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Hybrid analytical resolution approach based on ambiguity function for attitude determination

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Abstract: When satellite navigation receivers are equipped with multiple antennas, they can deliver attitude information. In previous researches, carrier phase differencing measurement equations were built in the earth-centered, earth-fixed (ECEF) coordinate, and attitude angles could be obtained through the rotation matrix between the body frame (BF) and the local level frame (LLF). Different from the conventional methods, a hybrid algorithm is presented to resolve attitude parameters utilizing the single differencing (SD) carrier phase equations established in LLF. Assuming that the cycle integer ambiguity is known, the measurement equations have attitude analytical resolutions by using simultaneous single difference equations for two in-view satellites. In addition, the algorithm is capable of reducing the search integer space into countable 2D discrete points and the ambiguity function method (AFM) resolves the ambiguity function within the analytical solutions space. In the case of frequency division multiple access (FDMA) for the Russian Global Orbiting Navigation Satellite System (GLONASS), a receiver clock bias estimation is employed to evaluate its carrier phase. An evaluating variable and a weighted factor are introduced to assess the integer ambiguity initialization. By static and dynamic ground experiments, the results show that the proposed approach is effective, with enough accuracy and low computation. It can satisfy attitude determination in cases of GPS alone and combined with GLONASS.

Key words: Global positioning system (GPS), Russian Global Orbiting Navigation Satellite System (GLONASS), Analytical resolution, Attitude determination, Ambiguity function method (AFM)

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INTRODUCTION

Global positioning system (GPS) has been developed for the purpose of enabling accurate positioning and navigation anywhere on or near the surface of the Earth. In addition to the US system GPS-NAVSTAR, the Russian Global Orbiting Navigation Satellite System (GLONASS) is also in place. The European Galileo and Chinese COMPASS systems are currently under study and under construction. GLONASS is the Russian equivalent of the global positioning system. Like the GPS, the whole GLONASS was planned to consist of 24 satellites and 17 satellites are available now. GLONASS has great potential for precise navigation and geodetic applications. It has also been seen that there are many advantages in the integration of the two existing satellite systems (GPS and GLONASS). For example, their combination offers an increase in accuracy and integrity gained by adding more visible satellites. When the GLONASS system is completely deployed, at least 12 satellites will be visible in open areas at any time and the maximum number of in-view satellites can reach 20 in the best scenario. The increase in satellite availability will also make fast static and kinematic positioning much more feasible than it is with just each system alone (Hein *et al.*, 1997; Langley, 1997).

When the receivers of GPS or similar systems are equipped with multiple antennas with the baselines even as short as about 1 m, they can give attitude information. Therefore, the phase differences between signals received by the different antennas now constitute the key measurements. Since carrier phase difference measurements are ambiguous due to the unknown number of carrier signal cycles received, the estimated attitude is in principle ambiguous as well. Therefore, the resolution of the signal cycle ambiguity becomes a necessary task before determining the attitude (Chu and van Woerkom, 1997; Caporali, 2001). However, due to the fact that the GLONASS satellites transmit their signals using different frequencies, processing the GPS/GLONASS carrier phase measurement is much more complicated than processing only GPS data. In processing the GLONASS carrier phase, one of the critical issues is that the standard double-differencing (DD) procedure cannot cancel receiver clock terms in the DD carrier phase measurement. Consequently, the unknown parameters in the measurement equations include baseline components, DD ambiguities and relative receiver clock terms. As a consequence of this, the normal matrix becomes singular (Wang *et al.*, 2001). In order to remove this singularity, a number of modeling methods have been investigated.

As one of the important on-the-fly (OTF) approaches, the ambiguity function method (AFM) uses only the fractional value of the instantaneous carrier phase measurement, so the ambiguity function values are insensitive to the whole cycle change of the carrier phase or cycle slips (Li *et al.*, 2005; Wang *et al.*, 2007). In attitude determination, a fixed-length baseline is used to constrain the ambiguity resolution in the AFM process. Juang and Huang (1997) studied the AFM application in vehicle attitude determination and explored the modified Hopfield neural network approach to find the attitude. Xu *et al.*(2002) discussed the approach to apply the AFM to dynamic attitude determination. In addition, the GPS DD measurements are used in the above AFM process.

In this study, an integrated algorithm based on attitude analytical resolution and the OTF AFM is proposed. It computes the analytical solutions of dual nonlinear coupled equations using observations and assumed integer ambiguities of two satellites. Rather than in the 2D search space, some discrete points are gained, among which the maximum value is found. A mathematical description of the proposed approach and practical strategies for data processing are presented and tested using field data sets collected on single- and double-baselines.

GPS/GLONASS CARRIER PHASE DIFFERENCING MEASUREMENT

GPS carrier phase differencing measurement

The carrier phase observation equation for GPS can be written as

$$\varphi=\frac{1}{\lambda_{L1}}(\rho+\delta\rho+c\delta t_{\mathrm{r}}-c\delta t_{\mathrm{s}}+\delta\rho_{\mathrm{c}}+\delta\rho_{\mathrm{t}}-\delta\rho_{\mathrm{i}})-N+\varepsilon, \tag{1}$$

where φ is the GPS receiver carrier phase observation; $L1$ is one of the GPS carrier waves; λ_{L1} is the GPS $L1$ wavelength; ρ is the range between receiver antenna and a GPS satellite; $\delta\rho$ is the orbital error along the line of sight (LOS) from satellite to station; c is the speed of light in vacuum; δt_{r} is the receiver clock offset from GPS time; δt_{s} is the satellite clock offset from GPS time; $\delta\rho_{\mathrm{c}}$ is the line bias delay caused by the physical length of the cable; $\delta\rho_{\mathrm{t}}$ is the tropospheric delay; $\delta\rho_{\mathrm{i}}$ is the ionospheric delay; N is the carrier phase integer ambiguity; ε is the error term which includes measurement noise, multipath errors, etc.

A GPS-based attitude determination system usually consists of at least two radio frequency (RF) ports. Each port receives the GPS signals from an independent antenna. One can use two or more independent GPS receivers with $L1$ carrier phase output capability to construct an attitude determination system (Juang and Huang, 1997; Jang and Kee, 2006). One benefit of using a common clock reference is that the clock error is the same for all RF signals including carrier phase measurements, and thus it can be removed by single differences. For a baseline length of a few meters and the same GPS satellite, the orbital and atmospheric errors in Eq.(1) are practically the same. So the single differencing (SD) measurements between two antennas i, j are given as follows:

$$\Delta\varphi=\frac{1}{\lambda_{L1}}(\Delta\rho+c\Delta\delta t_{\mathrm{r}}+\Delta\delta\rho_{\mathrm{c}})-\Delta N+\Delta\varepsilon, \tag{2}$$

where Δ is the SD operator. SD does not eliminate the receiver clock error δt_{r} or the line bias $\delta\rho_{\mathrm{c}}$. The integer ambiguity term ΔN is not eliminated in the case of the single difference. The error term $\Delta\varepsilon$ includes multipath and receiver noise.

GLONASS carrier phase differencing measurement

GLONASS uses the frequency division multiple access (FDMA) technique to distinguish the signals from different satellites, rather than the code division multiple access (CDMA) technique used by GPS. The carrier phase observation equation for GLONASS satellite p can be written as

$$\varphi^p = \frac{1}{\lambda^p}\rho^p + \frac{c}{\lambda^p}(\delta t_s^p - \delta t_r) + \frac{c}{\lambda^p}(\delta\rho^p + \delta\rho_t^p - \delta\rho_i^p) - N^p + e^p + \varepsilon, \quad (3)$$

where the superscript p identifies the satellite; λ^p is the satellite p carrier wavelength; e^p is the hardware phase delay of the receiver. Due to the different GLONASS carrier frequencies, the value e^p of each channel is different (Leick *et al.*, 1995).

In the case of short baselines, the use of between-receiver carrier phase SD is proposed herein to resolve some problems. Similar to the GPS measurements, the SD procedures can considerably reduce some systematic errors existing in the GLONASS measurements, such as ionospheric delay, tropospheric delay and satellite orbit errors. For the same GLONASS satellite, the resulting SD mathematical model between the antennas i, j can be simplified as

$$\Delta\varphi^p = \frac{1}{\lambda^p}\Delta\rho^p - \frac{c}{\lambda^p}\Delta\delta t_r - \Delta N^p + \Delta e^p + \Delta\varepsilon, \quad (4)$$

where $\Delta\varphi^p$ is the SD carrier phase expressed in units of cycles; $\Delta\rho^p$ is the SD receiver-satellite range; $\Delta\delta t_r$ is the relative receiver clock error. For satellites p and q, the DD carrier phase is further written as

$$\nabla\Delta\varphi^{pq} = \frac{1}{\lambda^p}\Delta\rho^p - \frac{1}{\lambda^q}\Delta\rho^q - c\left(\frac{1}{\lambda^p} - \frac{1}{\lambda^q}\right)\Delta\delta t_r - \nabla\Delta N^{pq} + \nabla\Delta e^{pq} + \nabla\Delta\varepsilon. \quad (5)$$

where $\nabla\Delta$ denotes the DD operator.

Unlike GPS, the DD GLONASS carrier phases are sensitive to the receiver clock errors. Since the relative receiver clock term exists in Eq.(5), a receiver clock parameter has to be set up employing the baseline components and the DD ambiguities. This also means that the DD ambiguity parameters cannot be separated from the receiver clock parameter (Wang, 2000). In order to reduce the receiver clock errors in the DD carrier phase, such strategies as converting the original carrier phase into distances, the GLONASS mean frequency or the GPS $L1$ frequency have been proposed in (Xu *et al.*, 2002).

GLONASS receiver clock offset differencing

Considering Eq.(4), the GLONASS satellite p carrier phase measurement can also be expressed in Eq.(6). For the same receiver chip, the channel delay differencing value is zero. But the cable delay still exists, and here Δe^p denotes cable delay.

$$\lambda^p\Delta\varphi^p = \Delta\rho^p - c\Delta\delta t_r - \lambda^p\Delta N^p + \Delta e^p + \Delta\varepsilon. \quad (6)$$

Eq.(6) can be rewritten as

$$\lambda^p\Delta N^p = -\lambda^p\Delta\varphi^p + \Delta\rho^p - c\Delta\delta t_r + \Delta e^p + \Delta\varepsilon. \quad (7)$$

When the two receivers adopt different clocks, $\Delta\delta t_r$ and Δe^p have random shift values. If a common oscillator is used to generate time and frequency for the two receiver clocks, the term $\Delta\delta t_r$ of each receiver will have the same drift. However, this does not imply that the receiver clock offsets are identical. This is because the clock provides the two receivers with only a stream of pulses with no absolute time tag. Thus, the two initial receiver biases are different. This can also be seen by decomposing the $\Delta\delta t_r$ into two parts, the first part being an absolute initial bias from GLONASS time and the second part being the drift over time. Driving two receivers with a common oscillator ensures only that the second part is identical, but there is no guarantee that the first part is the same for both receivers. The term $\Delta\delta t_r$ is, therefore, a non-zero constant term (Keong, 1999).

In further investigation, the origin of this phenomenon is the front-end circuit of GPS/GLONASS receivers, since the phase lock loop (PLL) and multilevel frequency mixing are used in the RF circuit. When the frequency dividing circuit is power-on, the counter value is random, i.e., the initial phase is random although it is in synchronization with the local clock. Therefore, the term $\Delta\delta t_r$ will not be changed any more after the receiver is power-on because the $\Delta\delta t_r$ initial value relates only to the frequency divider.

The total SD residual bias can be expressed as

$$\Delta\tau = \Delta\delta t_{\mathrm{r}} - \frac{1}{c}\Delta e^{p}, \tag{8}$$

$\Delta\tau$ for GPS/GLONASS is constant over time with zero-mean white noise. At the same time, $\Delta\tau$ is largely absorbed in the integer ambiguity and finally becomes a fractional part within a cycle. Therefore, the bias can be estimated using a low pass filter:

$$\Delta\tilde{\tau}_k = \frac{k-1}{k}\Delta\tilde{\tau}_{k-1} - \frac{1}{k}\Delta\tau_k, \tag{9}$$

where $\Delta\tilde{\tau}_k$ is the estimate of the residual bias and $\Delta\tau$ is the residual bias. In the common clock system, $\Delta\tau$ must be measured first after the receiver starts. And this procedure can be a part of an attitude determination self survey (Park and Crassidis, 2006).

ATTITUDE ANALYTICAL RESOLUTION METHOD FOR SINGLE- AND DOUBLE-BASELINES

Single differencing measurement equation in a local level frame

Generally, the relative coordinates of the receivers can be obtained in the earth-centered, earth-fixed (ECEF) frame on a real-time basis. However, the vehicle attitude parameters are built in the local level frame. Therefore, the 3D attitude parameters can be obtained through the transformation between ECEF and local level frame (LLF).

In this research, the differencing measurement equations have been built up directly in LLF. ***b*** is the baseline vector, and the carrier signal from satellite *i* goes through antenna A and then reaches antenna B (Fig.1). As the baseline length is several meters, both ionospheric delay and tropospheric delay are cancelled. The single-differenced carrier phase measurement, which is the projection of the baseline in the direction of LOS, can be expressed as

$$\lambda^i \Delta p^i = \boldsymbol{b}\cdot\boldsymbol{S}^i, \tag{10}$$

where Δp^i is the true carrier phase differencing between antennas A and B, and

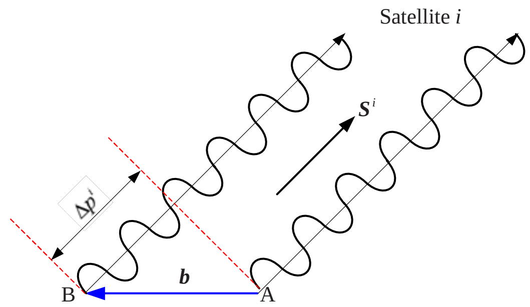

Fig.1 Carrier phase difference measurement model from two receiver antennas

$$\Delta p^i = \Delta\varphi^i + \Delta N^i + \frac{c}{\lambda^i}\Delta\tau. \tag{11}$$

Therefore, the SD measurement equation can be written as

$$\Delta\varphi^i = \frac{1}{\lambda^i}\boldsymbol{b}\cdot\boldsymbol{S}^i - \frac{c}{\lambda^i}\Delta\tau - \Delta N^i + \Delta\varepsilon^i. \tag{12}$$

Given the fact that ψ and θ are the yaw and pitch angles of ***b*** respectively, and that α^i and β^i are the elevation and azimuth angles of satellite *i* respectively, we have

$$\begin{cases} \boldsymbol{b} = \|\boldsymbol{b}\|(\cos\theta\sin\psi \quad \cos\theta\cos\psi \quad \sin\theta), \\ \boldsymbol{S}^i = (\cos\alpha^i\sin\beta^i \quad \cos\alpha^i\cos\beta^i \quad \sin\alpha^i). \end{cases} \tag{13}$$

The noise $\Delta\varepsilon^i$ can be omitted. Eq.(12) can be rewritten as

$$\begin{aligned}\Delta\varphi^i = {} & \frac{|\boldsymbol{b}|}{\lambda^i}[\sin\alpha^i\sin\theta + \cos\alpha^i\cos\theta\cos(\beta^i-\psi)] \\ & - \frac{c}{\lambda^i}\Delta\tau - \Delta N^i.\end{aligned} \tag{14}$$

Eq.(14) is a transcendental function, where ψ, θ and ΔN^i are unknown and ΔN^i is an integer theoretically (Xu *et al.*, 2002). $\Delta\tau$ is the receiver clock offset constant and can be obtained after the system starts. In addition, observed SD subtracts the $\Delta\tau$ term, and this process cannot affect the subsequent calculation. Herein, the equation without $\Delta\tau$ can be obtained finally:

$$\Delta\varphi^i = \frac{\|\boldsymbol{b}\|}{\lambda^i}[\sin\alpha^i \sin\theta + \cos\alpha^i \cos\theta\cos(\beta^i - \psi)] - \Delta N^i. \quad (15)$$

Similarly, the observation equations with m satellites can be described and have m+2 unknown parameters. If we keep tracking the satellites over the next epoch, integer ambiguity will remain constant. Then the real-time integer ambiguity and attitude angles that change with the movement of the carrier can be evaluated after many epochs.

In Eq.(12), for the GPS signal, $\lambda^i=\lambda_{L1}$; for the GLONASS signal, there are different λ^i values. Then for satellites j, k and single baseline $\|\boldsymbol{b}\|$, the DD carrier phase measurement equation is expressed as

$$\nabla\Delta\varphi^{jk} = \frac{1}{\lambda^j}(\boldsymbol{b}\cdot\boldsymbol{S}^j - c\Delta\tau) - \frac{1}{\lambda^k}(\boldsymbol{b}\cdot\boldsymbol{S}^k - c\Delta\tau) - \nabla\Delta N^{jk}. \quad (16)$$

If both satellites j, k are GPS satellites ($\lambda^j=\lambda^k=\lambda_{L1}$), the receiver clock bias can also be eliminated:

$$\nabla\Delta\varphi^{jk} = \frac{1}{\lambda_{L1}}\boldsymbol{b}\cdot(\boldsymbol{S}^j - \boldsymbol{S}^k) - \nabla\Delta N^{jk}. \quad (17)$$

Single baseline resolution for 2D attitude determination

Given the fact that the baseline length is fixed and ΔN^i is known, Eq.(15) in the 3D space can be demonstrated by a circle on a sphere with the center at point A and $\|\boldsymbol{b}\|$ as the radius. The line connecting the center of the circle to satellite i is vertical to the circle. The distance between the two centers of the circle and the sphere is

$$\lambda^i\Delta p^i = \lambda^i(\Delta\phi^i + \Delta N^i). \quad (18)$$

Given the fact that ΔN^i and ΔN^j for satellites i, j are known, the circles of satellites i, j are not concentric, and there are two intersections P_1 and P_2. One of them should be the correct attitude point, as shown in Fig.2. Similarly, the single-differenced integer ambiguity of the third satellite k is ΔN^k, and the third circle consequentially intersects the other two circles (Wang *et al.*, 2007). One of the two intersections certainly superposes the correct attitude angle. The correct attitude angle is P_1, as shown in Fig.2. More closed curves intersect any other closed curves at two intersections, and one of the intersections should be P_1.

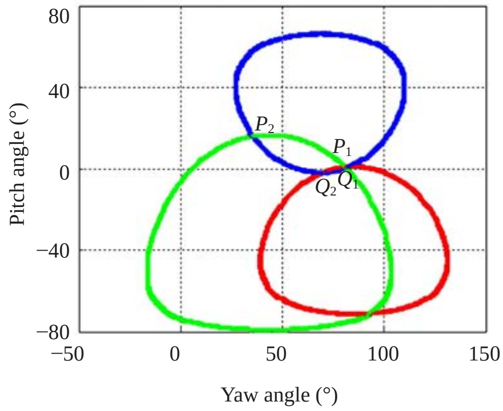

Fig.2 Intersections of three closed curves

The key issue in this problem is how to quickly obtain the intersections of two closed curves. To do this, the following equations are obtained from Eq.(15):

$$\begin{cases} \sin\beta^i\sin\theta + \cos\beta^i\cos\theta\cos(\alpha^i - \psi) \\ \quad = \lambda^i(\Delta\varphi^i + \Delta N^i)/\|\boldsymbol{b}\|, \\ \sin\beta^j\sin\theta + \cos\beta^j\cos\theta\cos(\alpha^j - \psi) \\ \quad = \lambda^j(\Delta\varphi^j + \Delta N^j)/\|\boldsymbol{b}\|. \end{cases} \quad (19)$$

Note that λ^i and λ^j represent the wavelengths of satellite i and satellite j, respectively, and these two satellites may belong to a different global positioning system or the same system, such as i for GPS and j for GLONASS or both for GLONASS. However, the process of Eq.(19) is very complicated and the final results are given as follows:

$$\psi = \pm\arcsin\left[\left(-g \pm \sqrt{g^2 - 4h}\right)/2\right], \quad (20)$$

$$\begin{cases} g = -2(e-d)(1-d) + 4f^2/[(e-d)^2 + 4f^2], \\ h = (1-d)^2/[(e-d)^2 + 4f^2], \end{cases}$$

$$\begin{cases} d = a\cos^2\beta^i + b\cos^2\beta^j + c\cos\beta^i\cos\beta^j, \\ e = a\sin^2\beta^i + b\sin^2\beta^j + c\sin\beta^i\sin\beta^j, \\ f = [a\sin(2\beta^i) + b\sin(2\beta^j) + c\sin(\beta^i + \beta^j)]/2, \end{cases}$$

$$\begin{cases} a = (\tan^2 \alpha^i)(1-(S^j)^2)/(S^i - S^j)^2, \\ b = (\tan^2 \alpha^j)(1-(S^i)^2)/(S^i - S^j)^2, \\ c = -(\tan \alpha^i)(\tan \alpha^j)(1-S^i S^i)/(S^i - S^j)^2, \end{cases}$$

$$\begin{cases} S^i = \lambda^i (\Delta\varphi^i + \Delta N^i)/(\|\boldsymbol{b}\| \sin \beta^i), \\ S^j = \lambda^j (\Delta\varphi^j + \Delta N^j)/(\|\boldsymbol{b}\| \sin \beta^j). \end{cases}$$

If the yaw angle ψ is known, the pitch angle θ can be obtained easily. According to the descriptions above, there will be eight solutions (ψ_k, θ_k) (k=1, 2, …, 8), but two of them are reasonable. The other six solutions can be cancelled by verification in the equations or by equation constraints, such as excessively large pitch values. Assume that there are l pairs of preliminary solutions from n pairs of integer ambiguity candidates, in which only one solution is correct by verification via the AFM.

Double baseline for 3D attitude determination

As mentioned above, it is easier to obtain the observation equations of double baselines:

$$\begin{cases} \sin\beta^i \sin\theta_1 + \cos\beta^i \cos\theta_1 \cos(\alpha^i - \psi_1) \\ \quad = \lambda^i (\Delta\varphi_1^i + \Delta N_1^i)/\|\boldsymbol{b}_1\|, \\ \sin\beta^j \sin\theta_1 + \cos\beta^j \cos\theta_1 \cos(\alpha^j - \psi_1) \\ \quad = \lambda^j (\Delta\varphi_1^j + \Delta N_1^j)/\|\boldsymbol{b}_1\|, \\ \sin\beta^i \sin\theta_2 + \cos\beta^i \cos\theta_2 \cos(\alpha^i - \psi_2) \\ \quad = \lambda^i (\Delta\varphi_2^i + \Delta N_2^i)/\|\boldsymbol{b}_2\|, \\ \sin\beta^j \sin\theta_2 + \cos\beta^j \cos\theta_2 \cos(\alpha^j - \psi_2) \\ \quad = \lambda^j (\Delta\varphi_2^j + \Delta N_2^j)/\|\boldsymbol{b}_2\|. \end{cases} \tag{21}$$

Here $\boldsymbol{b}_1=\|\boldsymbol{b}_1\|(\cos\theta_1\sin\psi_1 \ \ \cos\theta_1\cos\psi_1 \ \ \sin\theta_1)$ (ψ_1, θ_1 are yaw and pitch angles, respectively), $\boldsymbol{b}_2 = \|\boldsymbol{b}_2\|$ $\cdot(\cos\theta_2\sin\psi_2 \ \ \cos\theta_2\cos\psi_2 \ \ \sin\theta_2)$ (ψ_2, θ_2 are yaw and pitch angles, respectively), and the two baselines are orthogonal. According to Eq.(20), the yaw and pitch angles of $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ can be resolved, respectively.

Due to $\boldsymbol{b}_1 \perp \boldsymbol{b}_2$, the constraint relationship between the two baselines can be expressed as

$$\cos\theta_1 \cos\theta_2 (\cos\psi_1 \cos\psi_2 + \sin\psi_1 \sin\psi_2) + \sin\theta_1 \sin\theta_2 = 0. \tag{22}$$

A single-baseline can produce two resolutions and a double-baseline can produce four resolutions. According to the constraint condition equation, the correct resolution can be chosen directly. To obtain the 3D attitude parameters, the roll angle γ should be determined. According to geometric principle, γ is equal to $\angle B_2OE$ shown in Fig.3. It can be also proved that γ can be described below:

$$\cos\gamma = \cos\theta_2 \sin(\psi_2 - \psi_1). \tag{23}$$

The 3D attitude parameters (yaw angle ψ, pitch angle θ and roll angle γ) of a vehicle can be calculated by the measured yaw and pitch angles of two baselines (ψ_1, ψ_2) and (θ_1, θ_2) (Wang, 2008):

$$\begin{cases} \psi = \psi_1, \\ \theta = \theta_1, \\ \gamma = \arccos[\cos\theta_2 \sin(\psi_2 - \psi_1)]. \end{cases} \tag{24}$$

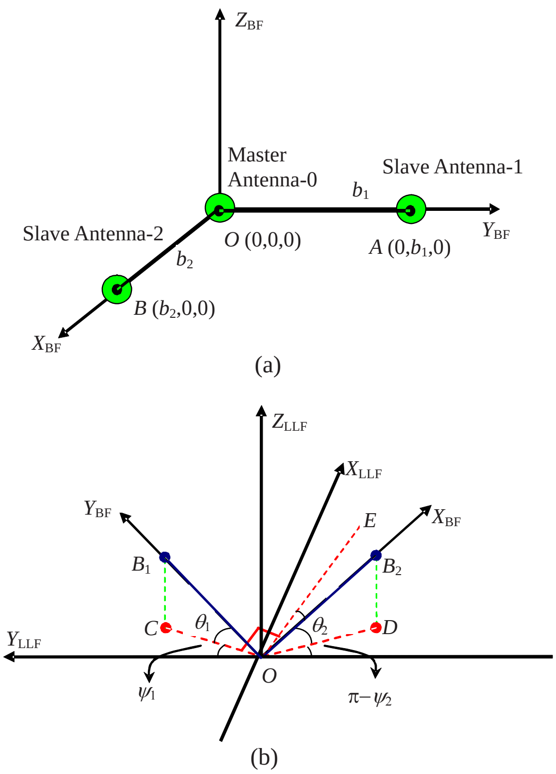

Fig.3 Relationship between (a) yaw angles and (b) pitch angles of double baseline

Integer ambiguity resolution

It is easier to solve attitude angles using Eq.(20) directly than to search for the maximum in the full 2D space. Note that a set of integer ambiguities of satellites i and j should be assumed previously. From n pairs of integer ambiguity candidates, there are m

pairs of preliminary solutions (ψ_1, θ_1), (ψ_2, θ_2), …, (ψ_m, θ_m). Then only one solution among them is correct, which can be passed via AFM (Counselman and Gourevitch, 1981; Caporali, 2001):

$$F(\psi,\theta)=\frac{1}{m}\sum_{i=1}^{m}\cos(2\pi)\{\Delta\varphi^i-\|\boldsymbol{b}\|[\sin\beta^i\sin\theta+\cos\beta^i\cos\theta\cos(\alpha^i-\psi)]/\lambda^i\}, \quad (25)$$

where $\psi\in[0°, 360°]$, $\theta\in[-90°, 90°]$.

Supposing that an analytic solution j is determined for epoch k, in respect of the satellite i, the float ambiguity could be given as (Wang *et al.*, 2007)

$$\Delta\hat{N}_j^i=\Delta\varphi_k^i-\|\boldsymbol{b}\|[\sin\alpha_k^i\sin\theta_{jk}+\cos\alpha_k^i\cos\theta_{jk}\cos(\beta_k^i-\psi_{jk})]/\lambda^i. \quad (26)$$

By Eq.(25), the ambiguity function is

$$F_k(\psi_j,\theta_j)=\frac{1}{M}\sum_{i=1}^{M}\cos(2\pi)\Delta\hat{N}_j^i, \quad (27)$$

where M is the satellite number.

Considering the measurement noise, a threshold T near 1 is necessary for filtering out the incorrect solution in Eq.(27), and the value is 0.8 experimentally in this study.

$$F_k(\psi_j,\theta_j)>T. \quad (28)$$

For M satellites tracked, the integer ambiguity vector is

$$\Delta\boldsymbol{N}_j=(\langle\Delta\hat{N}_j^1\rangle \ \ \langle\Delta\hat{N}_j^2\rangle \ \ \cdots \ \ \langle\Delta\hat{N}_j^M\rangle)^{\mathrm{T}}, \quad (29)$$

where $\langle\cdot\rangle$ denotes a rounding calculation.

For multi-epoch cases, it is necessary to weigh the data from different epochs. So an evaluation variable $E_k(\Delta\boldsymbol{N})$ and a weighted factor ζ are introduced to assess the effects of each epoch:

$$\begin{cases}E_0(\Delta\boldsymbol{N}_j)=0,\\ E_k(\Delta\boldsymbol{N}_j)=\zeta E_{k-1}(\Delta\boldsymbol{N}_j)+F_k(\psi_j,\theta_j).\end{cases} \quad (30)$$

The factor ζ can weigh the influence of different epochs on the integer ambiguity solution and the earlier the epoch appears, the smaller the power of ζ is, so the epoch has less effect on the solution.

If only one solution satisfies Eq.(28) at epoch k, the integer ambiguity can be determined promptly; if more solutions exist, according to Eq.(31), the optimum result comes out until one of the solutions is evaluated as being much better than the others. At the same time, the integer ambiguity with $E(\Delta\boldsymbol{N}_{\text{optimal}})$ is considered as the correct solution and the ambiguity initialization is finished.

$$\frac{E(\Delta\boldsymbol{N}_{\text{optimal}})}{E(\Delta\boldsymbol{N}_{\text{suboptimal}})}>\sigma. \quad (31)$$

In this approach, the residual bias $\Delta\tau$ should be pre-determined through the method in the subsection ‘GLONASS receiver clock offset differencing’. But $\Delta\tau$ may have undesirable accuracy and it can even be wrong. Here, in resolving the OTF ambiguity with seven observed satellites, $\Delta\tau$ has been changed in one epoch manually. In four epochs, the maximum ambiguity function values and the corresponding $\Delta\tau$ values (in unit of cycle) are sequentially (0.28, 0.992), (0.26, 0.993), (0.26, 0.993) and (0.29, 0.991). It is shown that the offset of $\Delta\tau$ does not exceed 5 mm, and that the peak value of the ambiguity function is very close to 1 (Fig.4). While the $\Delta\tau$ offset is increasing, the ambiguity function maximum is decreasing gradually and becomes obscure.

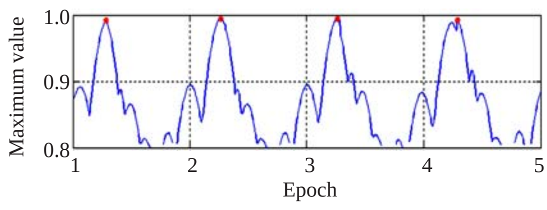

Fig.4 Influences of clock bias on the ambiguity function maximum

Analytical attitude determination algorithm flow

Based on the above analysis, a procedure for analytical attitude determination can be summarized as follows (Fig.5):

(1) Calculate the elevation and azimuth angles of GPS/GLONASS satellites.

(2) Choose two proper satellites, use the value of a single difference carrier phase (DCP) and resolve

the baseline attitude parameters according to Eq.(20).

(3) Verify the attitude parameters. For a 2D scenario, recalculate the attitude using the third satellite and then verify the results. For a 3D scenario, verify the attitude using the constraint relation between the two baseline vectors.

(4) Convert the two-baseline attitude values into 3D attitude parameters using Eq.(24).

(5) If cycle slips happen in this epoch, repair the cycle slips and redefine the integer ambiguity.

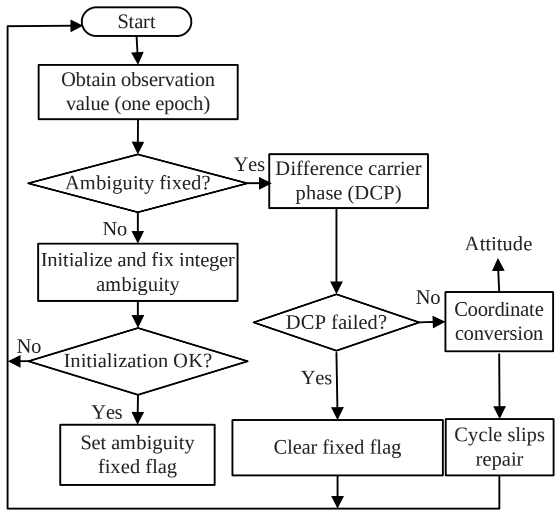

Fig.5 Algorithm flow of attitude determination

EXPERIMENT

A multi-antenna GPS/GLONASS attitude determination system with a set of low cost components was constructed. The hardware platform uses several parallel independent front ends and correlators. Besides the control function, the digital signal unit also offers the position, velocity, time and attitude (PVTA) function, including the self-survey function of attitude determination.

Static test

Keeping the two antennas steady based on the high-precision rotary table with the baseline of 1 m, a number of static experiments have been implemented to validate the method proposed above over a long time (Fig.6). Since the geometric distribution of GPS satellites keeps changing, we are required to detect the new integer ambiguity continuously.

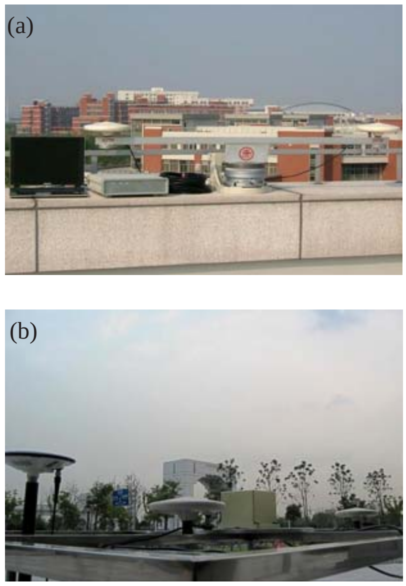

Fig.6 Antenna configuration of static (a) and kinematic (b) test

In Fig.7, the average attitude angles are 44.16° in yaw, and 0.49° in pitch. The yaw and the pitch angles always fluctuate around the average values. The results show the reliability of the attitude initialization method under various satellite distributions. Assuming that the measurement noise is at the level of 0.005 m, the measurement noise will introduce an error of $E_r\approx 0.005\times 57.3/||\boldsymbol{b}||=0.29°$ (Li *et al.*, 2004). In terms of precision, the error of the attitude solution is listed in Table 1. The standard deviations of yaw and pitch angles are 0.17° and 0.29°, respectively. So it can be concluded that the proposed algorithm is as accurate as the traditional ones.

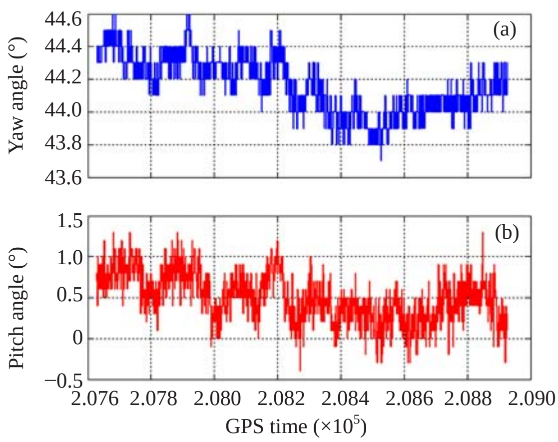

Fig.7 Yaw angle (a) and pitch angle (b) in the static experiment

Table 1 Attitude solution error in the static experiment

Error	$\Delta\psi$ (°)	$\Delta\theta$ (°)
Standard deviation	0.17	0.29
Minimum	−0.46	−0.89
Maximum	0.44	0.81

Dynamic test

In Fig.8, a vehicle was moving along a path on November 12, 2007 at Shanghai Jiao Tong University campus. And the direction of its motion (yaw angle) was recorded every 1 s. Two antennas with a baseline of 1 m were installed on the top of the vehicle.

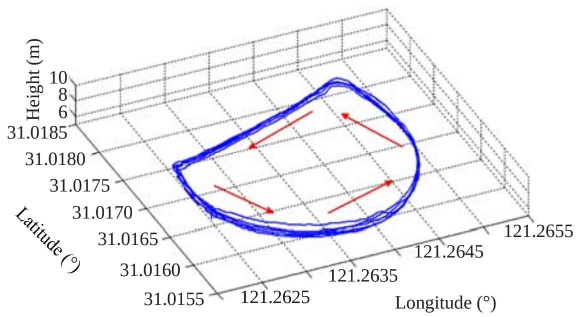

Fig.8 Motion path of the kinematic experiment

As shown in Fig.9, the recorded yaw is identical to the periodical motion of the vehicle. However, these records can only show the coherence between the measurement results, but fail to provide the absolute reference base. In our records, after losing satellite tracking, the maximal recovering time of attitude calculation is 30 s (Area A in Fig.9a). The integer searching time is about 10 ms while the traditional AFM approaches are within 80 ms. A comparison with the records of a high-precision inertial measurement unit (IMU) is also given for reference. Without the system error, the difference between the two systems is less than 0.18°. This indicates the high accuracy and reliability of the proposed algorithm.

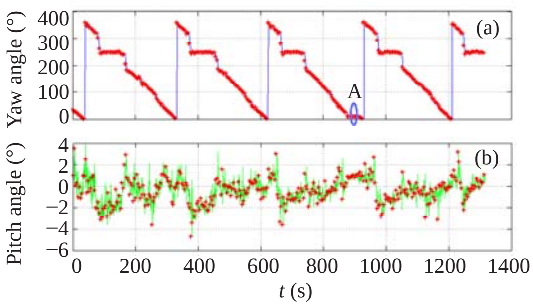

Fig.9 Results of the kinematic experiment. (a) Yaw angel; (b) Pitch angel

Additional GLONASS satellite impact on attitude determination

Generally speaking, the main advantage of using GLONASS satellite signals in addition to GPS signals is the increased number of available satellite signals. In the Shanghai area of China (about 31° N), the number of available visible GPS satellites is at least six, while an average of approximately 2~4 GLONASS satellites are in view. In addition, it is testified that the GLONASS augmentation helps to improve the accuracy of the combined system. Previously, when some obstructions or electronic jamming for GPS were inescapable, the final GPS signal could be lost, which means having to wait to re-initialize and then try to measure again (Lightsey and Madsen, 2003). Using the additional GLONASS signals could mean continuing to work in areas where it was previously not possible.

Although GPS and GLONASS are different GPSs, the accuracy of the two systems is on the same level and, furthermore, the SD method largely eliminates the system errors and bias. With regards to accuracy, the increased number of satellite signals is beneficial for the user. More satellites signals can help improve the accuracy of measured attitude. During the test period, the number of combined satellites did not fall below eight and in fact the availability of eight or more combined satellites was 98%. In Figs.10 and 11, the standard deviation of the yaw angle for SD measurement of six GPS satellites is 0.1° (1σ), while with eight GPS+GLONASS satellites (GGs) the standard deviation reaches 0.08° (1σ). When GLONASS is used in addition to GPS, the system is generally more stable and can improve accuracy.

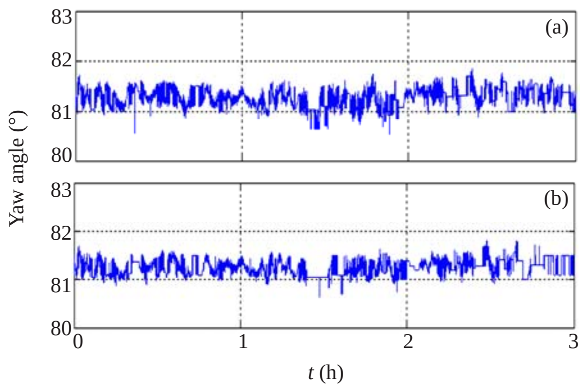

Fig.10 Yaw angle comparison between six GPSs and eight GGs. (a) Six GPSs measurement; (b) Six GPSs and two GLONASS measurement

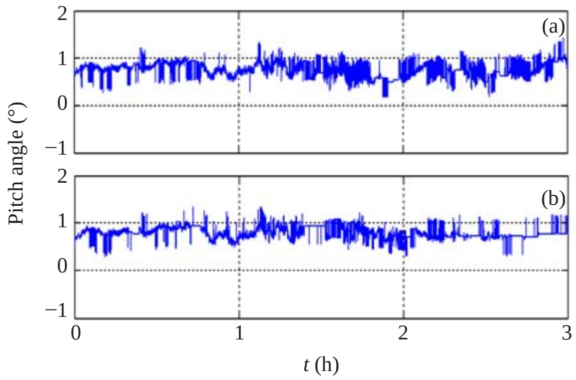

Fig.11 Pitch angle comparison between six GPSs and eight GGs. (a) Six GPSs measurement; (b) Six GPSs and two GLONASS measurement

In further investigation, redundancy improves the speed and reliability of the ambiguity resolution process, which is the key to precise real-time attitude. The performance of the carrier phase integer ambiguity OTF resolution is improved. In the case of six GPS satellites, in only 30% of cases can we realize the initialization in one epoch. After adding two GLONASS satellites, in 80% of cases we can realize the initialization in one epoch (Wang, 2008). The addition of GLONASS satellites can also help to maintain attitude continuity. Even when a sufficient number of GPS satellites are available, redundancy provided by a combined constellation can reduce the time-to-fix ambiguities. Using GPS alone and GGs, incorrect ambiguity solutions were detected in the tests. The good reliability results can be attributed to the hybrid approach, which uses repeated and stable search strategies to reduce the probability of selecting an incorrect set of integer ambiguities.

CONCLUSION

A combination approach of attitude analytical resolution and improved AFM for GPS/GLONASS 2D/3D attitude determination is presented. The proposed equations simplify the previous measurement equations under the conditions of a common reference clock for the receivers and the constraint of the spherical surface. The analytical solutions can be provided by an algebraic method or a direct computation method. As a result, the computation time for the candidate solutions is reduced greatly. This algorithm is only valid for SD carrier phase measurements and the receiver clock bias was estimated previously.

A number of experiments demonstrate that this improved approach significantly outperforms the traditional ones in terms of the computation load. Compared with the traditional DD approach, the efficiency of the improved one is better than that of the traditional one on average, with equivalent performance in reliability and accuracy.

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