Journal of Zhejiang University SCIENCE A
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Science Letters:

Average SNR of maximum ratio transmission with selection combining

Xian-yi RUI

(*School of Electronic and Information Engineering, Soochow University, Suzhou 215006, China*)

E-mail: xyrui@suda.edu.cn

Received July 25, 2009; Revision accepted Oct. 3, 2009; Crosschecked Nov. 11, 2009

Abstract: Two kinds of selection combining schemes including generalized selection combining (GSC) and generalized order selection combining (GOSC) are investigated. In the GSC scheme, L strongest diversity branches from a total of R diversity branches are selected and coherently combined by maximal ratio combining. GOSC means that the Lth strongest diversity branch from R diversity branches is selected for reception. Closed-form expressions for the average signal-to-noise ratios of maximum ratio transmission with GSC and GOSC are derived in Rayleigh fading channels.

Key words: Maximum ratio transmission (MRT), Generalized selection combining (GSC), Generalized order selection combining (GOSC), Average signal-to-noise ratio, Rayleigh fading

doi:10.1631/jzus.A0920451 **Document code:** A **CLC number:** TN92

INTRODUCTION

Maximum ratio transmission (MRT) is a popular and simple scheme to maximize the output signal-to-noise ratio (SNR) in multi-antenna systems (Lo, 1999; Dighe *et al*., 2003; Maaref and Aissa, 2009). Recently, the performance of multiple-input multiple-output (MIMO) systems with joint MRT and maximal ratio combining (MRC), denoted as MIMO MRC, has been investigated in various conditions (Kang and Alouini, 2004; Chen and Tellambura, 2005; Maaref and Aissa, 2007; Rui *et al*., 2007). However, the use of the MRC scheme at the receiver of MIMO MRC leads to an increase in system hardware complexity, cost and power consumption because more radio frequency chains are required than the selection combining scheme. Especially for a mobile terminal, this increase is not desirable. To reduce the complexity, selection combining (SC), including generalized selection combining (GSC) (Alouini and Simon, 2000; Ma and Pasupathy, 2004; Theofilakos *et al*., 2008; Ma *et al*., 2009) and generalized order selection combining (GOSC) (Ma and Zhang, 2006; Kwan and Leung, 2007), can be employed at the receiver. In the GSC scheme, L strongest diversity branches from a total of R diversity branches are selected and then they are coherently combined by the MRC technique. Alouini and Simon (2000) presented the performance analysis of the GSC scheme, and derived the closed-form expressions for the average SNR, outage probability and average error probability of a wide variety of modulation schemes. Ma *et al*.(2009) investigated the asymptotic performance of absolute-threshold- and normalized-threshold-based GSC schemes over generalized fading channels for high average signal-to-noise ratios. However, these results are obtained based on the system with a single antenna at the transmitter.

In the GOSC scheme, the Lth strongest diversity branch from a total of R diversity branches is selected at the receiver. When L=1, GOSC represents conventional SC. Performance analysis of the GOSC scheme is needed when performance loss occurs in the SC scheme where the receiver makes an error in selecting the strongest branch (Kwan and Leung, 2007).

In this letter, we analyze the performance of two kinds of SC schemes, GSC and GOSC, in MRT systems. Closed-form expressions for their average SNR are derived under independent and identically distributed (i.i.d.) Rayleigh fading.

SYSTEM MODEL

Consider an MRT system equipped with T transmit and R receive antennas in a flat Rayleigh fading environment. Let $\boldsymbol{H}=(h_{ij})_{R\times T}$ denote the $R\times T$ channel matrix whose elements are i.i.d. complex Gaussian random variables with zero mean and unit variance. We assume that the feedback channel from the receiver to the transmitter is perfect, and that the channel state information is perfectly available to the receiver. The received signal $\boldsymbol{y}_i$ from receive antenna i can be expressed as

$$\boldsymbol{y}_i=\sqrt{P_\mathrm{s}}\boldsymbol{h}_i^\mathrm{H}\boldsymbol{ws}+\boldsymbol{n}, \tag{1}$$

where $\boldsymbol{s}$ represents the transmitted symbol satisfying $|\boldsymbol{s}|=1$, $\boldsymbol{w}$ is the $T\times 1$ normalized weighting vector at the transmitter, P_s is the signal power, and $\boldsymbol{n}$ is an $R\times 1$ additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\sigma_n^2\boldsymbol{I}_R$, where $\boldsymbol{I}_R$ denotes the $R\times R$ identity matrix. The $T\times 1$ vector $\boldsymbol{h}_i=[h_{i1}, h_{i2}, \ldots, h_{iT}]^\mathrm{T}$ denotes the channel between the T transmit antennas and the receive antenna i and is the ith column of the channel matrix $\boldsymbol{H}$. According to the MRT principle (Lo, 1999), the $T\times 1$ normalized transmit weighting vector $\boldsymbol{w}$ can be given by

$$\boldsymbol{w}=\boldsymbol{h}_i/\|\boldsymbol{h}_i\|. \tag{2}$$

After substituting Eq.(2) into Eq.(1), we can easily obtain the corresponding output SNR η_i:

$$\eta_i=\frac{P_\mathrm{s}\boldsymbol{h}_i^\mathrm{H}\boldsymbol{h}_i}{\sigma_n^2}=\eta_0\sum_{j=1}^{T}\left|h_{ij}\right|^2, \tag{3}$$

where $\eta_0=P_\mathrm{s}/\sigma_n^2$ denotes the average SNR per receive antenna. The variable η_i in Eq.(3) is an i.i.d. chi-squared variable with $2T$ degrees of freedom, so its probability density function (PDF) can be written as

$$f(x)=\frac{1}{\eta_0^T(T-1)!}x^{T-1}\mathrm{e}^{-x/\eta_0}, \tag{4}$$

and the corresponding cumulative density function (CDF) of η_i is expressed by

$$F(x)=1-\mathrm{e}^{-x/\eta_0}\sum_{t=0}^{T-1}\frac{x^t}{\eta_0^t t!}. \tag{5}$$

Let $\eta_{(1)}\geq\eta_{(2)}\geq\ldots\geq\eta_{(R)}\geq 0$ be the order statistics obtained by arranging the η_r ($r=1,2,\ldots,R$) in decreasing order of magnitude. We note that although unordered η_r ($r=1,2,\ldots,R$) are independent, $\eta_{(r)}$ ($r=1,2,\ldots,R$) are not independent and their joint PDF can be written by (David, 1981)

$$f_{\eta_{(1)},\eta_{(2)},\cdots,\eta_{(R)}}(\eta_{(1)},\eta_{(2)},\cdots,\eta_{(R)})=R!\prod_{r=1}^{R}f(\eta_{(r)}). \tag{6}$$

In the following sections of this letter, the two kinds of SC schemes, including GSC and GOSC, will be considered at the receiver.

MRT WITH GENERALIZED SELECTION COMBINING

Analysis

At the receiver, GSC is employed and L_c strongest diversity branches are combined. Therefore, the instantaneous SNR for MRC is

$$\eta_\mathrm{GSC}=\sum_{k=1}^{L_\mathrm{c}}\eta_{(k)}=\sum_{k=1}^{L_\mathrm{c}}\sum_{r=k}^{R}x_k=\sum_{k=1}^{L_\mathrm{c}}kx_k+L_\mathrm{c}\sum_{r=L_\mathrm{c}+1}^{R}x_r, \tag{7}$$

where

$$\eta_{(k)}\triangleq\sum_{r=k}^{R}x_r, \tag{8}$$

and

$$\begin{cases}x_k=\eta_{(k)}-\eta_{(k+1)}\geq 0,\\ x_R=\eta_{(R)}.\end{cases} \tag{9}$$

The Jacobian of the above transformation satisfies $|J(x_1,x_2,\ldots,x_R)|=1$. Then using the distribution theory for transformation of random vectors, the joint PDF of x_r ($r=1,2,\ldots,R$) can be expressed by

$$\begin{aligned}&f_{x_1,x_2,\cdots,x_R}(x_1,x_2,\cdots,x_R)\\&=\frac{f_{\eta_{(1)},\eta_{(2)},\cdots,\eta_{(R)}}(\eta_{(1)},\eta_{(2)},\cdots,\eta_{(R)})}{\left|J(x_1,x_2,\cdots,x_R)\right|}\\&=f_{\eta_{(1)},\eta_{(2)},\cdots,\eta_{(R)}}(\eta_{(1)},\eta_{(2)},\cdots,\eta_{(R)}).\end{aligned} \tag{10}$$

Substituting Eqs.(4) and (6) into Eq.(10), we can obtain

$$\begin{aligned} & f_{x_1,x_2,\cdots,x_R}(x_1,x_2,\cdots,x_R) \\ & = \frac{R!}{[\eta_0^T(T-1)!]^R}\prod_{r=1}^{R}\eta_{(r)}^{T-1}\exp\left(-\sum_{r=1}^{R}\frac{\eta_{(r)}}{\eta_0}\right) \\ & = \alpha_1\exp\left(-\sum_{r=1}^{R}\sum_{k=r}^{R}\frac{x_k}{\eta_0}\right)\prod_{r=1}^{R}\left(\sum_{k=r}^{R}x_k\right)^{T-1} \\ & = \alpha_1\exp\left(-\sum_{r=1}^{R}\frac{rx_r}{\eta_0}\right)(x_1+x_2+\cdots+x_R)^{T-1} \\ & \quad \cdot(x_2+x_3+\cdots+x_R)^{T-1}\cdots x_R^{\ T-1}, \end{aligned} \tag{11}$$

where $\alpha_1 = R!/[\eta_0^T(T-1)!]^R$. By applying the multinomial theorem, Eq.(11) can be rewritten as

$$\begin{aligned} & f_{x_1,x_2,\cdots,x_R}(x_1,x_2,\cdots,x_R) \\ & = \alpha_1\exp\left(-\sum_{r=1}^{R}\frac{rx_r}{\eta_0}\right)\sum_{I_1,I_2,\cdots,I_{R-1}}x_1^{n_1}x_2^{n_2}\cdots x_R^{n_R}, \end{aligned} \tag{12}$$

where

$$\sum_{I_1,I_2,\cdots,I_{R-1}} = \sum_{I_1}\frac{(T-1)!}{n_{1,1}!\cdots n_{1,R}!}\sum_{I_2}\frac{(T-1)!}{n_{2,2}!\cdots n_{2,R}!}\cdots\sum_{I_{R-1}}\frac{(T-1)!}{n_{R-1,R-1}!\cdots n_{R-1,R}!},$$

and

$$I_m = \{n_{m,m},\cdots,n_{m,R} \mid n_{m,m}+\cdots+n_{m,R} = T-1\}, \quad m = 1,2,\cdots,R-1, \tag{13}$$

$$\begin{cases} n_r = \sum_{m=1}^{r} n_{m,r}, \; r = 1,2,\cdots,R, \\ n_{R,R} = T-1, \end{cases} \tag{14}$$

$$\sum_{r=1}^{R} n_r = \sum_{r=1}^{R}\sum_{m=1}^{r} n_{m,r} = R(T-1). \tag{15}$$

Therefore, the average SNR $\bar{\eta}_{\text{GSC}}$ of MRT with the GSC scheme can be given by

$$\begin{aligned} & \bar{\eta}_{\text{GSC}} \\ & = \int_0^{+\infty}\cdots\int_0^{+\infty}\left(\sum_{k=1}^{L_c}kx_k + L_c\sum_{r=L_c+1}^{R}x_r\right)f(x_1,\cdots,x_R)\mathrm{d}x_1\cdots\mathrm{d}x_R \\ & = \sum_{k=1}^{L_c}kA_k + L_c\sum_{r=L_c+1}^{R}A_r, \end{aligned} \tag{16}$$

where

$$\begin{aligned} A_k & = \int_0^{+\infty}\cdots\int_0^{+\infty}x_k f_{x_1,x_2,\cdots,x_R}(x_1,x_2,\cdots,x_R)\mathrm{d}x_1\mathrm{d}x_2\cdots\mathrm{d}x_R \\ & = \alpha_1\sum_{I_1,I_2,\cdots,I_{R-1}}\prod_{r=1,r\neq k}^{R}\int_0^{+\infty}x_r^{n_r}\mathrm{e}^{-rx_r/\eta_0}\mathrm{d}x_r\int_0^{+\infty}x_k^{n_k+1}\mathrm{e}^{-kx_k/\eta_0}\mathrm{d}x_k \\ & = \alpha_1\sum_{I_1,I_2,\cdots,I_{R-1}}\prod_{r=1,r\neq k}^{R}(\eta_0/r)^{n_r+1}n_r!(\eta_0/k)^{n_k+2}(n_k+1)! \\ & = \frac{\alpha_1\eta_0}{k}\sum_{I_1,I_2,\cdots,I_{R-1}}\left(\prod_{r=1}^{R}\frac{n_r!}{r^{n_r}}\right)(n_k+1). \end{aligned} \tag{17}$$

We note that when T×1 as a special case, the above equation can be simplified into $A_k=\eta_0/k$. Combining Eq.(16), we can obtain the simple closed-form expression:

$$\bar{\eta}_{\text{GSC}} = L_c\eta_0\left(1+\sum_{r=L_c+1}^{R}\frac{1}{r}\right). \tag{18}$$

This is the same as in Alouini and Simon (2000).

Numerical example

As an example, Fig.1 plots the normalized average SNR $\bar{\eta}_{\text{GSC}}/\eta_0$ versus the number of selected branches L_c for MRT systems with GSC. In this case, R=8 and η_0=0 dB. From this figure, we can observe that the average SNR improves as the number of the strongest combined paths L_c increases. In Fig.1, the curve with T=1 represents a special case of single-input multiple-output (SIMO) systems with GSC at the receiver (Alouini and Simon, 2000). The agreement between the simulated and analytical results verifies the accuracy of our analytical formula in Eq.(16).

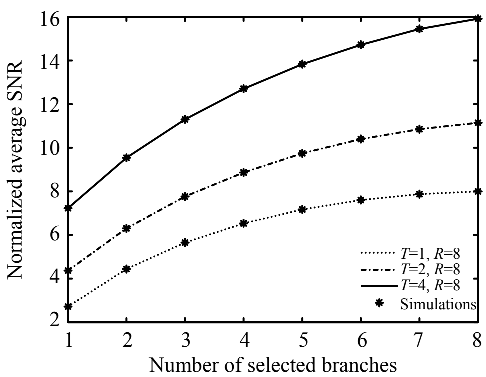

Fig.1 Average SNR of MRT with generalized selection combining

MRT WITH GENERALIZED ORDER SELECTION COMBINING

Analysis

In this subsection, the closed-form expression for average SNR of MRT systems with GOSC is derived when the Lth strongest diversity branch is selected at the receiver. Based on order statistic (David, 1981), PDF of MRT systems with GOSC can be given by

$$f_L(x)=\alpha_2[F(x)]^{R-L}[1-F(x)]^{L-1}f(x), \tag{19}$$

where $\alpha_2=R!/[(L-1)!(R-L)!]$. Substituting Eqs.(4) and (5) into Eq.(19), we can obtain

$$f_L(x)=\frac{\alpha_2 x^{T-1}\mathrm{e}^{-Lx/\eta_0}}{\eta_0^T(T-1)!}\left[1-\mathrm{e}^{-x/\eta_0}\nu\left(\frac{x}{\eta_0}\right)\right]^{R-L}\left[\nu\left(\frac{x}{\eta_0}\right)\right]^{L-1}, \tag{20}$$

where $\nu(x)=\sum_{t=0}^{T-1}x^t/t!$. By using the binomial theorem, Eq.(20) can be rewritten as

$$\begin{aligned}f_L(x)&=\frac{\alpha_2 x^{T-1}\mathrm{e}^{-Lx/\eta_0}}{\eta_0^T(T-1)!}[\nu(x/\eta_0)]^{L-1}\\&\quad\cdot\sum_{k=0}^{R-L}\binom{R-L}{k}(-1)^k\mathrm{e}^{-kx/\eta_0}[1-\nu(x/\eta_0)]^k\\&=\frac{\alpha_2}{\eta_0^T(T-1)!}[\nu(x/\eta_0)]^{L+k-1}\\&\quad\cdot\sum_{k=0}^{R-L}\binom{R-L}{k}(-1)^k\mathrm{e}^{-(L+k)x/\eta_0}x^{T-1}\\&=\frac{\alpha_2}{(T-1)!}\sum_{k=0}^{R-L}\sum_{m=0}^{(L+k-1)(T-1)}\left[\binom{R-L}{k}\right.\\&\quad\left.\cdot\frac{(-1)^k\beta_m(T;L+k-1)}{\eta_0^{T+m}\mathrm{e}^{(L+k)x/\eta_0}}x^{T+m-1}\right],\end{aligned} \tag{21}$$

where $\binom{N}{n}=\frac{N!}{n!(N-n)!}$ denotes the binomial coefficient, and $\beta_m(T;k)$ is the coefficient of x^m, $m=0,1,\ldots,k(T-1)$ in the expansion of $\left(\sum_{t=0}^{T-1}x^t/t!\right)^k$. Therefore, the average SNR $\bar{\eta}_{\text{GOSC}}$ of MRT systems with GOSC at the receiver can be given by

$$\begin{aligned}\bar{\eta}_{\text{GOSC}}&=\int_0^{+\infty}xf_L(x)\mathrm{d}x\\&=\frac{\alpha_2}{(T-1)!}\sum_{k=0}^{R-L}\sum_{m=0}^{(L+k-1)(T-1)}\left[\binom{R-L}{k}\right.\\&\quad\left.\cdot\frac{(-1)^k\beta_m(T;L+k-1)}{\eta_0^{T+m}}\int_0^{+\infty}\mathrm{e}^{-(L+k)x/\eta_0}x^{T+m}\mathrm{d}x\right]\\&=\frac{\eta_0\alpha_2}{(T-1)!}\sum_{k=0}^{R-L}\sum_{m=0}^{(L+k-1)(T-1)}\left[\binom{R-L}{k}\right.\\&\quad\left.\cdot\frac{(-1)^k\beta_m(T;L+k-1)(T+m)!}{(L+k)^{T+m+1}}\right].\end{aligned} \tag{22}$$

When the conventional SC with $L=1$ is employed, the average SNR expression in Eq.(22) can be simplified into

$$\bar{\eta}_{\text{GOSC}}=\frac{\eta_0 R}{(T-1)!}\sum_{k=0}^{R-1}\sum_{m=0}^{k(T-1)}\binom{R-1}{k}\frac{(-1)^k\beta_m(T;k)(T+m)!}{(k+1)^{T+m+1}}. \tag{23}$$

Numerical example

The analysis of MRT systems with GOSC at the receiver can be used to examine the effect of an incorrect best antenna selection. Fig.2 shows the normalized average SNR $\bar{\eta}_{\text{GOSC}}/\eta_0$ for MRT with the Lth strongest diversity branch being selected from $R=8$ diversity branches at the receiver. It can be seen that the larger L (which means the worse diversity branch selected to receive the signals) results in the smaller normalized average SNR. The agreement between the simulated and analytical results verifies the accuracy of Eq.(22).

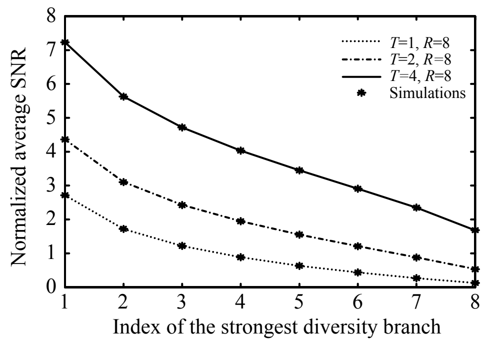

Fig.2 Average SNR of MRT with generalized order selection combining

CONCLUSION

In this letter we present the performance analysis of MRT with SC schemes over Rayleigh fading channels. Closed-form expressions for the average SNR of the systems with GSC and GOSC schemes are derived.

References

Alouini, M.S., Simon, M.K., 2000. An MGF-based performance analysis of generalized selection combining over Rayleigh fading channels. *IEEE Trans. Commun.*, **48**(3):401-415. [doi:10.1109/26.837044]

Chen, Y., Tellambura, C., 2005. Performance analysis of maximum ratio transmission with imperfect channel estimation. *IEEE Commun. Lett.* **9**(4):322-324. [doi:10.1109/LCOMM.2005.1413621]

David, H.A., 1981. Order Statistics (2nd Ed.). Wiley, New York, p.9-20.

Dighe, P.A., Mallik, R.K., Jamuar, S.S., 2003. Analysis of transmit-receive diversity in Rayleigh fading. *IEEE Trans. Commun.*, **51**(4):694-703. [doi:10.1109/TCOMM.2003.810871]

Kang, M., Alouini, M.S., 2004. A comparative study on the performance of MIMO MRC systems with and without cochannel interference. *IEEE Trans. Commun.*, **52**(8):1417-1425. [doi:10.1109/TCOMM.2004.833027]

Kwan, R., Leung, C., 2007. General order selection combining for Nakagami and Weibull fading channels. *IEEE Trans. Wirel. Commun.*, **6**(6):2027-2033. [doi:10.1109/TWC.2007.05810]

Lo, T.K.Y., 1999. Maximum ratio transmission. *IEEE Trans. Commun.*, **47**(10):1458-1461. [doi:10.1109/26.795811]

Ma, Y., Pasupathy, S., 2004. Efficient performance evaluation for generalized selection combining on generalized fading channels. *IEEE Trans. Wirel. Commun.*, **3**(1):29-34. [doi:10.1109/TWC.2003.821130]

Ma, Y., Zhang, D., 2006. Generalized Selection Multiuser Scheduling with Channel Estimation Errors. IEEE Conf. on Global Communications, p.1-5. [doi:10.1109/GLOCOM.2006.455]

Ma, Y., Dong, X., Yang, H.C., 2009. Asymptotic performance of threshold-based generalized selection combining. *IEEE Trans. Veh. Technol.*, **58**(5):2579-2585. [doi:10.1109/TVT.2008.2010050]

Maaref, A., Aissa, S., 2007. Eigenvalue distributions of Wishart-type random matrices with application to the performance analysis of MIMO MRC systems. *IEEE Trans. Wirel. Commun.*, **6**(7):2678-2689. [doi:10.1109/TWC.2007.05990]

Maaref, A., Aissa, S., 2009. Optimized rate-adaptive PSAM for MIMO MRC systems with transmit and receive CSI imperfections. *IEEE Trans. Commun.*, **57**(3):821-830. [doi:10.1109/TCOMM.2009.03.060678]

Rui, X., Jin, R., Geng, J., 2007. Performance analysis of MIMO MRC systems in the presence of self-interference and co-channel interferences. *IEEE Signal Processing Lett.*, **14**(11):801-803. [doi:10.1109/LSP.2007.903254]

Theofilakos, P., Kanatas, A.G., Efthymoglou, G.P., 2008. Performance of generalized selection combining receivers in *K* fading channels. *IEEE Commun. Lett.*, **12**(11):816-818. [doi:10.1109/LCOMM.2008.080779]