Journal of Zhejiang University SCIENCE A
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
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Saturation throughput analysis of RTS/CTS scheme in an error-prone WLAN channel

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Received Nov. 11, 2008; Revision accepted Mar. 23, 2009; Crosschecked Oct. 18, 2009

Abstract: In this paper, a new three-dimensional Markov model is proposed for the estimation of saturation throughput of RTS/CTS (request to send/clear to send) scheme in an error-prone wireless local area network (WLAN) channel. The model takes account of the effect of bit error on all the frames, and station short and long retry limits. Saturation throughput was re-analyzed using the new Markov model and numerical results closely matched those from simulation, confirming the accuracy of the new model. Evaluation of the influence of different parameters on throughput showed that the saturation throughput is sensitive to channel bit error rates and packet length, especially in high bit error conditions.

Key words: Saturation throughput, RTS/CTS, Error-prone WLAN channel, Markov model
doi:10.1631/jzus.A0820786 **Document code:** A **CLC number:** TN925.93

INTRODUCTION

In recent years, the saturation throughput of distributed coordination function (DCF) mode in IEEE 802.11 medium access control (MAC) protocol (IEEE 802.11 WG, 2007) has been estimated by many researchers. Most of them used a bi-dimensional Markov chain (Bianchi, 1998; 2000) to model each station's MAC behavior. Wu *et al*.(2002) improved the model under ideal channel conditions by considering the retry limit. Their numerical results showed that the retry limit is a key parameter that cannot be omitted in WLAN MAC performance analysis.

However, the analyses in several studies (Bianchi, 1998; 2000; Tay and Chua, 2001; Wu *et al*., 2002) were based on the assumption of ideal channel conditions. In real radio channels, bit error may occur because of fading, noise and path loss, and can introduce substantial complexity to the design and performance analysis. Many researchers have investigated the effect of bit errors on system performance (He *et al*., 2002; Hadzi-Velkov and Spasenovski, 2003; Yeo and Agrawala, 2003; Dong and Varaiya, 2005; Ni *et al*., 2005; AlSabbagh *et al*., 2007; 2008; Peng *et al*., 2008). Yeo and Agrawala (2003) extended the saturation model of Tay and Chua (2001) which used mathematical approximations with average values, and analyzed the capacity and variability of the MAC protocol in error channel conditions. Most of these models were based on the Markov model (Bianchi, 1998; 2000), but He *et al*.(2002) and Dong and Varaiya (2005) did not consider the effect of frame retry limit. Ni *et al*.(2005) only analyzed the bit error effect on throughput in basic access mode. Peng *et al*.(2008) proposed a backoff mechanism to improve the system performance under basic access mode. Both Hadzi-Velkov and Spasenovski (2003) and AlSabbagh *et al.* (2007) considered a constant frame error probability only for data frames, and assumed that the control frames were always successfully received, an assumption that applies only in low bit error rate conditions. Hadzi-Velkov and Spasenovski (2003) and AlSabbagh *et al*.(2007; 2008) used only one retry limit parameter in their analyses.

The 802.11 protocol defines two retry counters: station short retry count (SSRC) for frames which are less than or equal to the RTSThreshold, and station long retry count (SLRC) for longer frames. This means that SSRC is used when a station is contending for a medium, while SLRC is used when a station is accessing a reserved medium. These two retry counters have different preset retry limits, which means that a station has different retry times when it encounters different failures. In RTS/CTS scheme, SSRC is used in RTS/CTS transfer, and SLRC is used in DATA/ACK transfer after the CTS is received correctively. Therefore, the models of Hadzi-Velkov and Spasenovski (2003) and AlSabbagh *et al.*(2007; 2008) with only one retry limit are inaccurate. In this paper, we present a new three-dimensional (3D) Markov model to re-estimate the saturation throughput of RTS/CTS scheme in an error-prone channel. The new model is described in Section 2, and the calculation of saturation throughput is derived in Section 3. In Section 4 the model is validated, and the effects of system parameters on saturation throughput are analyzed. Section 5 concludes the paper.

MARKOV CHAIN MODEL

In the following analysis, we assume a fixed number n of contending stations working at saturation condition. The RTSThreshold is set to the length of RTS frame. We assume that the data packet is longer than the RTSThreshold; therefore, RTS/CTS mechanism is always used. RTS/CTS failure causes the increment of SSRC, and DATA/ACK failure causes the increment of SLRC. The short retry limit is m_s and long retry limit is m_l. Let $m_{max}=m_s+m_l$.

The stochastic processes $s_1(t)$ and $s_2(t)$ are used to represent SSRC and SLRC respectively, and $b(t)$ is used to represent the backoff counter for a given station at slot time t. Since the backoff counter is frozen when the channel is sensed busy, the slot time here is expressed as the sum of a constant value σ and a variable time interval between two consecutive backoff time counter decrements (Bianchi, 1998).

Fig.1 (left side) shows the 3D process $\{s_1(t), s_2(t), b(t)\}$ in a backoff procedure. To simplify the description of the station's behavior, we use $\{s_1(t), s_2(t)\}$ to substitute this backoff procedure (the dotted line ellipse), which is shown in the right side of Fig.1. The simplified process is shown in Fig.2. Figs.1 and 2 together describe the station's behavior. Since the frame error probabilities P_{es} and P_{el} are independent of the states $s_1(t)$ and $s_2(t)$ of the station, the 3D process $\{s_1(t), s_2(t), b(t)\}$ is a discrete-time Markov chain.

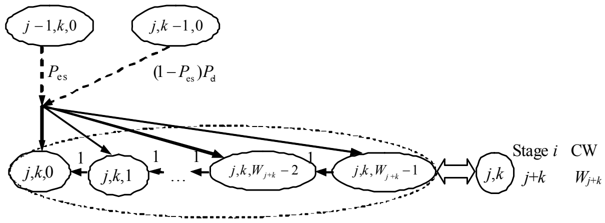

Fig.1 The Markov process in a backoff procedure

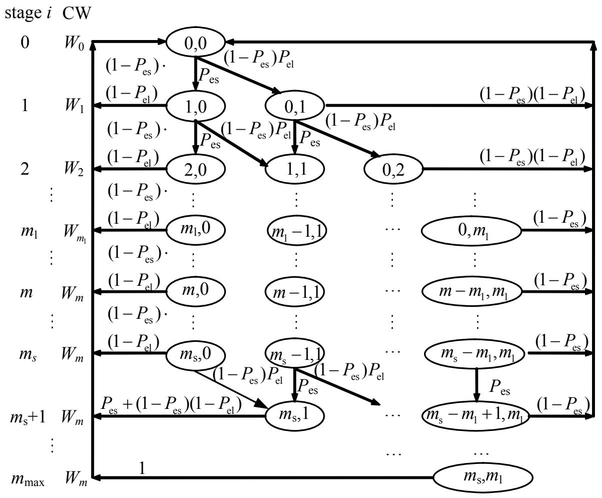

Fig.2 Simplified Markov chain model for one station

The value of the station's backoff stage i is equal to the sum of the values of $s_1(t)$ and $s_2(t)$. W_i, the value of contention window in stage i, is described as

$$W_i = \begin{cases} 2^{j+k} \cdot W, & 0 \le j+k \le m, \\ 2^m \cdot W, & m < j+k \le m_{max}, \end{cases} \quad (1)$$

where W=CW$_{min}$+1, $2^m W$=CW$_{max}$+1. i and W_i are also listed in Fig.2.

When the station uses RTS/CTS mechanism under an error-prone channel, both the control frame and data frame may encounter transfer failures. We use P_{es} and P_{el} to represent the RTS/CTS failure probability and DATA/ACK failure probability, respectively. P_{es} is the combination of the collision probability p_{coll} and the RTS/CTS error probability p_{errs} caused by bit error, while P_{el} is the DATA/ACK error probability p_{errl}, which is caused by bit error. They can be expressed as

$$\begin{cases} P_{\text{es}} = p_{\text{coll}} + (1 - p_{\text{coll}}) \cdot p_{\text{errs}}, \\ P_{\text{el}} = p_{\text{errl}}. \end{cases} \quad (2)$$

Let $b_{j,k,l} = \lim_{t\to\infty} P\{s_1(t) = j,\ s_2(t) = k,\ b(t) = l\}$, $j \in [0,\ m_s],\ k \in [0,\ m_l],\ l \in [0,\ W_{j+k} - 1]$ denote the stationary distribution of the Markov model. The non-null one-step transition probabilities in this model are:

$$b_{j,k,l} = \frac{W_{j+k} - l}{W_{j+k}} \cdot b_{j,k,0}, \quad 0 \le j \le m_s, \quad 0 \le k \le m_l,\ 0 \le l \le W_{j+k} - 1, \quad (3)$$

$$b_{j,0,0} = b_{j-1,0,0} \cdot P_{\text{es}}, \quad 0 < j \le m_s, \quad (4)$$

$$b_{0,k,0} = b_{0,k-1,0} \cdot (1 - P_{\text{es}}) \cdot P_{\text{el}}, \quad 0 < k \le m_l, \quad (5)$$

$$b_{j,k,0} = b_{j,k-1,0} \cdot (1 - P_{\text{es}}) \cdot P_{\text{el}} + b_{j-1,k,0} \cdot P_{\text{es}}, \quad 0 < j \le m_s, \quad 0 < k \le m_l. \quad (6)$$

Let $a=P_{\text{es}}$, $d = (1 - P_{\text{es}}) \cdot P_{\text{el}}$, and

$$\boldsymbol{B} = \begin{bmatrix} b_{0,0,0} & 0 & \cdots & 0 \\ b_{1,0,0} & b_{0,1,0} & & \vdots \\ \vdots & \vdots & & 0 \\ b_{m_l,0,0} & b_{m_l-1,1,0} & & b_{m_l-1,m_l,0} \\ \vdots & \vdots & & \vdots \\ b_{m_s,0,0} & b_{m_s-1,1,0} & & b_{m_s-m_l,m_l,0} \\ 0 & b_{m_s,1,0} & & b_{m_s-m_l+1,m_l,0} \\ \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & b_{m_s,m_l,0} \end{bmatrix}.$$

According to Eqs.(3)~(6), all the elements in $\boldsymbol{B}$ can be expressed by $b_{0,0,0}$, which is $\boldsymbol{B}=b_{0,0,0}\boldsymbol{X}$, and $\boldsymbol{X}$ can be expressed as

$$X_{qr} = \begin{cases} C_{q-1}^{r-1} \cdot a^{q-r} \cdot d^{r-1}, & 0 \le q - r \le m_s, \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

Normalizing the stationary distribution of the Markov chain to 1, we can express $b_{0,0,0}$ as

$$b_{0,0,0} = 2 \cdot [(W_0 + 1\ \cdots\ \underbrace{W_m + 1\ \cdots\ W_m + 1}_{m_{\max} - m + 1})_{1\times(m_{\max}+1)} \boldsymbol{X} (1\ \cdots\ 1)^{\text{T}}_{1\times(m_l+1)}]^{-1}. \quad (8)$$

Let τ be the probability that a station transmits in a generic slot time. As any transmission occurs only when the backoff window is equal to zero, it is

$$\tau = \sum_{j=0}^{m_s} \sum_{k=0}^{m_l} b_{j,k,0} = b_{0,0,0} \cdot \left[(1\ \cdots\ 1)_{1\times(m_{\max}+1)} \boldsymbol{X} (1\ \cdots\ 1)^{\text{T}}_{1\times(m_l+1)} \right]. \quad (9)$$

In the stationary state, the collision probability p_{coll} can be expressed as the probability that at least one of the $n-1$ remaining stations transmits at the same time, which is

$$p_{\text{coll}} = 1 - (1 - \tau)^{n-1}. \quad (10)$$

Eqs.(2), (9), and (10) determine the unique values of τ and p_{coll}.

SATURATION THROUGHPUT ANALYSIS

Let P_{tr} be the probability that at least one transmission occurs in the considered slot time, and P_s be the probability that exactly one transmission occurs, conditioned on P_{tr}. For a WLAN of n contending stations, they are given by

$$P_{\text{tr}} = 1 - (1 - \tau)^n, \quad (11)$$

$$P_s = \frac{n \cdot \tau \cdot (1 - \tau)^{n-1}}{1 - (1 - \tau)^n}. \quad (12)$$

In the error-prone channel, a frame transmission may encounter the following situations: (1) successful transmission, (2) RTS-CTS error caused by collision, (3) RTS-CTS error caused by channel noise with no collision, and (4) DATA-ACK error caused by noise. The corresponding probabilities are

$$\begin{cases} P_1 = P_{\text{tr}} \cdot P_s \cdot (1 - p_{\text{errs}}) \cdot (1 - p_{\text{errl}}), \\ P_2 = P_{\text{tr}} \cdot (1 - P_s), \\ P_3 = P_{\text{tr}} \cdot P_s \cdot p_{\text{errs}}, \\ P_4 = P_{\text{tr}} \cdot P_s \cdot (1 - p_{\text{errs}}) \cdot p_{\text{errl}}. \end{cases} \quad (13)$$

In each case, the time occupied is

$$\begin{cases} T_1 = T_{\text{RTS}} + \text{SIFS} + \delta + T_{\text{CTS}} + \text{SIFS} + \delta \\ \quad + T_{\text{Payload}} + \text{SIFS} + \delta + T_{\text{ACK}} + \text{DIFS} + \delta, \\ T_2 = T_{\text{RTS}} + \delta + \text{CTS\_timeout} + \text{DIFS}, \\ T_3 = T_{\text{RTS}} + \delta + \text{CTS\_timeout} + \text{DIFS}, \\ T_4 = T_{\text{RTS}} + \text{SIFS} + \delta + T_{\text{CTS}} + \text{SIFS} + \delta \\ \quad + T_{\text{Payload}} + \delta + \text{ACK\_timeout} + \text{DIFS}, \end{cases} \quad (14)$$

where δ is the propagation delay, T_{RTS}, T_{CTS}, T_{Payload}, and T_{ACK} stand for the transmission time of RTS, CTS, DATA and ACK frame, respectively. CTS_timeout and ACK_timeout are calculated as SIFS+T_{ACK}+δ, and T_{ACK} is the time required to transmit an ACK frame, including preamble, PLCP header and any additional PHY dependent information, at the lowest PHY mandatory rate.

Therefore, the normalized saturation throughput S of the RTS/CTS access mechanism in an error-prone channel can be expressed as follows:

$$S = \frac{P_1 \cdot E[P]}{(1-P_{\text{tr}}) \cdot \sigma + P_1 \cdot T_1 + P_2 \cdot T_2 + P_3 \cdot T_3 + P_4 \cdot T_4}, \quad (15)$$

where $E[P]$ is the average packet length and σ is the duration of an empty slot time.

SIMULATION RESULTS

To validate our new model, we have compared its results with those obtained from Hadzi-Velkov and Spasenovski (2003) and with simulation results from ns-2. The system parameters are those specified for the DSSS PHY layer (IEEE 802.11 WG, 2007). The channel bit rate C is 1 or 11 Mb/s. The propagation delay (δ) is assumed to be a constant value, 1 μs. The packet payload ($E[P]$) is 8224 bits.

The frame error probabilities p_{errs} and p_{errl} are functions of bit error rate (BER) and frame length:

$$\begin{cases} p_{\text{errs}} = 1-(1-\text{BER})^{\text{RTS+CTS}}, \\ p_{\text{errl}} = 1-(1-\text{BER})^{H+E[P]+\text{ACK}}. \end{cases} \quad (16)$$

Figs.3a and 3b show the results of saturation throughput. Here, we use 'new model' to represent our proposed model, and 'old model' to represent the model of Hadzi-Velkov and Spasenovski (2003). The symbols represent the simulation results we obtained in ns-2 with different BERs.

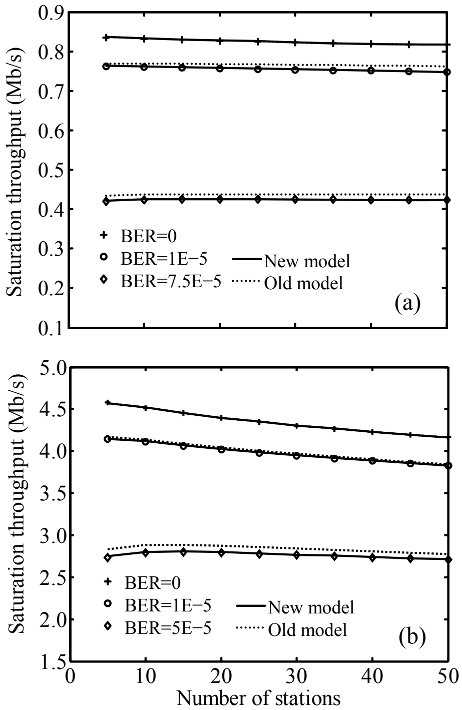

Fig.3 Saturation throughput: analysis versus simulation for *C*=1 Mb/s (a) and *C*=11 Mb/s (b)
New model: our proposed; Old model: Hadzi-Velkov and Spasenovski (2003)'s

When BER is set to 0, the results from the new and old models are the same (Figs.3a and 3b). Actually, zero BER represents the ideal channel condition, and in such a situation these two models change to the model of Wu *et al.*(2002). When BER is greater than zero, results from our new Markov model are very close to those obtained from simulation, and are more accurate than those in Hadzi-Velkov and Spasenovski (2003). In the RTS/CTS scheme, using RTS to contend the medium can help stations detect collisions more quickly. Thus, the saturation throughput of RTS/CTS scheme changes little with increasing network scale. However, Fig.3b shows that the saturation throughput decreases slightly as the network scale increases in high data rate condition. This is because with the increase of data rate, the time spent on MAC frame transmission decreases, while the time of PHY header transmission, DIFS, SIFS, and σ remain the same. The ratio of this additional time spending to total time spending increases, and more collisions mean more additional time spending, resulting in the slight decrease in saturation throughput. The old model overestimates the saturation throughput because it does not consider the bit error effect on control frames, and ignores one of the two retry limits. To prove this, we calculate the error between model results and ns-2 simulation for C=1 Mb/s (Fig.4).

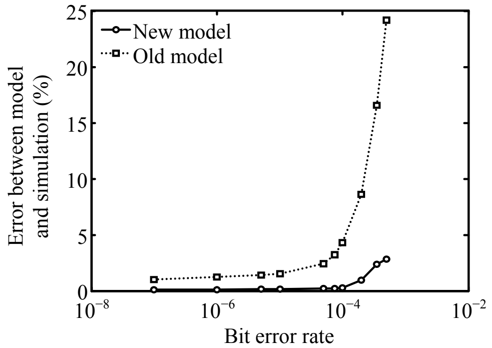

Fig.4 Error between model results and ns-2 simulation with different bit error rates for C=1 Mb/s

Fig.4 shows that, with the increase of BER, the error between the old model and the simulation increases much faster than the error between our new model and simulation. This is mainly caused by the effect of bit error on control frames. When BER is greater than 1E−5, the RTS/CTS error rate is greater than 0.01, which is no longer negligible. Therefore, the model of Hadzi-Velkov and Spasenovski (2003) is not accurate enough. By comparison, our model works well under high BER conditions. The error between our model and the simulation is less than 0.5% when the BER is less than 1E−4, and even under the higher BER condition, it is less than 3%, which shows that our model is precise enough to predict the performance of RTS/CTS access mode in an error-prone WLAN channel.

The impact of BER on saturation throughput is shown in Fig.5 when n=10 and 40. Different values of n have little effect on throughput, while the change of BER has a significant effect. When BER increases from 1E−8 to 5E−4, the saturation throughput degrades to almost zero. When BER is less than 1E−5, the throughput changes little. This is because the collision probability p_{coll} is much greater than the frame error probabilities p_{errs} and p_{errl}, and most of the transmission failure is caused by the collision. While the BER is larger, the frame error probability becomes the main reason for transmission failures, and causes the dramatic decrease in saturation throughput.

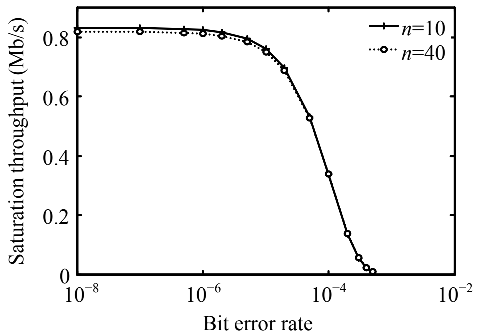

Fig.5 Saturation throughput versus BER for C=1 Mb/s

Fig.6 shows the impact of packet length on saturation throughput with different BER and n. With smaller BER, the saturation throughput increases with the increase in packet length as the collisions are the key factor in transmission failures and RTS/CTS helps to improve this. However, with the increase in packet length, the data frame becomes more and more sensitive to bit error, especially in the high bit error rate conditions. In such situations, the transmission error becomes dominant over the collision error. Therefore, the throughput decreases when packet length grows beyond a certain point.

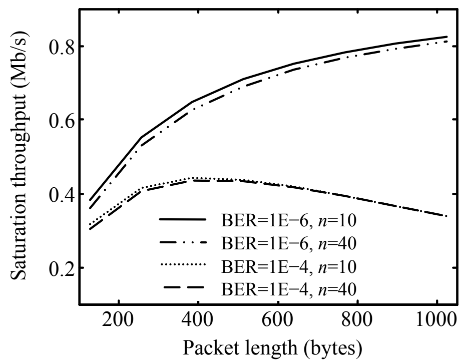

Fig.6 Saturation throughput versus packet length for C=1 Mb/s

We computed the optimal packet length at different bit error rates (Fig.7). Here, the optimal packet length refers to the packet length at the MAC layer which can maximize the saturation throughput for a given channel BER and scale. The optimal packet length decreases when the channel BER increases, and is insensitive to the number of stations.

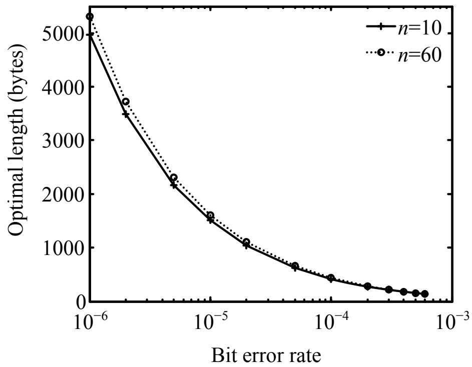

Fig.7 Optimal packet length at MAC layer with different bit error rates

CONCLUSION

In this paper, we presented an analytical 3D Markov chain model to calculate the saturation throughput of RTS/CTS scheme in an error-prone WLAN channel. The simulation results showed that our model is more accurate than previous models. We also evaluated the influence of different bit error rates and different packet lengths on saturation throughput. The study showed that the channel bit error rate affects system performance significantly when it is greater than 1E−5. Moreover, in the high bit error rate condition, an optimal frame size exists, which can maximize system saturation throughput.

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