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Plastic limit load analysis for pressure pipe with incomplete welding defects based on the extended Net Section Collapse Criteria*

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Abstract: With von Mises yield criterion, the loading range of Net Section Collapse (NSC) Criteria is extended from combined tension and bending loadings to combined bending, torsion and internal pressure loadings. A new theoretical analyzing method of plastic limit load for pressure pipe with incomplete welding defects based on the extended NSC Criteria is presented and the correlative formulas are deduced, the influences of pipe curvature, circumferential length and depth of incomplete welding defects on the plastic limit load of pressure pipe are considered as well in this method. Meanwhile, according to the orthogonal experimental design method, the plastic limit loads are calculated by the finite element method and compared with the theoretical values. The results show that the expressions of plastic limit load of pressure pipe with incomplete welding defects under bending, torsion and internal pressure based on extended NSC criteria are reliable. The study provides an important theoretical basis for the establishment of safety assessment measure towards pressure pipe with incomplete welding defects.

Keywords: Net Section Collapse (NSC) Criteria, Incomplete welding defects, Pressure pipe, Plastic limit load, Finite element
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1 Introduction

Industrial pressure pipe is widely used in the domain of petroleum, chemicals, energy, steel and other industries. It is a pressure equipment of high risk for the medium it transports is always inflammable, explosive, poisonous or strong corrosive. Thus, its safety is a key point to safeguard the normal operation of the whole production system. The defects are inevitable since pressure pipes are usually adopting manual arc welding whose quality is difficult to control, and incomplete welding defects are in the majority, which is a potential threat of the safety. Therefore, reasonable evaluation of incomplete welding defects is significant to ensure the safety of pressure pipes.

The failure model of pipes, made of low and medium strength steels, is frequently dominated by plastic collapse (Ahn *et al*., 2002; Hasegawa *et al*., 2009), so incomplete welding defects are usually simplified to local thinning defects and analyzed (Choi *et al*., 2003; Shim *et al*., 2004; Zheng *et al*., 2004; Kim *et al*., 2006; Kamaya *et al*., 2008) or evaluated (GB/T 19624-2004; Smith and Dwivedy, 2007) by the plastic limit load analysis method. The theoretical basis of the above method is Net Section Collapse (NSC) Criteria (Kaninen *et al*., 1978) which was applied only to structures under combined tension and bending loadings. However, in practice, besides tension and bending loadings, torsion is also a non-negligible loading because of spatial arrangement. The analysis method without taking torsion into consideration is not perfect and will affect the conclusion of safety assessment. Meanwhile, pipe curvature has a great impact on plastic limit load (Chen *et al*., 2000) on account of special structure of pressure pipe which is different from pressure vessel, but all the present methods did not consider this geometry factor impact on plastic limit load.

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Therefore, this paper established an extended NSC Criteria for structure under combined bending, torsion and internal pressure loadings. A new theoretical analyzing method of plastic limit load for pressure pipe with incomplete welding defects based on the extended NSC Criteria is presented and the correlative formulas are deduced. The influence of pipe curvature, circumferential length and depth of incomplete welding defects on plastic limit load is considered as well in this method. After verification with finite element method results, it shows that the above-mentioned method is reasonable. The study provides an important theoretical basis for the establishment of safety assessment measure towards pressure pipe with incomplete welding defects.

2 Theory

2.1 Introduction of Net Section Collapse Criteria

NSC Criteria considers that the pipe with circumferential defect will produce large plastic deformation under bending or tension loading. When the stress of uniform distribution in the net section of defect structure is up to the flow stress of material, it is up to the plastic limit state of the pipe, and the corresponding load is plastic limit load. Now NSC Criteria is the main method to analyze the plastic limit load for pressure pipe with defects in international scope (Rahman, 1998; Rahman and Wilkowski, 1998).

2.2 Extension of NSC Criteria and plastic limit load analysis based on the extended NSC Criteria

2.2.1 Extension of NSC Criteria and assumption of analysis

Current NSC Criteria is applied only to pipes under bending and tension loading, but there is torsion existing in pipeline because of the spatial arrangement, so the pipe with defects usually subjects to combined internal pressure P, bending moment M_B and torsion moment M_τ (Fig. 1).

For the pressure pipe with incomplete welding defects under combined bending, torsion and internal pressure loadings, it is assumed as follows according to limit analysis principle (Hu *et al.*, 1998):

1. Ideal rigid-plastic material assumption

It is assumed that the material is ideal rigid-plastic; its yield stress is equal to flow stress σ_f which we take the average of yield strength σ_s and tensile strength σ_b.

2. Small deformation assumption

Pipe deformation under applied load is not considered in limit analysis process, the pipe section always keeps circular plane.

3. Uniform shear stress assumption

The shear stress in the net section is evenly distributed when the pipe with incomplete welding defects reaches the plastic state under torsion loading, etc.

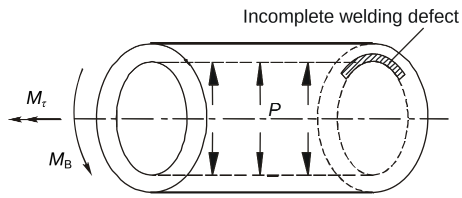

Fig. 1 Actual loading model of pipe with incomplete welding defects

2.2.2 Plastic limit load analysis based on extended NSC Criteria

1. Plastic limit load analysis of pipe without defects

The limit load of pipe without defects under pure internal pressure or pure bending moment is (GB/T 19624-2004)

$$P_{L0}^* = 2\sigma_f \ln(r_o / r_i) / \sqrt{3}, \tag{1}$$

$$M_{B0}^* = 4\sigma_f (r_o^3 - r_i^3) / 3, \tag{2}$$

where P_{L0}^* is the plastic limit pressure of pipe without defects under pure internal pressure, M_{B0}^* is the plastic limit bending moment of pipe without defects under pure bending moment, σ_f is the flow stress of pipe material, r_o is the external radius of pipe, and r_i is the internal radius of pipe.

According to assumption 3, the material enters the plastic state when all the shear stresses reach the limit value τ_f. The limit value τ_f is determined by von Mises yield criteria, so we have $\tau_f = \sigma_f / \sqrt{3}$, and then the limit torsion of pipe without defects under pure torsion moment is

$$\begin{aligned} M_{\tau 0}^* &= \int_{r_i}^{r_o} \tau_f 2\pi r^2 \mathrm{d}r = 2\pi \tau_f (r_o^3 - r_i^3) / 3 \\ &= 2\pi \sigma_f (r_o^3 - r_i^3) / 3\sqrt{3}, \end{aligned} \tag{3}$$

where $M^*_{\tau 0}$ is the plastic limit torsion of pipe without defects under pure torsion moment.

2. Plastic limit load analysis of pipe with incomplete welding defects

As shown in Fig. 2, the stress distribution model of the defective pipe in plastic limit state was established based on assumption, and the equivalent stress on net section is equal to σ_f. There are also uniform shear stress τ and circumferential stress σ_θ on the section which is in 3D stress state, so the normal stress on the section is not σ_f, and the neutral axis is deviated from the center of the section. There are two different situations in this model: one is that all of the defect areas is in the tensile stress zone above the neutral axis, i.e., $\theta+\beta<\pi$ (Fig. 2a), the other is that part of the defect areas is in the compressive stress zone below the neutral axis, i.e., $\theta+\beta\ge\pi$ (Fig. 2b). θ is the one-half of the angle of the circumferential extent of the defect, β is the neutral angle, σ_1 is the tensile stress of the net section above the neutral axis, and σ_2 is the compressive stress of the net section below the neutral axis. Their values depend on σ_f, σ_θ, τ and the geometry dimension of the defective structure, and can be determined by von Mises yield criteria.

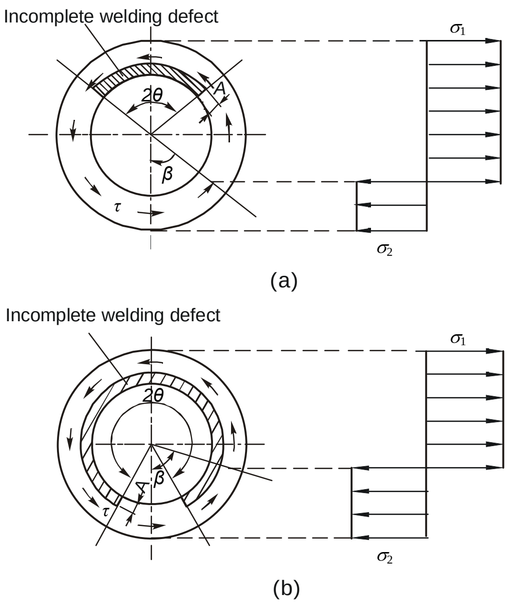

Fig. 2 Mechanical model of defective pipe in plastic limit state under combined bending, torsion and internal pressure loadings. (a) $\theta+\beta<\pi$; (b) $\theta+\beta\ge\pi$. *A* is the depth of incomplete welding defect

To be convenient, define n_θ as the dimensionless circumferential stress, m_B as the dimensionless bending moment and m_τ as the dimensionless torsion, respectively.

$$n_\theta=\sigma_\theta/\sigma_f=pr/(t\sigma_f),\quad m_B=M_B/M^*_{B0},\quad m_\tau=M_\tau/M^*_{\tau 0},\tag{4}$$

where σ_θ is the circumferential stress, p is the internal pressure, r is the mean radius, and t is the thickness of the pipe wall.

According to torsion equilibrium, the uniform shear stress τ caused by applied torsion can be obtained as

$$\tau=\frac{\sigma_f m_\tau}{\sqrt{3}\left(1-\frac{A}{t}\frac{\theta}{\pi}\right)},\tag{5}$$

where τ is the uniform shear stress caused by applied torsion, A is the depth of incomplete welding defect, and θ is the half crack angle.

When the equivalent stress of any point on the net section is equal to flow stress σ_f, the pipe with incomplete welding defect under combined internal pressure, bending and torsion loading reaches the plastic limit state. According to von Mises yield criteria, the equivalent stress σ_{eq} is equal to the flaw stress σ_f,

$$\begin{aligned}\sigma_{eq}&=\frac{1}{\sqrt{2}}\sqrt{\sigma^2+\sigma_\theta^2+(\sigma-\sigma_\theta)^2+6\tau^2}\\&=\sqrt{\sigma^2+\sigma_\theta^2+\sigma\sigma_\theta+3\tau^2}=\sigma_f,\end{aligned}\tag{6}$$

where σ is axial normal stress.

Let $\sigma_\theta=n_\theta\sigma_f$ and substituting Eq. (5) into Eq. (6), then

$$\sigma^2-n_\theta\sigma_f\sigma+n_\theta^2\sigma_f^2+\sigma_f^2m_\tau^2\Big/\left(1-\frac{A\theta}{t\pi}\right)^2-\sigma_f^2=0.\tag{7}$$

The solution of the above equation is the section normal stresses σ_1, σ_2 in the limit state:

$$\frac{\sigma}{\sigma_{\mathrm{f}}}=\frac{n_\theta \pm \sqrt{4-3n_\theta^2-4m_\tau^2\Big/\left(1-\dfrac{A\theta}{t\pi}\right)^2}}{2}, \quad (8)$$

hence,

$$\frac{\sigma_1}{\sigma_{\mathrm{f}}}=\frac{n_\theta}{2}+\sqrt{1-\frac{3}{4}n_\theta^2-m_\tau^2\Big/\left(1-\frac{A\theta}{t\pi}\right)^2}, \quad (9)$$

$$\frac{\sigma_2}{\sigma_{\mathrm{f}}}=\frac{n_\theta}{2}-\sqrt{1-\frac{3}{4}n_\theta^2-m_\tau^2\Big/\left(1-\frac{A\theta}{t\pi}\right)^2}. \quad (10)$$

The normal stress σ_1 is tensile stress and σ_2 is generally compressive stress, σ_1 is larger than $|\sigma_2|$; if the internal pressure is zero, σ_1 and σ_2 have the same absolute magnitude but opposite direction; if both internal pressure and torsion moment are zero, then the absolute magnitude of σ_1 and σ_2 is equal to flow stress σ_{f}, and the mechanical model of pipe with incomplete welding defect in limit state is in accordance with the original NSC Criteria in this condition.

(1) Plastic limit load analysis of pipe with incomplete welding defects when $\theta+\beta<\pi$

As shown in Fig. 2a, when the pipe with incomplete welding defects enters the plastic limit state under combined internal pressure, bending and torsion loadings, according to axial force equilibrium condition which means applied axial force N is equal to the sum of the normal stress, the axial force N can be obtained as

$$\begin{aligned}N&=2\int_0^\theta\int_{r_{\mathrm{i}}+A}^{r_{\mathrm{o}}}\sigma_1 r\mathrm{d}r\mathrm{d}\alpha+2\int_\theta^{\pi-\beta}\int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}}\sigma_1 r\mathrm{d}r\mathrm{d}\alpha\\&+2\int_{\pi-\beta}^{\pi}\int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}}\sigma_2 r\mathrm{d}r\mathrm{d}\alpha=\sigma_1\theta[r_{\mathrm{o}}^2-(r_{\mathrm{i}}+A)^2] \quad (11)\\&+(\pi-\beta-\theta)\sigma_1(r_{\mathrm{o}}^2-r_{\mathrm{i}}^2)+\beta\sigma_2(r_{\mathrm{o}}^2-r_{\mathrm{i}}^2).\end{aligned}$$

Due to the existence of shear stress and circumferential stress, σ_1 and σ_2 may not be equal, and not be equal to flow stress σ_{f} either.

For thin wall pipe,

$$\frac{r_{\mathrm{o}}^2-(r_{\mathrm{i}}+A)^2}{2}\approx r(t-A),\quad \frac{r_{\mathrm{o}}^2-r_{\mathrm{i}}^2}{2}\approx rt.$$

Eq. (11) can be further simplified to

$$N\approx 2rt\left[(\pi-\beta-\theta A/t)\sigma_1+\beta\sigma_2\right]=0. \quad (12)$$

After further simplification, the neutral axis central angle β can be obtained as

$$\beta=\frac{(\pi-\theta A/t)\sigma_1}{\sigma_1-\sigma_2}. \quad (13)$$

In the same manner, according to moment equilibrium condition, the bending moment M_{B} and the normal stress σ_1, σ_2 on the net section of the pipe must satisfy the following equation:

$$\begin{aligned}M_{\mathrm{B}}&=2\sigma_1\frac{r_{\mathrm{o}}^3-(r_{\mathrm{i}}+A)^3}{3}\sin\theta\\&+2\sigma_1\frac{r_{\mathrm{o}}^3-r_{\mathrm{i}}^3}{3}(\sin\beta-\sin\theta)-2\sigma_2\frac{r_{\mathrm{o}}^3-r_{\mathrm{i}}^3}{3}\sin\beta.\end{aligned} \quad (14)$$

For thin wall pipe, Eq. (14) can be simplified to

$$M_{\mathrm{B}}\approx 2r^2t\left[\sigma_1\left(\sin\beta-\frac{A}{t}\sin\theta\right)-\sigma_2\sin\beta\right]. \quad (15)$$

In dimensionless terms, Eq. (15) becomes

$$m_{\mathrm{B}}=\frac{M_{\mathrm{B}}}{M_{\mathrm{B0}}^*}=\frac{\left(1-\dfrac{A}{t}\right)\sigma_1\sin\beta-\sigma_2\sin\beta}{2\sigma_{\mathrm{f}}}. \quad (16)$$

Substituting Eqs. (9) and (10) into Eqs. (13) and (16) leads to

$$\left(\frac{m_{\mathrm{B}}+\dfrac{A}{4t}n_\theta\sin\theta}{\sin\beta-\dfrac{A}{2t}\sin\theta}\right)^2+\frac{3}{4}n_\tau^2+\frac{m_\tau^2}{\left(1-\dfrac{A}{t}\dfrac{\theta}{\pi}\right)^2}=1, \quad (17)$$

$$\beta=\frac{\left(\pi-\dfrac{A}{t}\theta\right)\left(\dfrac{n_\theta}{2}+\sqrt{1-\dfrac{3}{4}n_\theta^2-m_\tau^2\Big/\left(1-\dfrac{A}{t}\dfrac{\theta}{\pi}\right)^2}\right)}{2\sqrt{1-\dfrac{3}{4}n_\theta^2-m_\tau^2\Big/\left(1-\dfrac{A}{t}\dfrac{\theta}{\pi}\right)^2}}. \quad (18)$$

The above two equations depict the relationship between various loadings and the solving formula of neutral axis central angle β in plastic limit state.

(2) Plastic limit load analysis of pipe with

incomplete welding defects when $\theta+\beta\geq\pi$

As shown in Fig. 2b, when $\theta+\beta\geq\pi$, most of defect area is in tensile stress area, the rest is in compressive stress area.

With the same method mentioned above, the relation between β and σ_1, σ_2 can be obtained:

$$\beta=\frac{(1-A/t)\sigma_1\pi}{\sigma_1(1-A/t)-\sigma_2}. \tag{19}$$

For thin wall pipe, the applied limit bending moment M_B is

$$M_B=2r^2t\sin\beta\left[(1-A/t)\sigma_1-\sigma_2\right]. \tag{20}$$

In dimensionless terms, Eq. (20) becomes

$$m_B=\frac{M_B}{M_{B0}}=\frac{\left(1-\frac{A}{t}\right)\sigma_1\sin\beta-\sigma_2\sin\beta}{2\sigma_f}. \tag{21}$$

Introducing Eqs. (9) and (10) into Eqs. (19) and (21) leads to the relation formula of various loadings and the solving formula of neutral axis central angle β

$$\left(\frac{\frac{m_B}{\sin\beta}+\frac{A}{4t}n_\theta}{1-\frac{A}{2t}}\right)^2+\frac{3}{4}n_\theta^2+\frac{m_\theta^2}{\left(1-\frac{A}{t}\frac{\theta}{\pi}\right)^2}=1, \tag{22}$$

$$\beta=\pi\frac{\left(1-\frac{A}{t}\right)\left(\frac{n_\theta}{2}+\sqrt{1-\frac{3}{4}n_\theta^2-m_\tau^2\Big/\left(1-\frac{A}{t}\frac{\theta}{\pi}\right)^2}\right)}{\left(2-\frac{A}{t}\right)\sqrt{1-\frac{3}{4}n_\theta^2-m_\tau^2\Big/\left(1-\frac{A}{t}\frac{\theta}{\pi}\right)^2}-\frac{A}{t}\frac{n_\theta}{2}}. \tag{23}$$

3 Finite element analysis process and validation

To validate the theoretical analysis method and expression of plastic limit load based on the extended NSC Criteria, numerical calculation and verification is performed with finite element analysis software, and the influence of pipe curvature, circumferential length and depth of incomplete welding defects on limit load are considered as well.

3.1 Calculation model

Calculating object (Fig. 3) is welded by two straight pipes made of 20 steel (Chinese steel grade, the corresponding steel in ASME is 1020), the length of the model is three times of the pipe diameter to avoid the end profile restraint effect, and we take 1/2 pipe as the calculation model. Limit analysis is mainly to satisfy the force equilibrium in global region, and it is insensitive to stress concentration because high stress caused by geometrical discontinuity is going to transfer through yield zone, thus the welding defect can be simplified to rectangular slot but without circular arc transition in the root of defect.

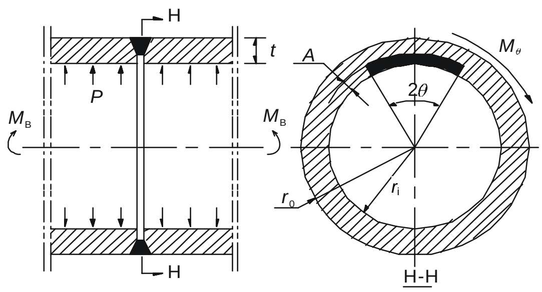

Fig. 3 Structural diagram of pipe with incomplete welding defect

Speaking of the finite element analysis method, this study uses the SOLID95 element in the model near the defect and the SOLID45 element in the model far away from the defect. To improve calculation accuracy and save calculation time, the meshing near the defect is more dense than the region far away from the defect, as shown in Fig. 4.

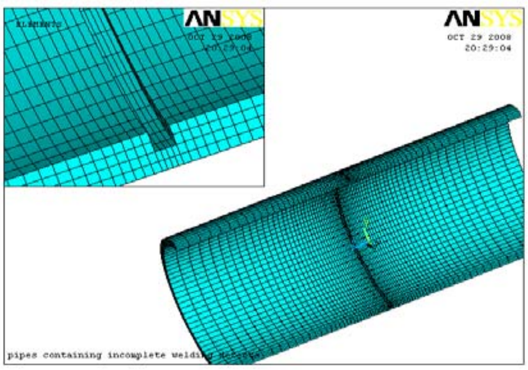

Fig. 4 1/2 model of defective pipe and meshing

3.2 Material properties and boundary conditions

In the finite element analysis process, material properties are defined by real stress-strain curve which is obtained by tensile test (Han *et al*., 2008). As shown in Fig. 5, the elastic modulus E=2.1×10^5 MPa, Poisson's ratio μ=0.3, and the equal-strength welding is used which means the welding material and the mother material are the same.

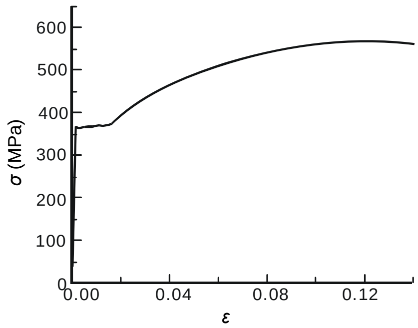

Fig. 5 Real stress-strain curve of 20 steel

When imposing restriction on the pipe, one pipe end is complete constrained, and the other end is free. Internal pressure loading is exerted on the internal surface of the pipe directly; but when exerting bending and torsion loadings, we need to defined MPC184 elements (a constrain element can be used in situations that require some types of kinematic constraint to be imposed) first; the MPC184 elements are generated between a central node near the end surface and nodes in the end surface. Then the loading can be exerted on the central node (Fig. 6).

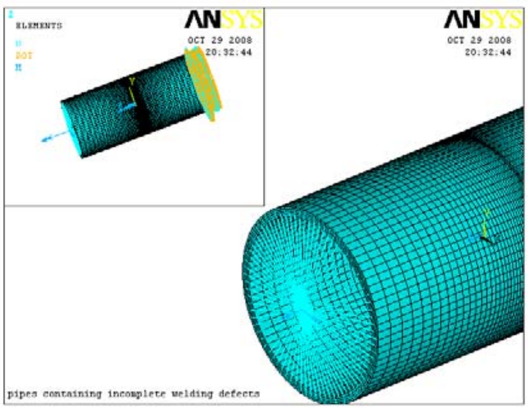

Fig. 6 Loading on finite element model

3.3 Setting of nonlinear analysis and determination of plastic limit load

This study takes the Newton-Raphson iterative as the iterative method in the procedure. AUTO option and large-deformation option are chosen in solution option, and convergence examination is proceeded by VALUE. TOLER after force convergence criteria is chosen.

The plastic limit load is determined by double tangent principle in the stress-strain curve of the maximum strain node, as shown in Fig. 7.

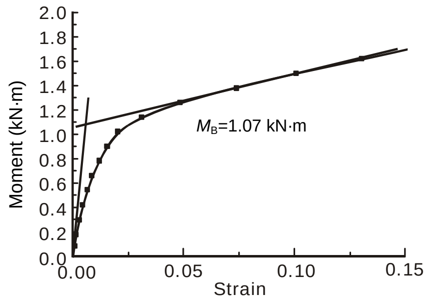

Fig. 7 Double tangent principle to determine the plastic limit moment of model No.18 under 0.8P_{L0}
P_{L0} is the plastic limit pressure of the defective pipe under single internal pressure loading

3.4 Theoretical value of plastic limit load comparison with FEM computational result

Both orthogonal experimental design and limit load analysis theory are used to study the influence of various factors to limit load. For the sake of convenience, we take dimensionless a, b, and k as three factors of the orthogonal experimental. a, b, and k represent the relative circumferential length, relative depth of incomplete welding defects and pipe curvature, respectively, and each factor is divided into 5 levels.

Dimensionless circumferential length of incomplete welding defect $a=\theta/\pi$, we take 0.2, 0.4, 0.6, 0.8, 1.0 as 5 levels.

Dimensionless depth of incomplete welding defect $b=A/t$, we take 0.1, 0.2, 0.4, 0.6, 0.8 as 5 levels.

Pipe curvature $k=r_o/r_i$, we take 1.05, 1.10, 1.15, 1.20, 1.25 as 5 levels.

This study firstly calculates the limit internal pressure P_L, the limit bending moment M_B and the limit torque M_τ under each corresponding single load, then the corresponding limit bending moment M_B under combined bending, torsion and internal pressure loadings whose internal pressure is 0, 0.2, 0.4, 0.6, 0.8, 1.0 times the limit internal pressure and whose torsion is 0, 0.2, 0.4, 0.6, 0.8, 1.0 times the limit torsion. Because of space limitation, only partial results are listed in Table 1.

Meanwhile, the results of model Nos. 1–4 are given in Fig. 8 to compare theoretical value with finite element result. The four models have different a, b, and k. In Fig. 8, THC is the theoretical value of limit load, FEM is finite element calculated value of limit load, n is the times of torsion value by limit torsion under single torsion loading, and P/P_L is the times of internal pressure value by limit internal pressure under single internal pressure loading.

As shown in Fig. 8, it is related among the three limit loadings, such as applied internal pressure, torsion increase, and bending moment deduces. Finite element analysis results are slightly smaller than theoretical analysis results. There is about 5% error between them which is quite acceptable. So the expressions of plastic limit load of pressure pipe with incomplete welding defects under bending, torsion and internal pressure by extended NSC Criteria is reasonable.

4 Conclusions

1. With von Mises yield criterion, the loading range of NSC Criteria is extended from combined tension and bending loadings to combined bending, torsion and internal pressure loadings. A new theoretical analyzing method of plastic limit load for pressure pipe with incomplete welding defects based on the extended NSC Criteria is presented and the correlative formulas are deduced.

Table 1 Comparison between theoretical value and FEM value of plastic limit load*

No.	Pipe size		Defect size		Type	$M_{\tau0}$ (kN·m)	P_{L0} (MPa)	M_{B0} (kN·m)	M_B (kN·m)							
									$0.2M_{\tau0}$				$0.4M_{\tau0}$			
	r_o	r_i	b	a					$0.2P_{L0}$	$0.4P_{L0}$	$0.6P_{L0}$	$0.8P_{L0}$	$0.2P_{L0}$	$0.4P_{L0}$	$0.6P_{L0}$	$0.8P_{L0}$
1	79.5	75.5	0.1	0.2	Theoretical	31.01	21.63	33.85	32.25	29.33	23.89	14.65	29.95	26.78	20.77	10.11
					FEM	28.95	19.48	31.47	31.70	28.96	22.70	14.03	29.24	25.99	20.10	10.02
2	79.5	75.5	0.2	0.6	Theoretical	27.85	21.59	30.93	29.74	27.46	22.72	14.07	27.09	24.50	19.02	8.34
					FEM	26.24	20.91	29.73	28.97	26.85	22.02	13.99	26.96	24.02	18.76	7.98
3	44.5	40.5	0.2	0.2	Theoretical	9.14	39.42	9.86	9.40	8.57	7.01	4.32	8.70	7.80	6.06	2.92
					FEM	7.87	37.76	8.97	9.12	8.24	6.87	3.96	8.16	7.54	5.94	2.78
4	28.5	25.0	0.6	0.6	Theoretical	2.11	52.60	2.04	1.80	1.59	1.31	0.91	1.45	1.22	0.87	0.32
					FEM	1.98	50.84	1.86	1.68	1.42	1.26	0.86	1.39	1.08	0.78	0.29
5	16.0	13.0	0.8	0.2	Theoretical	0.71	85.27	0.67	0.65	0.59	0.49	0.31	0.58	0.53	0.40	0.17
					FEM	0.64	76.74	0.60	0.58	0.51	0.42	0.29	0.53	0.48	0.35	0.13
6	16.0	13.0	0.1	0.6	Theoretical	0.78	86.68	0.87	0.83	0.76	0.62	0.38	0.77	0.69	0.53	0.25
					FEM	0.70	80.15	0.81	0.79	0.68	0.59	0.32	0.71	0.62	0.49	0.20
7	28.5	25.0	0.4	0.2	Theoretical	3.04	54.68	3.18	3.04	2.79	2.29	1.42	2.79	2.51	1.95	0.92
					FEM	2.86	50.21	2.96	2.96	2.68	2.14	1.23	2.65	2.40	1.76	0.84
8	44.5	40.5	0.4	0.6	Theoretical	7.24	39.02	7.76	7.17	6.59	5.71	4.41	6.26	5.63	4.63	3.03
					FEM	7.06	38.74	7.55	7.02	6.43	5.28	4.19	6.08	5.51	4.49	2.98
9	23.5	19.5	0.6	0.2	Theoretical	2.15	77.37	2.17	2.07	1.91	1.57	0.99	1.89	1.70	1.32	0.60
					FEM	1.96	69.63	1.95	1.96	1.84	1.36	0.85	1.77	1.68	1.23	0.52
10	23.5	19.5	0.8	0.6	Theoretical	1.27	65.97	0.94	0.75	0.60	0.41	0.16	0.47	0.30	0.04	0.00
					FEM	1.16	60.88	0.85	0.69	0.54	0.36	0.12	0.40	0.22	0.02	0.00

* Because of space limitation, only partial results are given; $M_{\tau0}$ is the plastic limit torsion of the defective pipe under single torsion loading; M_{B0} is the plastic limit moment of the defective pipe under single bending moment loading

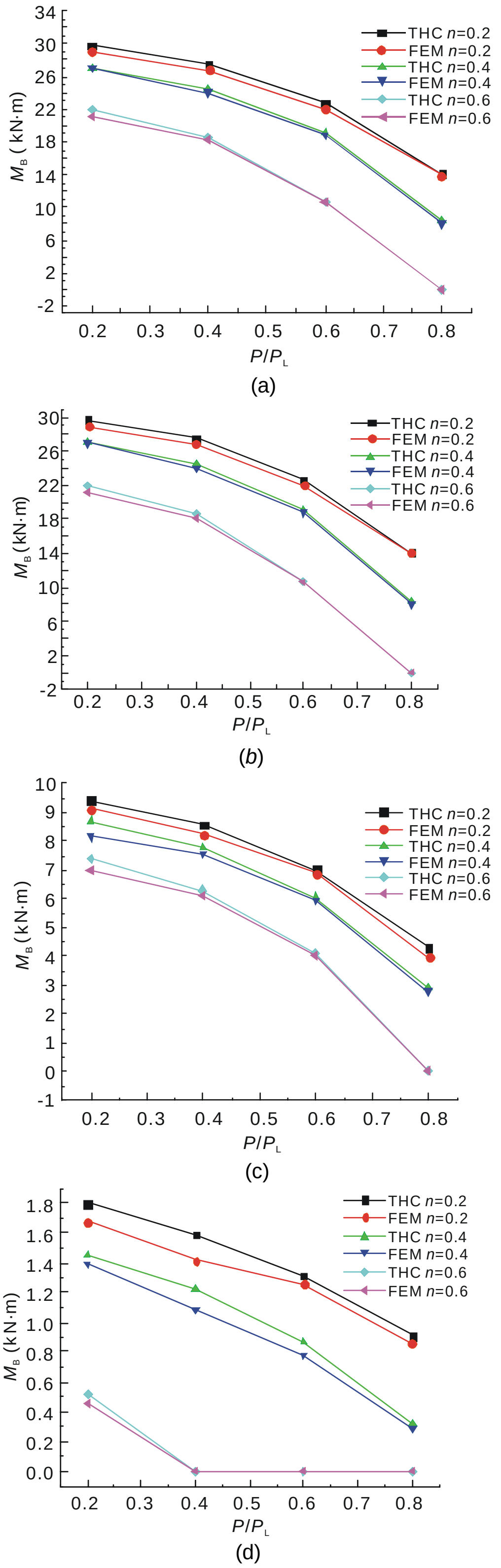

Fig. 8 Comparison between theoretical results and FEM results. (a) No. 1 (*a*=0.2, *b*=0.1, *k*=1.05); (b) No. 2 (*a*=0.6, *b*=0.2, *k*=1.05); (c) No. 3 (*a*=0.2, *b*=0.2, *k*=1.10); (d) No. 4 (*a*=0.6, *b*=0.6, *k*=1.15)

2. The combined orthogonal experimental design and limit load analysis theory are used to study the influence of relative circumferential length, relative depth of incomplete welding defects and pipe curvature on plastic limit load.

3. The comparison between finite element analysis and theoretical analysis showed that the results are in good agreement because that the error is distributed in 5%. The study provides an important theoretical basis to establish a safety assessment method for pressure pipe with incomplete welding defects.

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