting gas journal bearings*

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Abstract: Taking a small pressure change in the gas film of self-acting gas-lubricated journal bearings into account, the corresponding nonlinear Reynolds equation is linearized through appropriate approximation and a modified Reynolds equation is derived and solved by means of the finite difference method (FDM). The gas film pressure distribution of a self-acting gas-lubricated journal bearing is attained and the load capacity is calculated. The numerical solution has a better agreement with experimental data than a direct numerical solution for different values of the bearing number. It is of interest to note that the eccentricity ratio, at which the new numerical solution is in better agreement with experimental data, is different when the bearing number is changing. The new numerical solution is slightly larger when the eccentricity ratio is smaller, and becomes slightly smaller when the eccentricity ratio is larger.

Key words: Self-acting gas journal bearings, Reynolds equation, Nonlinearity, Finite difference method (FDM)
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INTRODUCTION

Gas bearings operate with the pressure generated for lubricating film. Because gas bearings have such characteristics as low friction, high precision, and low pollution, they have been successfully used in many commercial applications, such as navigation systems, computer disk drives, high-precision instruments and sensors, dental drills, machine tools and turbo-compressors (Gross, 1980). There was much research into gas bearings in the 1960s. In recent years much attention has been paid to gas bearings with the advent of power micro electromechanical systems (MEMS) research (Epstein *et al*., 1997; Epstein and Senturia, 1997; Gad-el-Hak, 1999; Ehrich and Jacobson, 2003; Wang *et al*., 2005).

The Reynolds equation is the fundamental equation with which the performance of a hydrodynamic gas-lubricated bearing is investigated. Due to gaseous compressibility, the Reynolds equation is nonlinear and its analytical solution is usually difficult to obtain. Therefore a numerical method is an effective means to study gas-lubricated journal bearings. All numerical methods used will be treated in two classes: analytical-numerical and direct-numerical (Castelli and Pirvics, 1968). In the former class either some features of the particular bearing under study are taken advantage of, or some approximations are made in order to partially solve the problem by analytical means. Harrison (1913) first presented solutions for an infinitely long gas-lubricated slider and journal bearings. His work on the infinite journal bearing was improved upon by Katto and Soda (1952) who gave an approximate analytical solution. Ausman (1961) published the linearized pH solution for the small motion of a journal bearing of finite length. Methods of the latter class approach the problem immediately with numerical approximations, the accuracy of which is often easier to be controlled. Elrod and Burgdorfer (1959)

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presented solutions for the infinite journal bearings, i.e., length-to-diameter ratio is infinite ($L/D=\infty$). Raimondi (1961) obtained solutions for the full journal bearings with L/D ratioes of 2, 1, and 0.5. Cheng and Pan (1965) adopted the finite element method (FEM) in analyzing the stability of gas-lubricated, plain journal bearings of finite length. Malik *et al.*(1989) obtained the response of a journal bearing accelerated from an equilibrium state by solving the time-transient Reynolds equation and the equations of motion of a journal bearing by means of the FEM. Bert and Malik (1997) and Malik and Bert (1994) analyzed steady-state and transient characteristics of gas-lubricated journal bearings with a differential quadrature method (DQM). Piekos and Breuer (1999) and Piekos (2000) used an orbit method with a pseudospectral technique for treating the Reynolds equation. Cioc *et al*.(2003; 2004) calculated the flow of gas journal bearings considering inertial effects using the space-time conservation element and solution element (CE/SE) method. Qi *et al.*(2006) transformed the Reynolds equation into a standard elliptic partial differential equation form and gained solutions using Matlab's partial differential equation (PDE) solver.

Because an increased amount of time is spent on solving the nonlinear Reynolds equation with direct numerical approaches, it is essential to find a simple and convenient method for the problem. Due to little change of pressure in the gas film of the self-acting journal bearing, the corresponding nonlinear Reynolds equation can be linearized through proper approximation. The modified Reynolds equation is obtained and solved using the FDM method. The gas film pressure distribution of the self-acting gas-lubricated journal bearing is attained and the load capacity is calculated. Finally, the new numerical solution shows better agreement with experimental data than the direct numerical solution, which proves its validity.

REYNOLDS EQUATION FOR SELF-ACTING GAS JOURNAL BEARINGS

The Reynolds equation, which determines pressure distribution of the bearings, can be derived from the motion equation, continuity equation, and energy equation for gas lubrication film. By assuming steady conditions and neglecting the inertial terms, body forces, x-direction and y-direction components of viscosity force induced by flow gradient, as shown in Fig.1, the Reynolds equation is derived as

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\mu}\frac{\partial p}{\partial y}\right)=6u_0\frac{\partial(\rho h)}{\partial x}+12\frac{\partial(\rho h)}{\partial t}, \tag{1}$$

where ρ is gas density, μ is gas absolute viscosity, h is gas film thickness, p is gas film pressure, u_0 is x-direction component of gas velocity, x-direction (the axial direction) is perpendicular to the plane xOz, t is time variable.

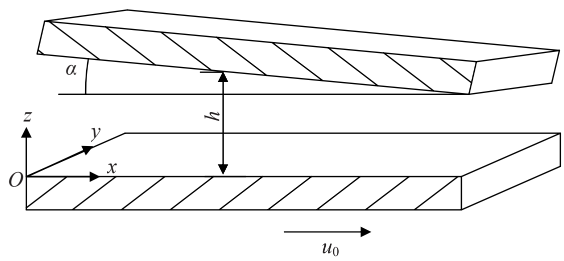

Fig.1 Sketch of gas-lubricated journal bearings' model

The Reynolds equation relating pressure, density, surface velocity, and film thickness is a 2D elliptic differential one, which is fundamental in the gas-lubricated journal bearing field. The solution to the Reynolds equation provides the pressure distribution throughout the film, given the geometry, property, and state parameters. Once the pressure distribution is known, the other properties of the gas film are readily determined.

The gas-lubricated film obeys a polytropic relation (Gross, 1980)

$$p\rho^{-n}=\text{constant}, \tag{2}$$

where p stands for the gas film pressure and ρ the density of gases. n is the polytropic gas-expansion exponent, whose value lies between 1 and γ. γ represents the ratio of the specific heat per unit weight for constant pressure to the one for constant volume, that is, $\gamma=c_p/c_V$. When flow is isothermal, Eq.(2) with n=1 derives from the equation of state.

The viscosities of common gases increase with temperature but are comparatively insensitive to moderate temperature and pressure changes. Within a gas lubricating film, temperature and pressure changes are normally small, and viscosity is considered to be constant.

Therefore, by assuming constant temperature, Eq.(1) becomes

$$\frac{\partial}{\partial x}\left(ph^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(ph^3\frac{\partial p}{\partial y}\right)=6\mu u_0\frac{\partial(ph)}{\partial x}. \quad (3)$$

The self-acting gas-lubricated journal bearing is illustrated in Fig.2, where c is the radial clearance, e the eccentricity, L the bearing length, O_b the center of the bearing, O_j the center of the journal, r the journal radius, W the applied load, W_H in the direction of the line of centers, W_V normal to the direction of the line of centers, θ, ζ the circumferential coordinate and axial coordinate, respectively, ϕ the attitude angle, ω the angular velocity of the journal. The axial direction y is perpendicular to the plane xOz, Eq.(3) becomes

$$\frac{\partial}{\partial \theta}\left(ph^3\frac{\partial p}{\partial \theta}\right)+r^2\frac{\partial}{\partial y}\left(ph^3\frac{\partial p}{\partial y}\right)=6\mu u_0 r\frac{\partial(ph)}{\partial \theta}. \quad (4)$$

The non-dimensional form for Eq.(4) is

$$\frac{\partial}{\partial \theta}\left(PH^3\frac{\partial P}{\partial \theta}\right)+\frac{\partial}{\partial \zeta}\left(PH^3\frac{\partial P}{\partial \zeta}\right)=\Lambda\frac{\partial}{\partial \theta}(PH), \quad (5)$$

where

$$\begin{cases} x=r\theta,\ y=r\zeta, \\ u_0=r\omega,\ P=p/p_\mathrm{a}, \\ \Lambda=\dfrac{6\mu\omega}{p_\mathrm{a}}\left(\dfrac{r}{c_0}\right)^2, \\ h=c_0(1+\varepsilon\cos\theta)=c_0H, \end{cases}$$

where H, P is the non-dimensional gas film thickness and gas film pressure, respectively, r is journal radius, ω is journal angular velocity, Λ is bearing number, p_a is ambient pressure, ε is eccentricity ratio and c_0 is radial clearance.

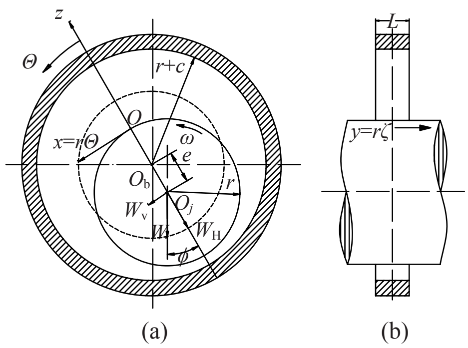

Fig.2 Schematic diagram of self-acting gas-lubricated journal bearings. (a) Cross section; (b) Axial section

The boundary conditions for Eq.(5) are

$$\begin{aligned} &P\left(\theta,\pm\frac{L}{2r}\right)=1, \\ &P(0,\zeta)=P(2\pi,\zeta), \\ &P(\theta,\zeta)=P(\theta,-\zeta), \end{aligned} \quad (6)$$

where

$$\theta\in[0,2\pi],\ \zeta\in\left[-\frac{L}{2r},\frac{L}{2r}\right].$$

Eq.(5), a non-linear PDE with respect to non-dimensional pressure P, can be solved directly by the finite difference method (FDM). However, as mentioned above, the disadvantages of the direct-numerical method are: increased amount of computer time, the need for an efficient computer system and the instability in the convergence of iteration process.

MODIFIED REYNOLDS EQUATION

Because the Reynolds equation is a non-linear partial differential one, the calculation method of the direct-numerical solution is more complex and occupies much more computer time. In order to simplify the calculation, the Reynolds equation is linearized as follows. First Eq.(5) becomes

$$\begin{aligned} &H^3\left(\frac{\partial^2 P}{\partial\theta^2}+\frac{\partial^2 P}{\partial\zeta^2}\right)+3H^2\frac{\partial H}{\partial\theta}\frac{\partial P}{\partial\theta}-\Lambda\frac{\mathrm{d}H}{\mathrm{d}\theta} \\ &=-\frac{H^3}{P}\left[\left(\frac{\partial P}{\partial\theta}\right)^2+\left(\frac{\partial P}{\partial\zeta}\right)^2\right]+\Lambda\frac{H}{P}\frac{\partial P}{\partial\theta}. \end{aligned} \quad (7)$$

Considering the compressibility, low viscosity and low pressure of gas-lubricated journal bearings, $\left(\frac{\partial P}{\partial \theta}\right)^2$, $\left(\frac{\partial P}{\partial \zeta}\right)^2$ are considered to be small. The pressure gradient of gas film, $\frac{\partial P}{\partial \theta}$, $\frac{\partial P}{\partial \zeta}$ have the same trend as the eccentricity ratio ε, that is, $\frac{\partial P}{\partial \theta}$, $\frac{\partial P}{\partial \zeta}$ increase as ε increases. Therefore, it is appropriate to make the following approximation:

$$\left(\frac{\partial P}{\partial \theta}\right)^2 + \left(\frac{\partial P}{\partial \zeta}\right)^2 \approx \varepsilon \frac{\partial P}{\partial \theta} + \varepsilon \frac{\partial P}{\partial \zeta}. \tag{8}$$

Harrison (1913), Katto and Soda (1952) substituted $p_a c$ for ph in deriving a linearized ph solution of the Reynolds equation, and the same approximation is adopted here. Therefore, the non-dimensional pressure P in the denominator of the right hand in Eq.(7) is replaced by $\frac{P_0}{1+\varepsilon\cos\theta} = \frac{1}{H}$. Eq.(7) becomes

$$\begin{aligned} &H^3\left(\frac{\partial^2 P}{\partial \theta^2} + \frac{\partial^2 P}{\partial \zeta^2}\right) + 3H^2 \frac{\partial H}{\partial \theta}\frac{\partial P}{\partial \theta} - \Lambda \frac{\mathrm{d}H}{\mathrm{d}\theta} \\ &\quad + H^2\left(\varepsilon \frac{\partial P}{\partial \theta} + \varepsilon \frac{\partial P}{\partial \zeta}\right) - \Lambda \frac{\partial P}{\partial \theta} = 0. \end{aligned} \tag{9}$$

As soon as Eq.(9) is solved by the means of FDM, the characteristics of gas bearings, such as pressure distribution, load capacity, are obtained.

The load capacity of the self-acting gas-lubricated journal bearings consists of two parts: one is W_{H} in the direction of the center line, and the other is W_{V} normal to the direction of the center line, as shown in Fig.2.

$$\begin{cases} W = \sqrt{W_{\mathrm{H}}^2 + W_{\mathrm{V}}^2}, \\ W_{\mathrm{H}} = 2\int_0^{L/2}\int_0^{2\pi} p\cos\theta r\mathrm{d}\theta\mathrm{d}y, \\ W_{\mathrm{V}} = 2\int_0^{L/2}\int_0^{2\pi} p\sin\theta r\mathrm{d}\theta\mathrm{d}y. \end{cases} \tag{10}$$

After discretization of the solution region, Eq.(10) becomes:

$$\begin{cases} W_{\mathrm{H}} = p_{\mathrm{a}} \cdot 2\pi r L \sum_{i=1}^{n-1}\sum_{j=1}^{m-1} P_{i,j}\cos\theta_i \Delta\theta\Delta\zeta, \\ W_{\mathrm{V}} = p_{\mathrm{a}} \cdot 2\pi r L \sum_{i=1}^{n-1}\sum_{j=1}^{m-1} P_{i,j}\sin\theta_i \Delta\theta\Delta\zeta, \end{cases} \tag{11}$$

where n, m are the circumferential and axial nodes of gas-lubricated journal bearings, respectively.

LOAD CAPACITY CALCULATION AND COMPARISON

To demonstrate the validity of the new numerical solution, numerical results are compared with experimental data in the literature. Experimental data for gas journal bearings has been rare in recent years, but it can be found in many researches of the 1960s. The parameters here have the same values as those in (Whitley and Betts, 1959), as shown in Table 1.

Table 1 Parameters of gas journal bearings

Parameter	Value
Journal radius r (mm)	25.4
Radial clearance c_0 (mm)	12.7×10^{-3}
Bearing width L (mm)	76.2
Lubricant	Argon
Absolute viscosity μ (N·s/m^2)	2.29×10^{-5}
Ambient pressure P_{a} (N/m^2)	1.033×10^{5}

Fig.3 demonstrates the pressure distribution of self-acting gas-lubricated journal bearings. It can be seen that pressure distribution for gas bearings is different from that for oil bearings because of gas compressibility: (1) There is a large pressure peak value in the convergent region while a small pressure peak value in the divergent region; i.e., the magnitude of pressure in the convergent region is larger than that in the divergent region; (2) The pressure in the convergent region ascends smoothly while that in the divergent region descends quickly; (3) Contrary to oil bearings, the magnitude of pressure change is small, which explains the smaller load capacity of gas bearings.

It is of interest to compare the calculated values with experimental data. The comparisons of load capacity versus eccentricity ratio between the direct numerical solution, the new numerical solution and the experimental data for different bearing numbers are shown in Figs.4a~4c.

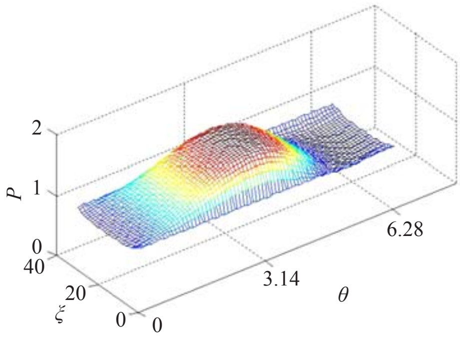

Fig.3 Non-dimensional pressure distribution of self-acting gas-lubricated journal bearings

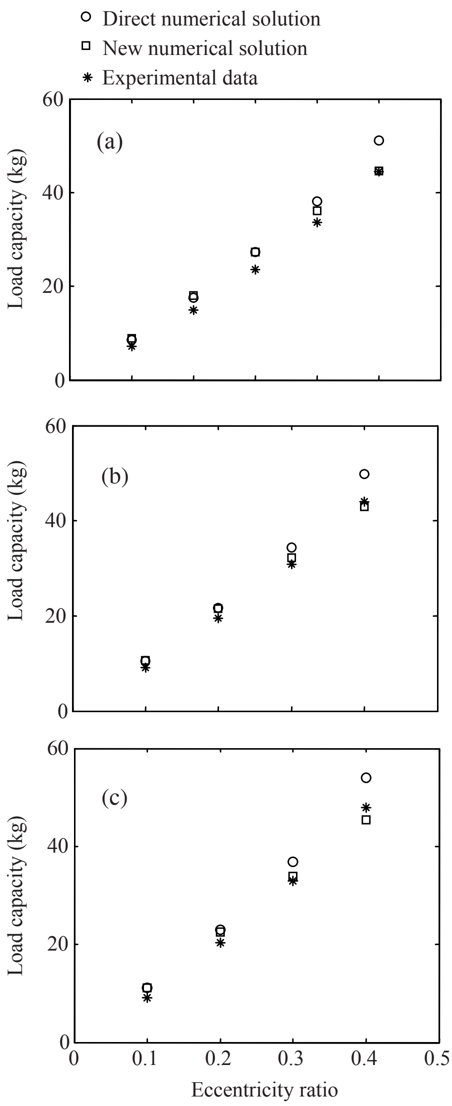

Fig.4 Comparison of load capacity when the bearing number Λ is (a) 1.63; (b) 3.26; (c) 4.89

It can be seen that the load capacity for self-acting gas-lubricated journal bearings increases with an increasing eccentricity ratio. For the same bearing number, both the value of the direct numerical solution and that of the new numerical solution are larger than that with experimental data. The difference between the new numerical solution and the experimental data is smaller than that between the direct numerical solution and the experimental data. Especially when the bearing number increases, the consistency between the new numerical solution and the experimental data is better.

In addition, the eccentricity ratio at which the new numerical solution is in better agreement with the experimental data is different when the bearing number is changing. The new numerical solution is larger than the experimental data when the eccentricity ratio is smaller than 0.3, and becomes smaller than the experimental data when the eccentricity ratio is 0.4.

CONCLUSION

As a non-linear PDE, the Reynolds equation for self-acting gas-lubricated journal bearings is difficult to obtain as an exact analytical solution. It is simple and convenient to transform the Reynolds equation into a linear partial differential one through proper approximation and attain the numerical solution using the FDM. The study demonstrates that the new numerical solution shows better agreement with the experimental data than the direct numerical solution, showing it is superior to the direct numerical solution. It is of interest to note that the eccentricity ratio at which the new numerical solution is in better agreement with the experimental data is different when the bearing number is changing. The new numerical solution is larger than the experimental data when the eccentricity ratio is smaller than 0.3, and becomes smaller than the experimental data when the eccentricity ratio is 0.4.

References

Ausman, J.S., 1961. An improved analytical solution for self-acting, gas-lubricated journal bearings of finite length. *Journal of Basic Engineering*, **83**:188-194.

Bert, C.W., Malik, M., 1997. Transient analysis of gas-lubricated journal bearing systems by differential quadrature. *Journal of Tribology*, **119**(1):91-99. [doi:10.1115/1.2832489]

Castelli, V., Pirvics, J., 1968. Review of numerical methods in gas bearing film analysis. *Journal of Lubrication Technology*, **90**(4):777-792.

Cheng, H.S., Pan, C.H.T., 1965. Stability analysis of gas lubricated, self-acting, plain, cylindrical journal bearings of finite length, using Galerkin's method. *Journal of Basic Engineering*, **87**(1):185-192.

Cioc, S., Dimofte, F., Keith, T.G., Fleming, D.P., 2003. Computation of pressurized gas bearings using the CE/SE method. *Tribology Transactions*, **46**(1):128-133. [doi:10.1080/10402000308982609]

Cioc, S., Keith, T.G., Dimofte, F., Fleming, D.P., 2004. Calculation of the flow in high-speed gas bearings including inertia effects using the CE/SE method. *Tribology Transactions*, **47**(3):402-412. [doi:10.1080/05698190490455636]

Ehrich, F.F., Jacobson, S.A., 2003. Development of high-speed gas bearings for high-power density microdevices. *Journal of Engineering for Gas Turbines and Power*, **125**(1):141-149. [doi:10.1115/1.1498273]

Elrod, H.G., Burgdorfer, A., 1959. Refinement of the Theory of Gas-lubricated Journal Bearings of Infinite Length. Proceedings of the First International Symposium on Gas Lubricated bearings, Washington, D.C.

Epstein, A.H., Senturia, S.D., 1997. Microengineering: macro power from micro machinery. *Science*, **276**(5316):1211. [doi:10.1126/science.276.5316.1211]

Epstein, A.H., Senturia, S.D., Al-Midani, O., 1997. Micro-heat engine, gas turbines and rocket engines—the MIT microengine project. AIAA Paper, Snowmass Village, Co., Paper 97-1773.

Gad-el-Hak, M., 1999. The fluid mechanics of microdevices—the freeman scholar lecture. *Journal of Fluids Engineering*, **121**(1):5-33. [doi:10.1115/1.2822013]

Gross, W.A., 1980. Fluid Film Lubrication. John Wiley & Sons, New York, U.S.

Harrison, W.J., 1913. The hydrodynamical theory of lubrication with special reference to air as a lubricant. *Transactions of Cambridge Philosophical Society*, **22**:34-54.

Katto, Y., Soda, N., 1952. Theory of Lubrication by Compressible Fluid with Special Reference to Air Bearing. Proceedings of the Second Japan National Congress for Applied Mechanics, p.267-270.

Malik, M., Bert, C.W., 1994. Differential quadrature solutions for steady-state incompressible and compressible lubrication problems. *Journal of Tribology*, **116**(2):296-302. [doi:10.1115/1.2927214]

Malik, M., Rahmatabadi, A.D., Jain, S.C., 1989. An assessment of the stability chart of linearized gas-lubricated plane journal bearing system. *Tribology Transaction*, **32**(1):54-60. [doi:10.1080/10402008908981862]

Piekos, E.S., 2000. Numerical Simulation of Gas-lubricated Journal Bearings for Microfabricated Machines. PhD Thesis, Massachusetts Institute of Technology, Cambridge, U.S.

Piekos, E.S., Breuer, K.S., 1999. Pseudospectral orbit simulation of non-ideal gas-lubricated journal bearings for microfabricated turbomachines. *Journal of Tribology*, **121**(3):604-609. [doi:10.1115/1.2834110]

Qi, S.M., Geng, H.P., Yu, L., 2006. New method for the calculation of the characteristics of aerodynamic bearings. *Journal of Mechanical Strength*, **28**(3):369-373 (in Chinese).

Raimondi, A.A., 1961. A numerical solution for the gas lubricated full journal bearing of finite length. *ASLE Transactions*, **4**:131-155.

Wang, J., Zhang, L., Xu, Z.J., 2005. Analysis of load capacity for aerodynamic bearings in power MEMS. *Journal of Chongqing University (Natural Science Edition)*, **28**(2):5-7 (in Chinese).

Whitley, S., Betts, C., 1959. Study of gas-lubricated, hydrodynamic, full journal bearing. *British Journal of Applied Physics*, **10**(10):455-463. [doi:10.1088/0508-3443/10/10/305]