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Analysis of 1D consolidation with non-Darcian flow described by exponent and threshold gradient*

Chuan-xun LI†1,2, Kang-he XIE[1], Kun WANG[1]
(*[1]MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering, Zhejiang University, Hangzhou 310027, China*)
(*[2]Department of Civil Engineering, Jiangsu University, Zhenjiang 212013, China*)
†E-mail: lichuanxun@yeah.net

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Abstract: Numerous experiments have shown that the water flow in fine-grained soils can obey an exponential relationship at small gradients and a linear relationship when the hydraulic gradient exceeds a certain limit. Based on the non-Darcian flow described by exponent and threshold gradient, the theory of 1D consolidation is modified in this paper to consider a linear variation in the vertical total stress with depth and the effect of ramp loading. The numerical solutions were derived in detail by the finite difference method for excess pore water pressure and the average degree of consolidation. Finally, the influence of various parameters on consolidation behavior was investigated. The results show that the rate of consolidation is reduced when non-Darcian flow described by exponent and threshold gradient is adopted in the theory of 1D consolidation. As well the distribution of vertical total stress has a great influence on the dissipation of excess pore water pressure, either for pervious top and pervious bottom (PTPB) or for pervious top and impervious bottom (PTIB). For the case of PTIB, the distribution of vertical total stress in a foundation has a great influence on the rate of consolidation; however, for the case of PTPB, the rate of consolidation is independent of the distribution of vertical total stress. The rate of consolidation is dependent on the ratio of the thickness of a soil layer to the equivalent head of the final average vertical total stress; the greater the value of this ratio, the slower the rate of consolidation. Finally, an increase in construction time reduces the consolidation rate of a foundation. Thus, consolidation behavior of 1D consolidation with non-Darcian flow has been thoroughly acquainted in this paper.

Key words: 1D consolidation, Non-Darcian flow, Ramp loading, Non-uniform distribution of vertical total stress
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1 Introduction

The linear relationship between flow velocity and hydraulic gradient is known as Darcy's flow law and was initially established for coarse-grained soils. It is still commonly used in consolidation theories for its simplicity. In practice, for fine-grained soils with low permeability under low hydraulic gradients, the deviation of water flow from the Darcy's law has been confirmed by a number of studies (Hansbo, 1960; Swartzendruber, 1962; Miller and Low, 1963; El-naggar and Krizek, 1973; Olsen, 1985; Dubin and Moulin, 1985). This phenomenon of deviation from Darcy's law has been called non-Darcian flow by Hansbo (1960). In addition, previous experiments have revealed that the water flow within the fine-grained soil may obey an exponential relationship at small gradients and a linear relationship when the hydraulic gradient exceeds a certain limit. Therefore, the non-Darcian flow relation proposed by Hansbo (1960) has been widely recognized. This relation was assumed to be (Fig. 1)

$$v=\begin{cases}\kappa i^m, & i\le i_1,\\ k(i-i_0), & i\ge i_1,\end{cases} \tag{1}$$

where v is the seepage velocity; k is the coefficient of

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permeability of the linear flow at high gradients; i_1 is the threshold hydraulic gradient for the linear relationship; m is the exponent of exponential flow at low gradients; $i_0=i_1(m-1)/m$; $\kappa=k/(mi_1^{m-1})$.

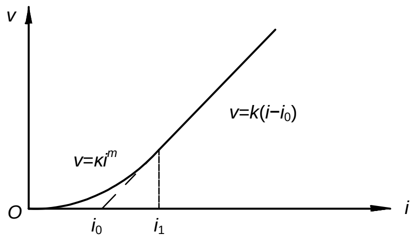

Fig. 1 Exponential correlation between flow velocity, *v*, and hydraulic gradient, *i*

Since Eq. (1), proposed by Hansbo (1960), has been verified for fine-grained soils by others, it has a theoretical significance in assessing the influence of non-Darcian flow on the consolidation behavior of soft soil foundations. The problem of 1D consolidation, based on non-Darcian flow, was first treated theoretically by Dubin and Moulin (1985). In their theoretical analysis, however, they exchanged the exponential part of the flow relation, $v=\kappa i^m$, for a linear part, $v=\kappa i$, for $i<i_1$ (Hansbo, 2003). Hansbo (2003) applied non-Darcian flow to 1D consolidation analysis and revealed a better agreement to the field observations. Moreover, based on non-Darcian flow, Teh and Nie (2002) analyzed the consolidation with the radial and vertical drainages of sand-drained ground and consequently investigated the influence of non-Darcian flow on the consolidation behavior. Xie *et al*. (2007) applied a semi-analytic method to solve the 1D consolidation equation, taking into consideration non-Darcian flow. Liu *et al*. (2009) studied 1D consolidation incorporating this non-Darcian flow using the finite volume method. According to Zhu and Yin (1998), in most cases the consolidation of a soil layer occurs simultaneously with the change of total stress that varies with time and depth due to a time-dependent external loading. All these previous work, however, did not deal with the simultaneous variation of the vertical total stress with both time and depth. In this paper, a comprehensive analysis was made of 1D consolidation, taking into consideration the non-Darcian flow described by Eq. (1), along with the change of vertical total stress with depth and time together.

2 Derivation of governing equations

2.1 Basic assumptions

To investigate the consolidation analysis of a soil layer under the above-mentioned depth-dependent ramp loading (Zhu and Yin, 1998) considering the non-Darcian flow law, the following assumptions are made:

1. Both the soil particles and the water in soil are incompressible.
2. The soil is homogeneous and fully saturated.
3. The deformation and the water flow within the soft soil foundation only occur in the vertical direction and the small strain assumption is incorporated.
4. The soil deformation is elastic and the compression coefficient is held constant during the process of consolidation.
5. The flow in the soil obeys non-Darcian law (as described in Eq. (1)) and the permeability parameters are constant during the consolidation process.
6. In accordance to the suggestion by Zhu and Yin (1998), as show in Fig. 2, the average vertical total stress in soil caused by the external load is assumed to change linearly with depth and gradually increase with time up to the final value and then to remain unchanged. Therefore, the average vertical total stress may be expressed as

$$\sigma(z,t)=\begin{cases}\left(\sigma_0+\dfrac{\sigma_1-\sigma_0}{H}z\right)\dfrac{t}{t_c}, & t\le t_c,\\ \sigma_0+\dfrac{\sigma_1-\sigma_0}{H}z, & t\ge t_c,\end{cases} \tag{2}$$

where H is the thickness of the soil layer; $\sigma(0, t)$ is the vertical total stress function at the top surface of foundation (at $z=0$), $\sigma(H, t)$ is the vertical total stress function at the bottom of foundation (at $z=H$); t_c is the construction time; σ_0 is the final vertical total stress at the top (at $z=0$ and $t=t_c$), whereas σ_1 is the final vertical total stress at the bottom (at $z=H$ and $t=t_c$), that is $\sigma_0=\sigma(0, t_c)$ and $\sigma_1=\sigma(H, t_c)$ (Fig. 2).

From Eq. (2) and Fig. 2, $t_c=0$ implies that the external load is applied instantly and the vertical total stress only changes with depth. In addition, $\sigma_1=\sigma_0$ indicates that the vertical total stress is assumed to be of uniform distribution in the vertical direction while changes with time only. If $t_c=0$ and $\sigma_1=\sigma_0$, the vertical total stress incorporated in this analysis is degenerated to that adopted by Terzaghi's theory.

2.2 Derivation of the governing equations

Considering a unit cell as shown in Fig. 3, the change of water quantity within the unit cell during an infinitesimal time period of dt can be expressed as

$$\mathrm{d}q=\frac{\partial v}{\partial z}\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}t. \tag{3}$$

Based on the above assumptions in subsection 2.1, the volume change of the unit cell during dt can be obtained:

$$\mathrm{d}V=m_{\mathrm{v}}\left(\frac{\partial u}{\partial t}-\frac{\partial \sigma(z,t)}{\partial t}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}t, \tag{4}$$

where m_{v} is the volume compressibility of soil.

According to the continuity condition that the change of water quantity should be equal to the volume change in soil, i.e., dq=dV. With Eqs. (1), (3) and (4), the equations governing 1D consolidation of soil with non-Darcian flow can be given as

$$\begin{cases}\dfrac{c_{\mathrm{v}}}{i_1^{m-1}}\left(\dfrac{1}{\gamma_{\mathrm{w}}}\dfrac{\partial u}{\partial z}\right)^{m-1}\dfrac{\partial^2 u}{\partial z^2}=\left(\dfrac{\partial u}{\partial t}-\dfrac{\partial \sigma(z,t)}{\partial t}\right), & i\le i_1,\\ c_{\mathrm{v}}\dfrac{\partial^2 u}{\partial z^2}=\left(\dfrac{\partial u}{\partial t}-\dfrac{\partial \sigma(z,t)}{\partial t}\right), & i\ge i_1,\end{cases} \tag{5}$$

where γ_{w} is the unit weight of water, $u(z, t)$ is the excess pore water pressure depended on z and t, $\sigma(z, t)$ is the vertical total stress depended on z and t, and c_{v} is the consolidation coefficient, i.e., $c_{\mathrm{v}}=k/(m_{\mathrm{v}}\gamma_{\mathrm{w}})$.

Drainage conditions in the vertical direction are either pervious top and pervious bottom (PTPB) or pervious top and impervious bottom (PTIB). So the drainage conditions can be written as

$$\text{PTIB:}\quad \begin{cases}u(0,t)=0,\\ \left.\dfrac{\partial u}{\partial z}\right|_{z=H}=0,\end{cases} \tag{6a}$$

$$\text{PTPB:}\quad \begin{cases}u(0,t)=0,\\ u(H,t)=0.\end{cases} \tag{6b}$$

The initial condition can be further given as

$$u(z,0)=\sigma(z,0), \tag{7}$$

where $\sigma(z, 0)$ is the vertical total stress at t=0.

Eq. (5) is the governing equation of 1D consolidation considering non-Darcian flow with the depth- and time-dependent total stress. If m=1, Eq. (5) is reduced to the traditional consolidation equation with Darcy's law (Zhu and Yin, 1998). Eq. (5) is a second order non-linear partial differential equation, and analytic solution could hardly be obtained. Therefore, the finite difference method is adopted to get the solutions of the governing equations in this study.

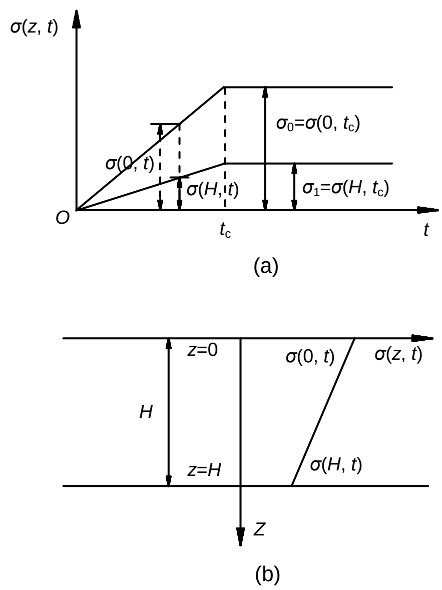

Fig. 2 Variation of the vertical total stress with time (a) and depth (b)

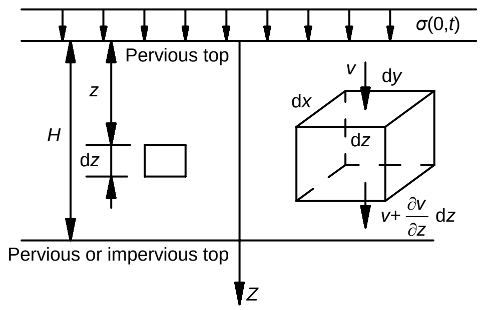

Fig. 3 Boundary condition, external load and unit body

3 Derivation of numerical solutions

3.1 Dimensionless process of the governing equation

For the convenience of the following studies, the dimensionless variables are introduced:

$$Z = \frac{z}{H}, \tag{8a}$$

$$T_\mathrm{v} = \frac{kt}{\gamma_\mathrm{w} m_\mathrm{v} H^2}, \tag{8b}$$

$$T_\mathrm{vc} = \frac{kt_\mathrm{c}}{\gamma_\mathrm{w} m_\mathrm{v} H^2}, \tag{8c}$$

$$U = \frac{u}{(\sigma_0 + \sigma_1)/2}, \tag{8d}$$

$$I = \frac{\gamma_\mathrm{w} Hi}{(\sigma_0 + \sigma_1)/2}, \tag{8e}$$

$$I_1 = \frac{\gamma_\mathrm{w} Hi_1}{(\sigma_0 + \sigma_1)/2}, \tag{8f}$$

$$n_0 = \frac{\sigma_0}{(\sigma_0 + \sigma_1)/2}, \tag{8g}$$

$$n_1 = \frac{\sigma_1}{(\sigma_0 + \sigma_1)/2}, \tag{8h}$$

$$Q(Z, T_\mathrm{v}) = \frac{\sigma(z,t)}{(\sigma_0 + \sigma_1)/2}, \tag{8i}$$

where Z a is dimensionless variable indicating the location in a soil layer; Z=0 and 1 indicate the locations of the top and the bottom surfaces of a foundation, respectively. T_v is the time factor of soil, which is the same as that in Terzaghi's theory. T_vc is the time factor of soil corresponding to the construction period t_c. U is a dimensionless variable indicating the magnitude of excess pore water pressure at any time and depth, and changes between 0 and 2; I is a dimensionless variable including the ratio of the thickness of a soil layer to the equivalent head of the final average vertical total stress and hydraulic gradient. When the thickness of a soil layer and the final average vertical total stress are given, variation of the value of I reflects the change of hydraulic gradient. I_1 is the value of I for the case of $i=i_1$. n_0 and n_1 are dimensionless variables indicating the distribution of vertical total stress in a foundation, and conform to this equation $n_0+n_1=2$. Q is a dimensionless variable indicating the magnitude of vertical total stress at any time and depth, and ranges from 0 to 2.

In terms of these dimensionless variables, Eqs. (5)–(7) can be rewritten as

$$\begin{cases} \dfrac{1}{I_1^{n-1}} \left(\dfrac{\partial U}{\partial Z}\right)^{m-1} \dfrac{\partial^2 U}{\partial Z^2} = \left(\dfrac{\partial U}{\partial T_\mathrm{v}} - \dfrac{\partial Q(Z,T_\mathrm{v})}{\partial T_\mathrm{v}}\right), & I \le I_1, \\ \dfrac{\partial^2 U}{\partial Z^2} = \left(\dfrac{\partial U}{\partial T_\mathrm{v}} - \dfrac{\partial Q(Z,T_\mathrm{v})}{\partial T_\mathrm{v}}\right), & I \ge I_1, \end{cases} \tag{9}$$

$$\begin{cases} U(0,T_\mathrm{v}) = 0, \\ \left.\dfrac{\partial U}{\partial Z}\right|_{Z=1} = 0, \end{cases} \tag{10a}$$

$$\begin{cases} U(0,T_\mathrm{v}) = 0, \\ U(1,T_\mathrm{v}) = 0, \end{cases} \tag{10b}$$

$$U(Z,0) = Q(Z,0), \tag{11}$$

where

$$Q(Z,T_\mathrm{v}) = \begin{cases} \left[n_0 + (n_1 - n_0)Z\right]\dfrac{T_\mathrm{v}}{T_\mathrm{vc}}, & T_\mathrm{v} \le T_\mathrm{vc}, \\ n_0 + (n_1 - n_0)Z, & T_\mathrm{v} \ge T_\mathrm{vc}. \end{cases} \tag{12}$$

3.2 Discretization of the governing equation

As stated above, the finite differential method is introduced to get the numerical solution for Eq. (9). Firstly, replace the (Z, T_v)-plane by a finite differential grid, the spatial domain $0 \le Z \le 1$ is divided into n equal segments from the top down by the proportional spacing ΔZ. If the nodal point of spatial domain is denoted as Z_i,

$$Z_i = i\Delta Z, \quad i=0, 1, \ldots, n, \tag{13}$$

meanwhile, the time axis is divided into a number of equal time increments ΔT_v. If the nodal point of time is denoted as $T_{\mathrm{v}j}$,

$$T_{\mathrm{v}j} = j\Delta T_\mathrm{v}, \quad j=0, 1, \ldots. \tag{14}$$

The Crank-Nicolson difference scheme is adopted for its relative stability. Thus, in terms of Crank-Nicolson difference scheme, Eq. (9) can be expressed as

$$\begin{aligned} &\alpha_i^j \lambda U_{i+1}^j - 2\left(\alpha_i^j \lambda + 1\right) U_i^j + \alpha_i^j \lambda U_{i-1}^j \\ &= -\alpha_i^{j-1} \lambda U_{i+1}^{j-1} + 2\left(\alpha_i^{j-1}\lambda - 1\right) U_i^{j-1} \\ &\quad - \alpha_i^{j-1} \lambda U_{i-1}^{j-1} - 2Q_i^j + 2Q_i^{j-1}, \end{aligned} \tag{15}$$

where i=0, 1, …, n and j=0, 1,…, U_i^j is the dimensionless expression for excess pore water pressure at $Z=i\Delta Z$ and $T_v=j\Delta T_v$, Q_i^j is the dimensionless expression for vertical total stress at $Z=i\Delta Z$ and $T_v=j\Delta T_v$, $\lambda=\Delta T_v/(\Delta Z)^2$, α_i^j is defined as

$$\alpha_i^j=\begin{cases}\left(\dfrac{1}{I_1}\dfrac{\left|U_{i+1}^j-U_{i-1}^j\right|}{2\Delta Z}\right)^{m-1}, & I_i^j<I_1,\\ 1, \ I_i^j\ge I_1,\end{cases} \tag{16}$$

where

$$I_i^j=\left|U_{i+1}^j-U_{i-1}^j\right|/\left(2\Delta Z\right).$$

In terms of the discrete nodal points, the boundary and initial conditions described by Eqs. (10a), (10b) and (11) can be rewritten as

$$\text{PTIB:}\quad \begin{cases}U_0^j=0,\\ U_{n+1}^j=U_{n-1}^j,\end{cases} \quad j=1,2,..., \tag{17a}$$

$$\text{PTPB:}\quad \begin{cases}U_0^j=0,\\ U_n^j=0,\end{cases} \quad j=1,2,..., \tag{17b}$$

$$U_i^0=Q_i^0,\quad i=1,2,...,n. \tag{18}$$

It can be seen from Eq. (15) that a computational problem will occur at the bottom for the PTIB model or at the middle section plane (at $z=H/2$) for the PTPB model at the case of n_0=1, where no water flows across. Since the v-i curves pass through the original point for both Darcy's flow and non-Darcian flow used in this study, to circumvent this mathematical difficulty, Darcy's flow law can be adopted at the impervious boundary (the bottom of a single-drained soil layer or the mid-plane of a double-drained layer). Considering the aforementioned matters, Eq. (15) can be rewritten in terms of matrix with the boundary Eq. (17a) and initial Eq. (18) conditions as

$$\boldsymbol{AU}=\boldsymbol{B}, \tag{19}$$

where $\boldsymbol{A}$ is an unknown tridiagonal matrix ($n\times n$) whose diagonal elements, upper diagonal elements and lower diagonal elements are

$$A_{i,i}=-2\left(\alpha_i^j\lambda+1\right),\ i=1,2,...,n-1, \tag{20a}$$

$$A_{i,i+1}=\alpha_i^j\lambda,\ i=1,2,...,n-1, \tag{20b}$$

$$A_{i,i-1}=\alpha_i^j\lambda,\ i=2,3,...,n-1, \tag{20c}$$

$$A_{nn}=-2(1+\lambda), \tag{20d}$$

$$A_{n(n-1)}=2\lambda. \tag{20e}$$

$\boldsymbol{U}$ is an unknown column matrix, whose elements U_i^j are dimensionless values of excess pore water pressure when $Z=i\Delta Z$, $T_v=j\Delta T_v$ (i=1,2,…,n). $\boldsymbol{B}$ is a known column matrix and its elements can be expressed as

$$\begin{aligned}B_i^{j-1}=&-\alpha_i^{j-1}\lambda U_{i+1}^{j-1}+2\left(\alpha_i^{j-1}\lambda-1\right)U_i^{j-1}\\ &-\alpha_i^{j-1}\lambda U_{i-1}^{j-1}-2\left(Q_i^j-Q_i^{j-1}\right),\\ &i=1,2,...,n-1,\end{aligned} \tag{21a}$$

$$B_n^{j-1}=2(\lambda-1)U_n^{j-1}-2\lambda U_{n-1}^{j-1}-2\left(Q^j-Q^{j-1}\right). \tag{21b}$$

If the boundary condition Eq. (17b) and initial condition Eq. (18) are considered, elements of the matrixes in Eq. (19) may be expressed as

$$A_{i,i}=-2(\alpha_i^j\lambda+1), \tag{22a}$$

$$A_{i,i+1}=\alpha_i^j\lambda, \tag{22b}$$

$$A_{i,i-1}=\alpha_i^j\lambda, \tag{22c}$$

$$i=1,2,...,n/2-1,n/2+1,...,n-1$$

$$A_{nn}=1,\ A_{n(n-1)}=0, \tag{22d}$$

$$A_{n/2,n/2}=-2(1+\lambda),\ A_{n/2,n/2-1}=A_{n/2,n/2+1}=\lambda, \tag{22e}$$

$$\begin{aligned}B_i^{j-1}=&-\alpha_i^{j-1}\lambda U_{i+1}^{j-1}+2\left(\alpha_i^{j-1}\lambda-1\right)U_i^{j-1}\\ &-\alpha_i^{j-1}\lambda U_{i-1}^{j-1}-2\left(Q_i^j-Q_i^{j-1}\right),\\ &i=1,2,...,n/2-1,n/2+1,...,n-1,\end{aligned} \tag{22f}$$

$$\begin{aligned}B_{n/2}^{j-1}=&-\lambda U_{n/2+1}^{j-1}+2(\lambda-1)U_{n/2}^{j-1}\\ &-\lambda U_{n/2-1}^{j-1}-2\left(Q^j-Q^{j-1}\right),\end{aligned} \tag{22g}$$

$$B_n^{j-1}=0. \tag{22h}$$

$\boldsymbol{U}$ is a column matrix with unknown elements U_i^j that are dimensionless values of excess pore water pressure at the time increment $j\Delta T_v$. $\boldsymbol{B}$ is a known column matrix and its elements U_i^{j-1} are dimensionless values of excess pore water pressure at the time increment $(j-1)\Delta T_v$. To develop the solution for

Eq. (19), $\boldsymbol{A}$ must be a constant matrix. If a time increment is small enough, the elements of $\boldsymbol{A}$ can be obtained by replacing the dimensionless value of excess pore water pressure at time increment $j\Delta T_v$ by that at time increment $(j-1)\Delta T_v$. The exact solutions for U_i^j can be obtained by repeating the above scheme. It can be seen that the solving conditions of Eq. (19) (Elnaggar and Krizek, 1973) has been satisfied during the process of iteration. The smaller the value of ΔT_v, the less the number of iterations.

3.3 Average degree of consolidation

The definition of average degree of consolidation in terms of deformation is the ratio of the settlement at any time to the final settlement:

$$U_s = \frac{S_t}{S_\infty}. \tag{23}$$

The settlement at any time follows that

$$S_t = m_v \int_0^H \left[\sigma(z,t) - u(z,t)\right] dz. \tag{24}$$

With Eqs. (2) and (24), the followings can be obtained:

$$S_t = \begin{cases} m_v \int_0^H \left\{ \left[\sigma_0 + \frac{(\sigma_1-\sigma_0)z}{H}\right] \frac{t}{t_c} - u(z,t) \right\} dz, & t \le t_c, \\ m_v \int_0^H \left\{ \left[\sigma_0 + \frac{(\sigma_1-\sigma_0)z}{H}\right] - u(z,t) \right\} dz, & t \ge t_c. \end{cases} \tag{25}$$

When excess pore water pressure has been reduced to zero, the final settlement of the foundation may follow from Eq. (25) as

$$S_\infty = m_v \int_0^H \left[\sigma_0 + (\sigma_1 - \sigma_0)\frac{z}{H}\right] dz. \tag{26}$$

Combining Eqs. (23), (25), and (26), the following relations can be obtained:

$$U_s = \begin{cases} \frac{T_v}{T_{vc}} - \int_0^1 U(Z,T_v) dZ, & T_v \le T_{vc}, \\ 1 - \int_0^1 U(Z,T_v) dZ, & T_v \ge T_{vc}. \end{cases} \tag{27}$$

The definition of average degree of consolidation in terms of stress is the ratio of the average effective stress at any time to the final average vertical total stress of the foundation, i.e.,

$$U_p = \frac{\int_0^H \left[\sigma(z,t) - u(z,t)\right] dz}{\int_0^H \sigma(z,\infty) dz}. \tag{28}$$

With Eqs. (2) and (28), the average degree of consolidation in terms of stress is given by

$$U_p = \begin{cases} \frac{T_v}{T_{vc}} - \int_0^1 U(Z,T_v) dZ, & T_v \le T_{vc}, \\ 1 - \int_0^1 U(Z,T_v) dZ, & T_v \ge T_{vc}. \end{cases} \tag{29}$$

From Eqs. (27) and (29), it is evident that the average degree of consolidation in terms of deformation is the same as that in terms of stress, i.e., $U_p=U_s$.

Since the analytic solution of excess pore water pressure cannot be developed, numerical integration must be adopted in Eqs. (27) and (29). Linear interpolation points may use the points in the finite differential grid. Thus, the average degree of consolidation can be expressed as

$$U_s = \begin{cases} \frac{T_v}{T_{vc}} - \frac{1}{n}\sum_{i=1}^{n} \frac{U_{i-1}+U_i}{2}, & T_v \le T_{vc}, \\ 1 - \frac{1}{n}\sum_{i=1}^{n} \frac{U_{i-1}+U_i}{2}, & T_v \ge T_{vc}, \end{cases} \tag{30}$$

where U_i is the dimensionless value of excess pore water pressure at $Z=i\Delta Z$.

4 Verification of the difference programming

When m=1, Eq. (1) is the same as Darcy's flow law. Under this situation, the theory of consolidation with non-Darcian flow (assumed in this study) turns into the consolidation theory given by Zhu and Yin (1998). To verify the accuracy of the difference programming, a comparison is made between the results of the present numerical method and the analytical method (Table 1).

Table 1 Comparison between the results of consolidation by the finite difference method and the analytic solution

T_v	Average degree of consolidation (%)			
	PTIB		PTPB	
	Results by FDM	Exact solution	Results by FDM	Exact solution
0.001	0.0320	0.0325	0.0485	0.0476
0.002	0.0892	0.0895	0.1358	0.1346
0.004	0.2496	0.2495	0.3824	0.3807
0.006	0.4556	0.4553	0.7014	0.6993
0.008	0.6980	0.6975	1.0791	1.0766
0.010	0.9716	0.9708	1.5073	1.5046
0.020	2.7062	2.7047	4.2594	4.2554
0.040	7.4945	7.4918	12.0412	12.0356
0.060	13.5461	13.5426	22.1051	22.0984
0.080	20.5704	20.5662	33.9646	33.9570
0.100	28.4029	28.3981	47.2848	47.2782
0.200	48.4095	48.4070	80.7996	80.7986
0.400	68.8512	68.8499	97.3324	97.3327
0.600	80.9876	80.9870	99.6294	99.6295
0.800	88.3929	88.3926	99.9485	99.9485
1.000	92.9138	92.9137	99.9928	99.9929

m=1, n_0=4/3, T_{vc}=0.1, Δz=0.01, λ=0.1

From Table 1, a good agreement was observed in the results between the finite difference method and the exact solution and, furthermore, that the maximal error is less than 0.02%. This confirms that the finite difference method is reliable in computing 1D consolidation with Darcy's flow. The main difference between Darcy's flow and non-Darcian flow (assumed here) lies in the choice of α_i^j. If the water flow in the soil obeys Darcy's law, $\alpha_i^j=\alpha_i^{j-1}=1$; otherwise, α_i^j, α_i^{j-1} are determined by Eq. (16). If the expressions of α_i^{j-1} and α_i^j can be ensured during the iteration process, moreover, the Crank-Nicolson difference scheme is relatively stable, and the results with non-Darcian flow by the finite difference method should be also reliable.

5 Analysis of consolidation behavior

As is well known, the number of iterations relies largely on the value of time increment. To reduce the number of iterations, the time increment should be kept small during the early stages of computation. The time step may gradually increase as the solution becoming smoother. Therefore, let n=100 and $\Delta T_v=1\times10^{-6}$ at the early stages of computation, when $T_v\geq0.1$, $\Delta T_v=1\times10^{-4}$.

Hansbo (1997; 2003) have reported that the value of m is equal to 1.5 for typical Swedish clays. Schmidt and Westmann (1973) followed the range of reported values of m at the low hydraulic gradient within 1–2. In this study, the value of m was assumed to range from 1 to 2. For the value of i_1, Hansbo (1960) stated that i_1 ranged from 4 to 10, while Dubin and Moulin (1985) reported that i_1 was between 8 and 35 (Teh and Nie, 2002). I_1 includes i_1 and the ratio of the thickness of the soil layer to the equivalent head of the average vertical total stress, I_1 was assumed to vary from 2 to 50 in this study. To investigate the influence of T_{vc} and n_0 on consolidation behavior, in this study, T_{vc} and n_0 were assumed to change between 0–1.0 and 0–2, respectively. Consolidation behavior with different parameters under different boundary conditions is cited in the following analysis.

5.1 Influence of *m* on consolidation behavior

As shown in Fig. 4, it is evident that the value of U based on non-Darcian flow (m>1) is greater than that based on Darcy's flow (m=1) at the same depth and the same time factor; furthermore, the greater m is, the greater the value of U is. Thus, the dissipation rate of excess pore water pressure with non-Darcian flow is slower than that with Darcy's flow for the case of PTPB.

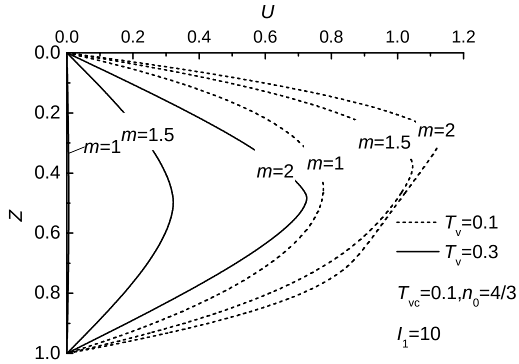

Fig. 4 Influence of *m* on the dissipation of excess pore water pressure for PTPB

Fig. 5 shows the influence of m on the average degree of consolidation for the case of PTPB. It can be seen that the rate of consolidation with non-Darcian flow is slower than that with Darcy's flow. With an increase in the value of m, this feature becomes more remarkable.

In practice, when n_0=4/3, the value of U with m=2 is smaller than that with m=1.5 at the bottom of the soil layer; whereas the value of U with m=2 is greater than that with m=1.5 at the upper part of the soil layer (Fig. 6). With an increase in time factor, however, this phenomenon will disappear. When T_v=5, the value of excess pore water pressure increases with an increase in the value of m in the whole soil layer under single drainage condition (Fig. 7).

The reason for this phenomenon is non-uniform distribution of vertical total stress and different dissipation rates of excess pore water pressure with different m. If $n_0 \leq 1$, as shown in Fig. 8, the value of excess pore water pressure would increase with an increase in the value of m in the whole soil layer during the whole consolidation process because there is no downward head difference.

In Fig. 9, m=1 implies that Darcy's law is adopted for PTIB case. In this case, the rate of consolidation of the soil layer is greater comparing to the case for m>1. In fact, m>1 indicates that the non-Darcian law is adopted. More simply, with an increase in the value of m under the single drainage condition, the rate of consolidation is greatly reduced.

5.2 Influence of I_1 on consolidation behavior

Fig. 10 shows the influence of I_1 on the dissipation of excess pore water pressure under double drainage conditions. It can be seen that the dissipation rate of pore water pressure is at its maximum for the case of I_1=2, whereas at its minimum for the case of I_1=50.

It can be seen from Fig. 11 that the dissipation of excess pore water pressure slows down with an increase in the value of I_1 for the case of PTIB. Moreover, a common feature is observed from both Figs. 6 and 11. When T_v=0.1, n_0=4/3, the value of U with I_1=50 is smaller than that with I_1=2 at the bottom of the soil layer for non-uniform distribution of initial vertical stress. With an increase in the value of T_v, such as when T_v=0.8, this phenomenon may disappear. If $n_0 \leq 1$, the value of excess pore water pressure

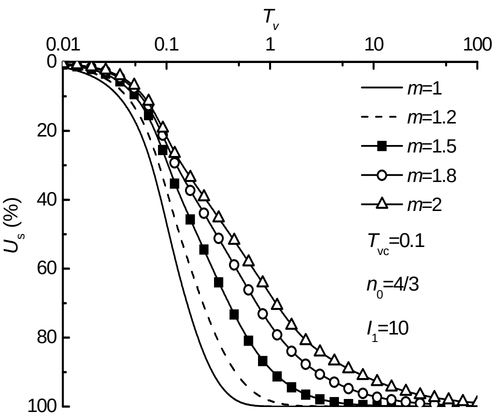

Fig. 5 Influence of *m* on the average degree of consolidation for PTPB

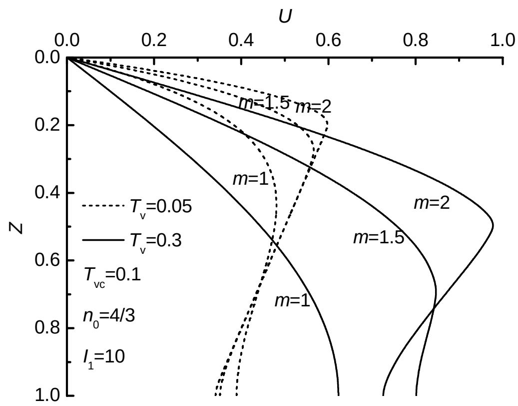

Fig. 6 Influence of *m* on the dissipation of excess pore water pressure for PTIB

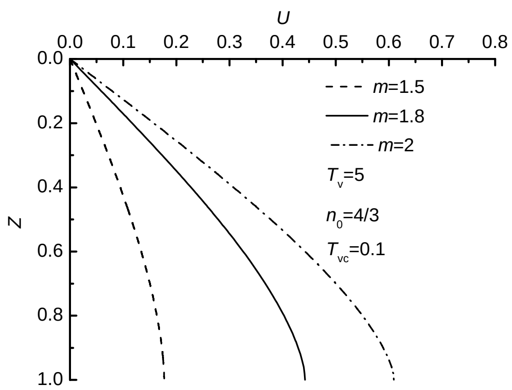

Fig. 7 Influence of *m* on the dissipation of excess pore water pressure for PTIB at T_v=5

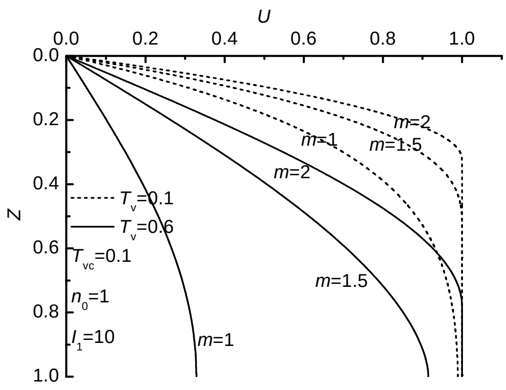

Fig. 8 Influence of *m* on the dissipation of pore water pressure for PTIB

increases with an increase in the value of I_1 in the whole soil layer during the whole consolidation process, because the downward head difference disappears.

Influence of I_1 on the average degree of consolidation under different boundary conditions can be seen from Figs.12 and 13. The rate of consolidation under any boundary condition (either PTIB or PTPB) decreases with an increase in the value of I_1. Meanwhile, the difference of average degree of consolidation between Darcy's flow and non-Darcian flow may gradually increase with an increase in the value of I_1.

According to Eq. (8f), the value of I_1 is proportional to the threshold hydraulic gradient for the linear flow and the ratio of the thickness of soil layer to the equivalent head of average vertical total stress. According to the study with Darcy's flow by Zhu and Yin (1998), the average degree of consolidation is independent of the value of average vertical total stress. However, the average degree of consolidation based on the non-Darcian flow (assumed here) is closely relevant to the value of average vertical total stress which may influence the value of I_1. The theory of 1D consolidation with non-Darcian flow considers the influence of the value of average vertical total stress on the rate of consolidation. At the same time, it should be noted that the thickness of the consolidation

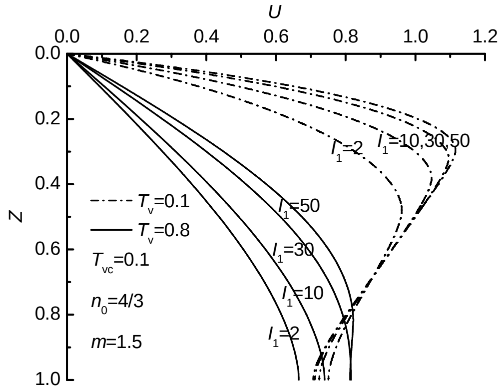

Fig. 11 Influence of I_1 on the dissipation of excess pore water pressure for PTIB

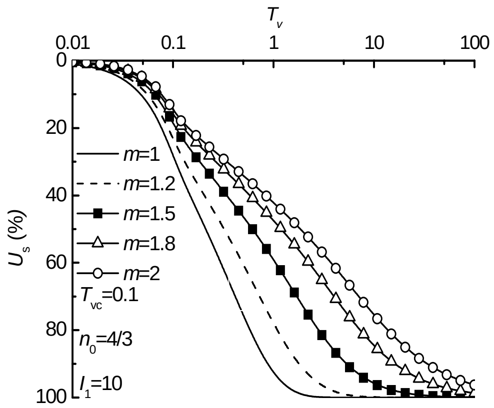

Fig. 9 Influence of *m* on the average degree of consolidation for PTIB

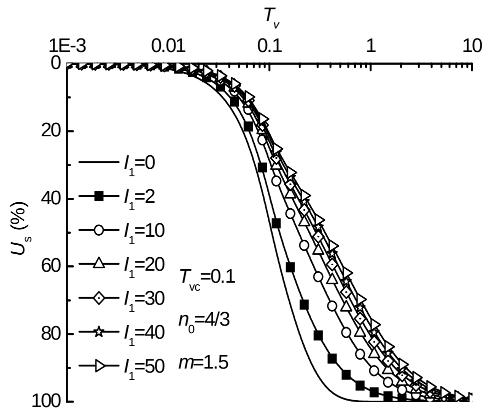

Fig. 12 Influence of I_1 on the average degree of consolidation for PTPB

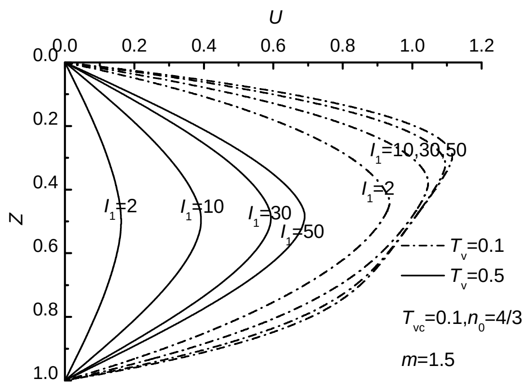

Fig. 10 Influence of I_1 on the dissipation of excess pore water pressure for PTPB

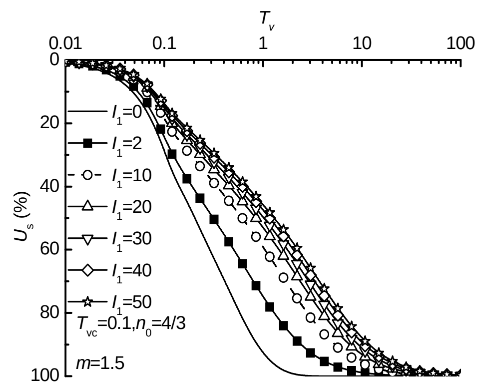

Fig. 13 Influence of I_1 on the average degree of consolidation for PTIB

soil layer may have a different effect on the rate of consolidation with Darcy's flow. As well known, with Darcy's flow thin sample has the same consolidation rate as the thicker one does at any same T_v. However, with non-Darcian flow, even if the value of I_1 is small, the similitude between thin samples and field layers is not satisfied any more. In fact, the rate of consolidation of thin samples is faster than that of thick ones at the same T_v.

5.3 Influence of n_0 on consolidation behavior

Fig. 14 shows the influence of the distribution of vertical total stress on the dissipation of excess pore water pressure under single drainage condition. When T_v=0.1, the value of U with n_0=2 is greater than that with n_0=0 at the upper soil layer. On the contrary, the value of U with n_0=2 is smaller than that with n_0=0 at the bottom of the soil layer for the reason of the non-uniform distribution of vertical total stress. When T_v=0.5, the value of U with n_0=2 is smaller than that with n_0=0 in the whole soil layer.

Influence of n_0 on the average degree of consolidation under single drainage condition can be seen from Fig. 15. For the case of PTIB, the rate of consolidation is at its maximum for the case of n_0=2 while at its minimum for the case of n_0=0. This feature coincides with 1D consolidation which is based on Darcy's flow.

As shown in Fig. 16, When T_v=0.1, the dissipation of excess pore water pressure under double drainage condition is closely related to the distribution of vertical total stress (i.e., the value of n_0). The value of excess pore water pressure increases with an increase in the value of n_0 at the upper soil layer (z<H/2); on the contrary, the value of excess pore water pressure decreases with an increase in the value of n_0 at the lower soil layer (z>H/2).

The curves of U_s vs. T_v with different n_0 become coincident with each other except for numerical computational error in Fig. 17. This confirms that the average degree of 1D consolidation based on non-Darcian flow is independent of the distribution of vertical total stress for the case of PTPB, as is the case based on Darcy's flow.

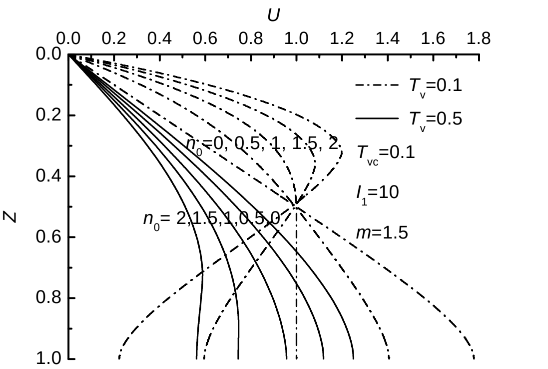

Fig. 14 Influence of n_0 on the dissipation of excess pore water pressure for PTIB

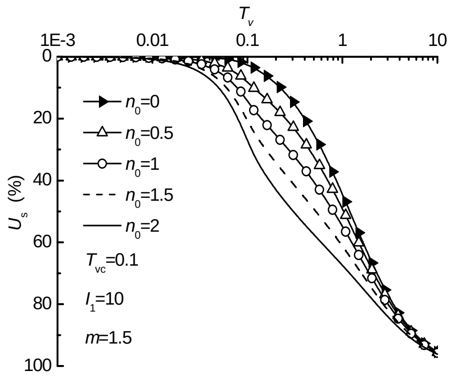

Fig. 15 Influence of n_0 on the average degree of consolidation for PTIB

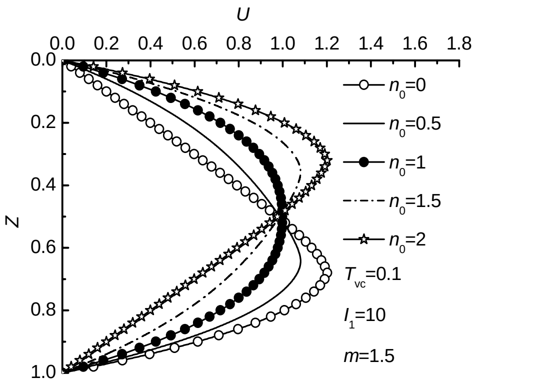

Fig. 16 Influence of n_0 on the dissipation of excess pore water pressure for PTPB

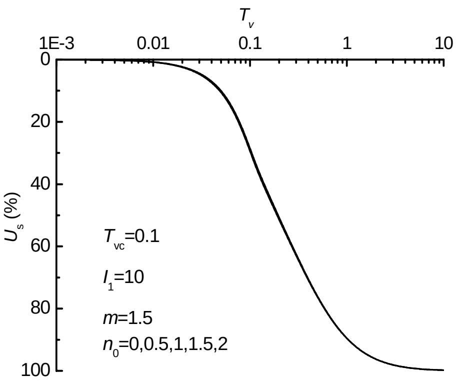

Fig. 17 Influence of n_0 on the average degree of consolidation for PTPB

5.4 Influence of T_{vc} on consolidation behavior

In Figs. 18 and 19, the influence of ramp loading rate on the average degree of consolidation under different drainage conditions is investigated. T_{vc}=0 implies that the load is applied instantly. Correspondingly, the rate of consolidation with non-Darcian flow is the maximum for the case of T_{vc}=0. With an increase in the value of T_{vc}, the rate of consolidation with non-Darcian flow is greatly reduced.

6 Conclusions

Based on the non-Darcian flow proposed by Hansbo (1960), the modified theory of 1D consolidation by considering variations of the vertical total stress with depth and construction time was developed and the consolidation behavior was investigated in detail. The following conclusions can be made:

1. The rate of consolidation based on non-Darcian flow is slower than that based on Darcy's flow. An increase in the values of m and I_1 reduces the rate of consolidation of the foundation with non-Darcian flow.

2. The dissipation of the excess pore water pressure is closely relevant to the distribution of the vertical total stress for both the cases of PTPB and PTIB.

3. The distribution of vertical total stress has a great influence on the rate of consolidation for the case of PTIB. The U-T_v curves for double drainage (PTPB), however, regardless of initial pressure distribution, coincide with each other except for numerical computational error. This confirms that the average degree of 1D consolidation based on non-Darcian flow is independent of the distribution of vertical total stress for the case of PTPB.

4. With consideration on non-Darcian flow, the rate of consolidation is related to the ratio of the thickness of the soil layer to the equivalent head of the average vertical total stress. The greater the average vertical stress (i.e., external load), the faster the rate of consolidation; the thicker the soil layer, even at the same T_v, the slower the rate of consolidation. Thus, the classical similitude with non-Darcian flow between laboratory samples and field layers is no longer satisfied.

5. The ramp loading rate has a great influence on the rate of consolidation. The faster the ramp loading rate, the faster the rate of consolidation.

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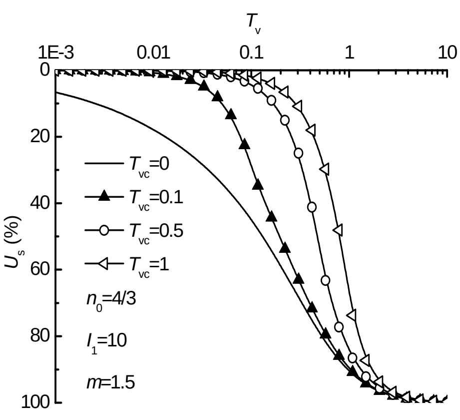

Fig. 18 Influence of ramp loading rate on the average degree of consolidation for PTIB

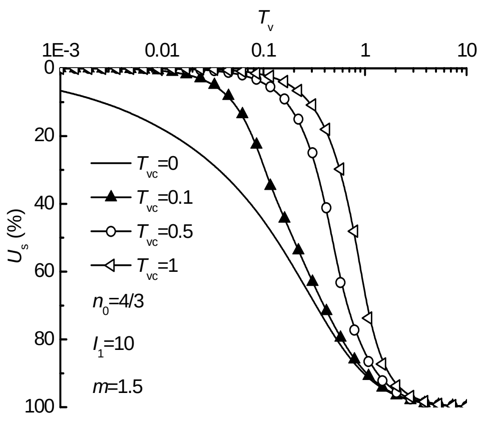

Fig. 19 Influence of ramp loading rate on the average degree of consolidation for PTPB

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