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Analysis and optimal synthesis of single loop spatial mechanisms*

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Abstract: In this work, a systematic approach is presented to obtain the input-output equations of a single loop 4-bar spatial mechanisms. The dialytic method along with Denavit-Hartenberg parameters can be used to obtain these equations efficiently. A genetic algorithm (GA) has been used to solve the problem of spatial mechanisms synthesis. Two types of mechanisms, e.g., RSCR and RSPC (R: revolute; S: spherical; C: cylindrical; P: prismatic), have illustrated the application of the GA to solve the problem of function generation and path generation. In some cases, the GA method becomes trapped in a local minimum. A combined GA-fuzzy logic (GA-FL) method is then used to improve the final result. The results show that GAs, combined with an adequate description of the mechanism, are well suited for spatial mechanism synthesis problems and have neither difficulties inherent to the choice of the initial feasible guess, nor a problem of convergence, as it is the case for deterministic methods.

Key words: Spatial mechanisms, Genetic algorithm-fuzzy logic (GA-FL) method, Denevit-Hartenberg parameters, Mechanism synthesis

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1 Introduction

Mechanisms are mechanical systems capable of producing a desired output motion for a given input motion. These mechanisms can be classified in three families: the first and simplest one is the planar mechanisms, which are limited to the transformation of planar motion; the second type and most versatile one is robot manipulators, which are capable of producing any motion in space but they are very complex to model and control; the third one is made of spatial mechanisms with limited degrees of freedom. Spatial mechanisms with limited degrees of freedom present an alternative way to impart a given motion to a body, which is cheap and easy to use. Moreover, this type of spatial mechanisms is reliable and requires very little energy due to the limited number of motors required to actuate the system. Therefore, the problem of designing the right spatial mechanism for a specified task is an important one. Sandor and Erdman (1984) defined three types of synthesis problems: function generation, path generation, and body guidance. In this work, we will be interested in the first two problems although the proposed method can easily be extended to solve the third one.

The input-output equations of a mechanism are usually obtained by algebraic methods that are specific to the geometry of a particular mechanism. Input-output equations are often very complex, presenting difficulties to solve and having more than

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one solution. Again, the existing methods depend on the mechanism studied and usually do not yield all the possible solutions. We have established the most general form of the input-output equation of a spatial mechanism with a single loop and one degree of freedom linkage with four joints. The geometric description adopted in this work uses the Denavit-Hartenberg parameters. This description allows for a general approach in the study of the dimensional synthesis of the proposed mechanisms. The kinematics of the mechanism is completely characterized by its topology, dimensions and location in a reference frame. The presented method is applicable to any single degree of freedom mechanism with a single loop.

To find the input-output equation, we used the dialytic method. The dialytic elimination method was introduced by Cayley in 1848 (Tsai, 1999). The method requires the derivation known as the elimination or resultant. The method will reduce any system of multivariable polynomial equations to a single polynomial with only one unknown. Raghavan and Roth (1998) developed a method based on dialytic elimination for finding all solutions to an arbitrary 6R mechanism. Their method reduces an inverse kinematics problem to finding the roots of a 16-degree polynomial. More recently, Lee and Shim (2003) presented the forward kinematics of the general Stewart-Gough platform using dialytic elimination.

In most cases, the user can only get a numerical solution. An alternative to this objective function is the structural error (Laribi *et al.*, 2004), which is based on the idea of finding a set of link dimensions verifying the loop closure equations. In this case, the geometric model is sufficient and there is no need to solve the obtained equations, but try to satisfy them. GA methods are well suited for this type of objective functions. The RSCR spatial mechanism (R: revolute; S: spherical; C: cylindrical), studied by Ananthasuresh and Kramer (1994), is the first one to illustrate the GA method. The function generation and the path generation are both solved for this type of mechanisms. Ananthasuresh and Kramer (1994) used the gradient method to solve the function generation problem for this mechanism. The results obtained in this work using the GA method are shown to be more accurate than those found in Ananthasuresh and Kramer (1994). Ananthasuresh and Kramer (1994) used a set of 12 design variables to describe the geometry of the RSCR mechanism. In this work, we used a geometric description based on the well-known Denavit-Hartenberg parameters. This geometric description allowed us to reduce the number of parameters required to describe the system from 12 to only 8 parameters, to solve the same problem. Hence, the optimization problem is simpler and faster, and the objective function is obtained in a systematic way using homogeneous matrices. The RSPC spatial mechanism (P: prismatic) is the second mechanism used to illustrate the application of the GA method in the dimensional synthesis of mechanisms. Recently, Fischer (2003) used this mechanism to illustrate a novel kinematics analysis. This type of mechanisms has an industrial application as an agitator mechanism in a washing machine (Fig. 1). To the best of our knowledge, no previous work has treated the synthesis problem for this type of mechanisms. The function generation and the path generation problems are solved. The RSPC type mechanism was used by several studies (Sandor and Erdman, 1984; Fischer, 2003). Although this work is based on the kinematics characteristics of the RSCR and the RSPC mechanisms (Chiang *et al.*, 1992), it can easily be extended to the family of single loop spatial mechanisms. This family includes the RRSC introduced by Permkumar *et al.* (1988), the RSSR introduced by Devanathan and Siddhanty (1984) and Rastegar and Tu (1992), and others (Sandor and Erdman, 1984; Sandor *et al.*, 1986; Dhall and Kramer, 1990; Permkumar and Kramer, 1990; Rastegar and Tu, 1996; Russell and Sodhi, 2001; 2003; Chung, 2004).

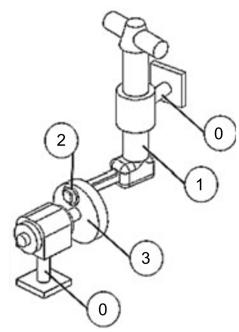

Fig. 1 Agitator mechanism in a washing machine (RSPC)

The contributions and understanding of spatial mechanisms in this study are: (1) an exhaustive description of 4-bar spatial mechanisms with a single-loop one degree of freedom; (2) a general geometric description of single-loop 4-bar spatial mechanisms; (3) a general method to solve the problems of identification of the input-output equation.

2 Geometric model of single loop spatial mechanisms

The input-output equations of a mechanism are usually obtained by algebraic methods, which are specific to the geometry of a particular mechanism. Input-output equations are often complex, presenting many difficulties to be solved and having more than one solution.

2.1 Different types of mechanisms

For a single kinematic loop with four bodies and four joints (Fig. 2), we can generate a large number of mechanisms. If we limit the types of joints as shown in Table 1, we can identify four classes of 48 spatial mechanisms.

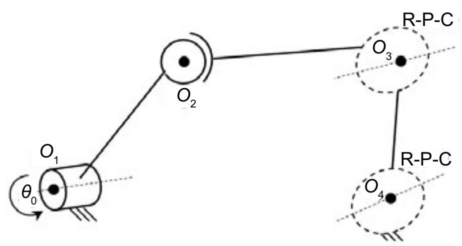

Fig. 2 A general 4-bar mechanism (θ_0: input angle, O_i: center of joint i)

Table 1 The four types of joints

Joint	Description
R	Revolute (one degree of freedom)
P	Prismatic (one degree of freedom)
C	Cylindrical (two degrees of freedom)
S	Spherical (three degrees of freedom)

An exhaustive description of different classes of a closed chain spatial mechanisms is presented in Table 2. These families are described topologically by four joints.

The mobility of a mechanism is given by the Kutzbach criterion (Gogu, 2005):

$$m = 6(i-1) - \sum\nolimits_{j=1}^{p}(6-f_j), \quad (1)$$

Table 2 The 4-bar mechanisms

Class notation	Description
R-R	R input joint and R output joint
R-C	R input joint and C output joint
R-P	R input joint and P output joint
R-S	R input joint and S output joint

where p is the number of joints; n $(n = i - 1)$ is the number of links (i denotes the total number of elements including the base); and f_j is the mobility of the joint.

Table 3 presents the possible linkages with different mobilities (m). It also enumerates the unknown variables appeared in closed loop equations (Eqs. (5a)–(5c)). The number of the variables in Table 3 is less than expected because we close the loop on the center of the spherical joint. Therefore, the spherical joint variables do not appear in the equations. The kinematics of all these mechanisms are described in the next section to express the input-output function.

Table 3 Unknown variables of the 4-bar mechanisms

Class notation	Unknown variable	Mobility
RSCR	d_2, θ_2, θ_3	1
RSPC	d_2, θ_3, d_1	1
RSRC	θ_2, θ_3, d_1	1
RSCC	$d_2, \theta_2, \theta_3, d_1$	2 (1 internal)
RPSC	θ_2, d_3, d_1	1
RRSC	θ_2, θ_3, d_1	1
RCSC	$\theta_2, d_3, \theta_3, d_1$	2 (1 internal)
RSCP	θ_2, θ_3, d_3	1
RCSP	d_2, θ_2, θ_3	1
RRCS	θ_2, θ_3	1
RPCS	d_2, θ_3, d_3	1
RCRS	d_2, θ_2, θ_3	1
RCPS	d_2, θ_2, d_3	1
RCCS	$d_2, \theta_2, d_3, \theta_3$	2 (1 internal)

d_i: the prismatic joint unknown; θ_i: the revolute joint unknown

2.2 Loop closer equation

The homogeneous transformation relating the tool frame to the fixed base frame is given by

$$^0\boldsymbol{T}_4 = {}^0\boldsymbol{T}_1^1\boldsymbol{T}_2^2\boldsymbol{T}_3^3\boldsymbol{T}_4 \quad (2)$$

where $^0\boldsymbol{T}_4 = \begin{bmatrix} \boldsymbol{I} & 0 \\ 0 & 1 \end{bmatrix}$. Here, $\boldsymbol{I}$ is the identity matrix.

The most general structure of this class of mechanisms is modeled geometrically by a maximum of 16 parameters, four parameters for each joint (Table 4).

Table 4 Denavitt-Hartenberg parameters of the four-bar mechanism

i	a_i	d_i	α_i	θ_i
0	a_0	d_0	α_0	θ_0
1	a_1	d_1	α_1	θ_1
2	a_2	d_2	α_2	θ_2
3	a_3	d_3	α_3	θ_3

Eq. (2) can be written as

$$^{0}\boldsymbol{T}_1^1\,\boldsymbol{T}_2 = {}^{0}\boldsymbol{T}_3^3\,\boldsymbol{T}_2. \tag{3}$$

Then we can deduce the expressions, in the most general form, of the coordinates of point O_2 (the center of the spherical joint), as follows:

$$^{0}\boldsymbol{T}_1^1\,\boldsymbol{T}_2 \begin{bmatrix} 0\\0\\0\\1 \end{bmatrix} = {}^{0}\boldsymbol{T}_3^3\,\boldsymbol{T}_2 \begin{bmatrix} 0\\0\\0\\1 \end{bmatrix}. \tag{4}$$

In Fig. 2,

$$\begin{aligned} & a_2 \cos\theta_1 \cos\theta_2 - a_2 \sin\theta_1 \cos\alpha_1 \sin\theta_2 \\ & \quad + d_2 \sin\theta_1 \sin\alpha_1 + a_1 \cos\theta_1 = \\ & \quad - a_3 \cos\theta_0 - d_3 \sin\theta_0 \sin\alpha_3 - a_0, \end{aligned} \tag{5a}$$

$$\begin{aligned} & a_2 \sin\alpha_1 \cos\theta_2 + a_2 \cos\theta_1 \cos\alpha_1 \sin\theta_2 \\ & \quad - d_2 \cos\theta_1 \sin\alpha_1 + a_1 \sin\theta_1 = \\ & \quad a_3 \cos\alpha_0 \sin\theta_0 - d_3 \cos\theta_0 \cos\alpha_0 \sin\alpha_3 \\ & \quad - d_3 \sin\alpha_0 \cos\alpha_3 - d_0 \sin\alpha_0, \end{aligned} \tag{5b}$$

$$\begin{aligned} & a_2 \sin\alpha_1 \sin\theta_2 + d_2 \cos\alpha_1 + d_1 + a_3 \sin\alpha_0 \\ & \quad \sin\theta_0 - d_0 \sin\alpha_0 \cos\theta_0 \sin\alpha_3 = \\ & \quad d_3 \cos\alpha_0 \cos\alpha_3 - d_0 \cos\alpha_0. \end{aligned} \tag{5c}$$

The order and the type of joints used in the construction of the spatial mechanism are not important. The objective is to identify the input-output function, $f(\theta_0, \theta_1)$ or $f(\theta_0, d_1)$.

Table 5 presents the possible associated linkages with different mobilities. The kinematics of all mechanisms are described by Eq. (4). The unknown variables for each mechanism are enumerated in Table 5. These scalar parameters represent the design variables of the mechanism, while Eqs. (5a)–(5c) express the input-output functions.

As a result of the rearrangement of Eq. (2) as Eq. (4), the entries of the left hand side matrix are functions of θ_1 and θ_2 and the entries of the right hand side matrix are functions of θ_0. This fact reduces the symbolic complexity of the resulting expressions. In addition, the use of this formulation allows us to describe the system in a linear form.

For a given spatial mechanism, each transformation matrix $^{i}\boldsymbol{T}_j$ contains one or more unknowns. We use q_i to represent these unknowns. q_i corresponds to d_i for prismatic joints and $S_i = \sin\theta_i$ and $C_i = \cos\theta_i$ for revolute joints, $i = 0, 1, 2, 3$.

A spatial mechanism consists of at most one spherical joint, and the obtained system can be expressed as

$$\boldsymbol{Q} \begin{bmatrix} q_2 \\ q_1 \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} q_3 \\ q_0 \\ 1 \end{bmatrix}, \tag{6}$$

where $\boldsymbol{Q}$ is a 3×4 matrix, whose entries are functions of parameters of the spatial mechanism. $\boldsymbol{P}$ is a 3×4 matrix, whose elements are independent of the joint variables and is a function of only q_3. Eq. (6) helps us eliminating the variables of the spherical joint.

Using the same procedure proposed by Raghavan and Roth (1998) and Mavroidis and Roth (1994) to process Eq. (6), we can eliminate the left hand side terms in terms of the right hand side. After substituting

$$S_i = \frac{2x_i}{1 + x_i^2}, \quad C_i = \frac{1 - x_i^2}{1 + x_i^2},$$

where $x_i = \tan(\theta_i/2)$, Eq. (6) can, therefore, be expressed as

$$\boldsymbol{\Sigma} \left[x_2^2, x_2, d_3, d_2, d_1, d_0, 1\right]^{\mathrm{T}} = 0, \tag{7}$$

where $\boldsymbol{\Sigma}$ is a 3×7 matrix, whose entries are linear combinations of q_1, q_0, and 1. The system given above is not square and it is converted into a square system using dialytic elimination. In particular, Eq. (7) is multiplied by x_2 to obtain a square system of the form:

$$\begin{gathered} \boldsymbol{\Sigma}' \boldsymbol{U}^{\mathrm{T}} = 0, \\ \boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2 d_3, x_2 d_2, x_2 d_1, x_2 d_0, x_2, d_3, d_2, d_1, d_0, 1\right]^{\mathrm{T}} = 0, \end{gathered} \tag{8}$$

where $\boldsymbol{U}$ is the unknown variables vector and $\boldsymbol{\Sigma}'$ is 12×12 matrix:

$$\boldsymbol{\Sigma}' = \begin{bmatrix} \boldsymbol{\Sigma} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{\Sigma} \end{bmatrix}, \tag{9}$$

Table 5 Polynomial system of the 4-bar mechanisms

Class notation	Unknown variables	Mobility	Polynomial system
RSCR	d_2, θ_2, θ_3	1	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_2, x_2, d_2, 1\right]^{\mathrm{T}} = 0$
RSPC	d_2, θ_3, d_1	1	$\boldsymbol{\Sigma} \left[d_2, d_1, 1\right]^{\mathrm{T}} = 0$
RSRC	θ_2, θ_3, d_1	1	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_1, x_2, d_1, 1\right]^{\mathrm{T}} = 0$
RSCC	$d_2, \theta_2, \theta_3, d_1$	2	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_2, x_2d_1, x_2, d_2, d_1, 1\right]^{\mathrm{T}} = 0$
RPSC	θ_2, d_3, d_1	1	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_3, x_2d_1, x_2, d_3, d_1, 1\right]^{\mathrm{T}} = 0$
RRSC	θ_2, θ_3, d_1	1	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_1, x_2, d_1, 1\right]^{\mathrm{T}} = 0$
RCSC	$\theta_2, d_3, \theta_3, d_1$	2	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_3, x_2d_1, x_2, d_3, d_1, 1\right]^{\mathrm{T}} = 0$
RSCP	θ_2, θ_3, d_3	1	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_3, x_2, d_3, 1\right]^{\mathrm{T}} = 0$
RCSP	d_2, θ_2, θ_3	1	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_2, x_2, d_2, 1\right]^{\mathrm{T}} = 0$
RRCS	θ_2, θ_3	1	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2, 1\right]^{\mathrm{T}} = 0$
RPCS	d_2, θ_3, d_3	1	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_3, x_2d_1, x_2, d_3, d_1, 1\right]^{\mathrm{T}} = 0$
RCRS	d_2, θ_2, θ_3	1	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_3, x_2d_1, x_2, d_3, d_1, 1\right]^{\mathrm{T}} = 0$
RCPS	d_2,θ_2,d_3	1	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_3, x_2d_1, x_2, d_3, d_1, 1\right]^{\mathrm{T}} = 0$
RCCS	$d_2, \theta_2, d_3, \theta_3$	2	$\boldsymbol{\Sigma}' \left[x_2^3, x_2^2, x_2d_3, x_2d_1, x_2, d_3, d_1, 1\right]^{\mathrm{T}} = 0$

where $\boldsymbol{O}$ is a 9×5 null matrix.

If $\boldsymbol{\Sigma}'\boldsymbol{U}^{\mathrm{T}} = 0$ is a homogeneous system of linear equations, then it is clear that 0 is a solution, which is known as the trivial solution. However, in Eq. (8), we have $\boldsymbol{U} \neq 0$, therefore, a non-trivial solution exists for our system if and only if $\det\boldsymbol{\Sigma}' = 0$.

3 Examples of the RSCR and RSPC mechanisms

3.1 Kinematic model of the RSCR spatial mechanism

Fig. 3 represents the schematic diagram of the RSCR mechanism. The definition of the Denavit-Hartenberg parameters (Denavit and Hartenberg, 1955) followed the same rules as in the case of open serial chains.

However, we close the loop by returning to the base. Therefore, the choice of the last coordinate system is no longer free as is the case for open serial chains. The parameters a_0, α_0, and d_0 are the fixed parameters for the base. Table 6 shows the Denavit-Hartenberg parameters for the RSCR mechanism where the variables are in bold face. The degrees of freedom of the spherical joint are given by the three angles α_2, α_3, and θ_3. In Fig. 3, the axis Z_2 is represented as parallel to Z_1, i.e., $\alpha_2 = 0$, only for the clarity of the figure, and does not alter the final loop closure equation.

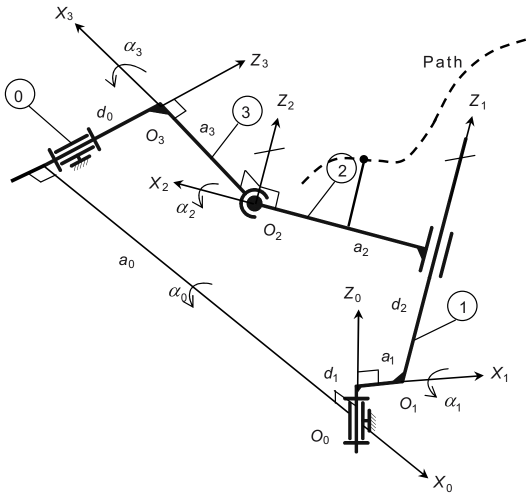

Fig. 3 The RSCR mechanism

Table 6 Denavit-Hartenberg parameters of the RSCR mechanism

i	a_i	d_i	α_i	$\boldsymbol{\theta_i}$
0	a_0	d_0	α_0	$\boldsymbol{\theta_0}$
1	a_1	d_1	α_1	$\boldsymbol{\theta_1}$
2	a_2	$\boldsymbol{d_2}$	0	$\boldsymbol{\theta_2}$
3	a_3	0	α_3	$\boldsymbol{\theta_3}$

The loop closure equation for this mechanism is given using homogeneous matrices as

$$ {}^0\boldsymbol{T}_1^1\boldsymbol{T}_2^2\boldsymbol{T}_3^3\boldsymbol{T}_4 = {}^0\boldsymbol{T}_4 = \begin{bmatrix} \boldsymbol{I} & 0 \\ 0 & 1 \end{bmatrix}. \qquad (10) $$

Eq. (10) is a 4×4 matrix equation that results in six independent scalar equations. The left term of Eq. (10) contains all the unknown geometric parameters of the mechanism, which are the Denavit-Hartenberg parameters a_i, α_i, d_i, and θ_i for $i = 0, 1, 2, 3$.

Using the loop closure equation of the mechanism, six scalar design equations are obtained at each position. The unknowns in these equations are the mechanism constant structural parameters and the joint variables $\theta_1, \theta_2, \theta_3$, and d_2, which vary from a position to another. Using the rearrangement of the equation presented in Section 2, Eq. (10) can be written as

$$^0\boldsymbol{T}_1^1\boldsymbol{T}_2 = {}^0\boldsymbol{T}_3^3\boldsymbol{T}_2, \tag{11}$$

which yields the following equations:

$$\begin{aligned} &a_2 \cos\theta_1 \cos\theta_2 - a_2 \sin\theta_1 \cos\alpha_1 \sin\theta_2 \\ &\quad + d_2 \sin\theta_1 \sin\alpha_1 + a_1 \cos\theta_1 \\ &\quad = -a_3 \cos\theta_0 - a_0, \qquad (12\text{a}) \\ &a_2 \sin\theta_1 \cos\theta_2 + a_2 \cos\theta_1 \cos\alpha_1 \sin\theta_2 \\ &\quad - d_2 \cos\theta_1 \sin\alpha_1 + a_1 \sin\theta_1 \\ &\quad = a_3 \sin\theta_0 \cos\alpha_0 - d_0 \sin\alpha_0, \qquad (12\text{b}) \\ &a_2 \sin\alpha_1 \sin\theta_2 + d_2 \cos\alpha_1 + d_1 \\ &\quad = -a_3 \sin\theta_0 \sin\alpha_0 - d_0 \cos\alpha_0. \qquad (12\text{c}) \end{aligned}$$

The input angle is θ_0, and the unknowns in the above equations are θ_1, d_2, and θ_2. In the general case, it is difficult to find explicitly θ_1 function of θ_0.

The objective is to obtain the input-output equation (θ_1 function of θ_0). This equation is obtained after consecutive eliminations of all other unknowns from the initial set of design equations. Using the same approach presented in Section 2, we can obtain the input-output function in a polynomial form, whose roots are used to compute the joint angle θ_1:

$$\begin{aligned} &\det\boldsymbol{\Sigma'} = f(\theta_0, \theta_1) = 0, \\ &f(\theta_0, \theta_1) = \\ &(2a_1a_0 + 2a_1a_3 \cos\theta_0) \cos\theta_1 + 2a_1d_0 \sin\alpha_0 \\ &+ (2a_1a_3 \cos\alpha_0 \sin\theta_0) \sin\theta_1 + d_0^2 \cos^2\alpha_0 \\ &-2a_3d_0 \cos\alpha_0 \sin\theta_0 \sin\alpha_0 - a_1^2 - a_0^2 - d_0^2 - a_3^2 \cos^2\theta_0 \\ &-a_3^2 \cos^2\alpha_0 - 2a_3a_0 \cos\theta_0 + a_3^2 \cos^2\theta_0 \cos^2\alpha_0 = 0. \end{aligned} \tag{13}$$

Eq. (13) is used to solve the problem of function generation for the RSCR mechanism when the axes Z_0 and Z_1 are parallel, i.e., the "cylindrical case", which corresponds to $\alpha_1 = 0$. The function generation problem for the RSCR mechanism was solved for this case in Ananthasuresh and Kramer (1994).

A second special case was considered in Ananthasuresh and Kramer (1994), i.e., the "cone case", which corresponds to the case when $a_1 = 0$, i.e., Z_0 and Z_1 intersect. In this work, we were not limited to a special case as shown in Ananthasuresh and Kramer (1994), but solved the general one for the RSCR mechanism (Appendix). The path generation problem was solved for this case (Ananthasuresh and Kramer, 1994), and could be formulated as follows: a given point on link (2), called the tracer point, has to generate a path as close as possible to a given path. For this case, we need to determine the orientation of the connecting link (2) to be able to determine the coordinates of the tracer point, P, with respect to the fixed frame. Indeed, for the given local coordinates of the tracer point on the connecting rod, one can find its global coordinates using the following equation:

$$\begin{bmatrix} X_P \\ Y_P \\ Z_P \\ 1 \end{bmatrix} = {}^0\boldsymbol{T}_2 \begin{bmatrix} x_P \\ y_P \\ z_P \\ 1 \end{bmatrix}, \tag{14}$$

where (X_P, Y_P, Z_P) are the coordinates of point P in the fixed reference frame (O_0, X_0, Y_0, Z_0), (x_P, y_P, z_P) are the coordinates of point P in the moving reference frame (O_2, X_2, Y_2, Z_2), and $^0\boldsymbol{T}_2$ is the homogeneous transformation matrix representing the position and orientation of link (2) with respect to the fixed coordinate system (link (0)).

3.2 Kinematic model of the RSPC spatial mechanism

Fig. 4 shows a 3D model of the RSPC mechanism. The axes of the revolute joints and cylindrical joints are both fixed. The input motion is the rotation of the revolute joint given by the angle θ_0 around Z_3. Table 7 shows the Denavit-Hartenberg parameters for the RSPC where the variables are in bold face. α_2 is taken equal to zero.

The loop closure equation for this mechanism is given using homogeneous matrices (Appendix) and the coordinates of point O_2 yield the three following

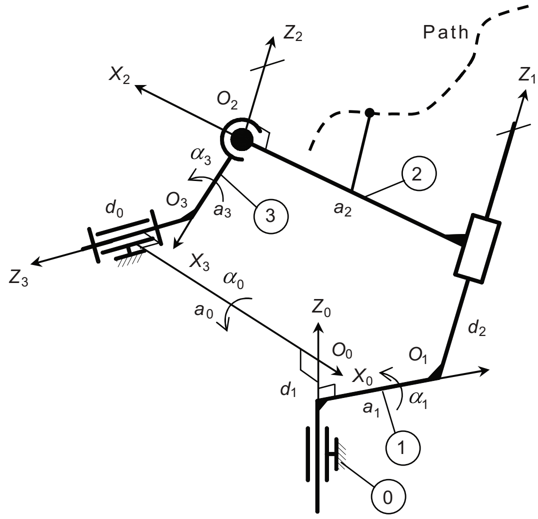

Fig. 4 The RSPC mechanism

Table 7 Denavit-Hartenberg parameters of the RSPC mechanism

i	a_i	d_i	α_i	θ_i
0	a_0	d_0	α_0	$\boldsymbol{\theta}_0$
1	a_1	$\boldsymbol{d_1}$	α_1	$\boldsymbol{\theta}_1$
2	a_2	$\boldsymbol{d_2}$	0	θ_2
3	a_3	0	α_3	$\boldsymbol{\theta}_3$

equations:

$$\begin{aligned}&a_2\cos\theta_1\cos\theta_2 - a_2\sin\theta_1\cos\alpha_1\sin\theta_2\\ &\quad + d_2\sin\theta_1\sin\alpha_1 + a_1\cos\theta_1\\ &\quad = -a_3\cos\theta_0 - a_0, \qquad (15a)\end{aligned}$$

$$\begin{aligned}&a_2\sin\theta_1\cos\theta_2 + a_2\cos\theta_1\cos\alpha_1\sin\theta_2\\ &\quad - d_2\cos\theta_1\sin\alpha_1 + a_1\sin\theta_1\\ &\quad = a_3\sin\theta_0\cos\alpha_0 - d_0\sin\alpha_0, \qquad (15b)\end{aligned}$$

$$\begin{aligned}&a_2\sin\alpha_1\sin\theta_2 + d_2\cos\alpha_1 + d_1\\ &\quad = -a_3\sin\theta_0\sin\alpha_0 - d_0\cos\alpha_0. \qquad (15c)\end{aligned}$$

The objective is to obtain the input-output equation. This equation is obtained using the same approach presented in Section 2, after consecutive eliminations of all other unknowns from the initial set of design equations. The determinant of the matrix $\boldsymbol{\Sigma}$ is set to zero and the input-output equation is obtained:

$$\det\boldsymbol{\Sigma} = f(\theta_0, \theta_1) = 0, \qquad (16)$$

$$\begin{aligned}f(\theta_0,\theta_1) =& \sin\alpha_1(a_2\cos\theta_2 + a_1 - a_3\sin\theta_1\sin\theta_0\cos\alpha_0\\ &+ d_0\sin\theta_1\sin\alpha_0 + a_3\cos\theta_1\cos\theta_0 + a_0\cos\theta_1) = 0. \qquad (17)\end{aligned}$$

For the case of the function generation, the objective function is based on Eq. (17). For the case of path generation, after solving Eq. (17) for all the variables of the mechanism, we determine the orientation of the connecting rod (2) to obtain the coordinates of the tracer point, P, with respect to the fixed frame. The coordinates of the tracer point on the connecting rod are given in the same manner as for the RSCR mechanism, i.e., Eq. (14).

The presented method has the advantage of being applicable to any four-bar spatial mechanism, since it is based solely on the loop closure equation and the classical Denavit-Hartenberg parameters. The use of the scheme of dialytic elimination of Section 2 offers more facility to yield to the algebraic formulation of the input-output function.

4 Results and discussion

4.1 Optimal synthesis

The design variables in the system are given by the set of parameters of the mechanism. These parameters are determined to satisfy a given task. In the case of the function generation, the design variables are

$$X_f^{\mathrm{RSCR}} = [a_0, d_0, \alpha_0, d_1, a_1, \alpha_1, a_2, a_3], \qquad (18)$$

$$X_f^{\mathrm{RSPC}} = [a_0, d_0, \alpha_0, a_1, \alpha_1, a_2, \theta_2, a_3]. \qquad (19)$$

For the problem of path generation, we need to add the local coordinates of the tracer point P and the location of the origin O to position in space the whole mechanism. The parameters needed to be added are

$$[x_P, y_P, z_P, X_0, Y_0, Z_0]. \qquad (20)$$

It is worth mentioning that the number of design variables is greatly reduced with respect to other geometrical descriptions found in the literature, hence improving the optimization process. Indeed, for the case of the RSCR mechanism, we needed only eight parameters to formulate the problem of function generation, whereas others (Ananthasuresh and Kramer, 1994) used 12 parameters to formulate the same problem. Moreover, this description allowed us to solve the general case for the RSCR mechanism, whereas only special cases were treated (Ananthasuresh and Kramer, 1994). The objective function to be optimized in the case of the function generation is given by

$$E = f(X) = \frac{1}{N_{\text{pt}}} \sum_{j=1}^{N_{\text{pt}}} \left[\left(\theta_1^j - \theta_1^{j\text{d}} \right)^2 \right], \qquad (21)$$

where N_{pt} is the number of synthesis points, and θ_1^j and $\theta_1^{j\text{d}}$ are the output angle and the desired output angle, respectively. In the case of the path generation, the function to be minimized is given by

$$E = f(X) = \frac{1}{N_{\text{pt}}} \sum_{j=1}^{N_{\text{pt}}} \left[(X_{P_j} - X_{P_j^{\text{d}}})^2 + (Y_{P_j} - Y_{P_j^{\text{d}}})^2 + (Z_{P_j} - Z_{P_j^{\text{d}}})^2 \right], \qquad (22)$$

where P_j and P_j^{d} are the positions of the points on the generated path and desired path, respectively. Therefore, E is the average error per point of the desired path.

4.2 Genetic algorithm optimization

The use of both deterministic and stochastic methods have been investigated for mechanisms synthesis. High complexity of the system introduces great difficulties in the use of deterministic methods (Chipperfield *et al.*, 1994). The GA is a stochastic global search method that mimics the metaphor of natural biological evolution. GAs operate on a population of potential solutions applying the principle of survival of the fittest to produce better and better approximations to a solution. At each generation, a new set of approximations is created by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using operators borrowed from natural genetics. This process leads to the evolution of populations of individuals that are better suited for their environment than individuals that were created, just as in natural adaptation. The GA differs substantially from more traditional searches and optimization methods (Vinod, 2004). The four most significant differences are: (1) GAs search a population of points in parallel, not a single point. (2) GAs do not require derivative information or other auxiliary knowledge; only the objective function and the corresponding fitness levels influence the directions of search. (3) GAs use probabilistic transition rules, not deterministic ones. (4) A number of potential solutions are obtained for a given problem and the choice of the final solution is left to the user.

It is worth mentioning that the last characteristic can be very useful in our case, since the user can choose among a set of potential solutions. Goldberg (1994) first introduced a GA that uses basic operators (selection, crossover, and mutation). However, it still can not be proven that simple (sometimes called canonical) GAs (selection+crossover+mutation) converge to the optimum of the fitness function (Lozano *et al.*, 1999) and have premature convergence and weak exploitation capabilities (Goldberg, 1994; Chelouah and Siarry, 2000; 2003; Renner and Ekart, 2003; Trabia, 2004). The main reason of premature convergence is a loss of diversity in the population. So an effective way to overcome the problem is to maintain the population diversity to explore new search domains continuously during the evolution process. The FL regulation technique (Laribi *et al.*, 2004) is regarded as an effective method to maintain population diversity to enhance the exploration of new search domains. So it effectively alleviates premature convergence and improves weak exploitation capacities of GAs. In our case, all the simulations were done using the following values for the genetic operators: the maximum number of generation MaxGen = 200; the number of individuals $N_{\text{ind}} = 100$; the mutation rate $F_1 = 0.7$; the shrinking mutation range factor $F_2 = 0.25$; the recombination rate $Opt_1 = 0.89$; the shrinking recombination range factor $Opt_2 = 0.12$; the generation gap $G_{\text{gap}} = 0.8$; and the bounding interval for each one of the design variables $I_x = [x_{\min}, x_{\max}]$.

In this work, we use the classical GA method to solve our problem. When this method fails to yield a satisfactory result, we use the GA-FL method developed in (Laribi *et al.*, 2004) to find the best choice of the bounding interval. The proposed method is made of a classical GA coupled with an FL controller (Tong-Tong, 1995). This controller monitors the variation of the design variables during the first run of the GA and modifies the initial bounding intervals to restart a second round of the GA. Fig. 5 shows the modified optimization algorithm and Fig. 6 shows the architecture of the FL controller. The process, which is the classical GA optimization, is triggered with an initial population chosen within the initial bounding intervals. The FL controller monitors the evolution of the different variables during the optimization and adjusts the bounding intervals for each design variable. These new intervals are then used to start a second round of optimization in order to improve the final result. The initial population is chosen randomly using the new calculated interval for each one of the variables. For more details, please refer to Laribi *et al.* (2004).

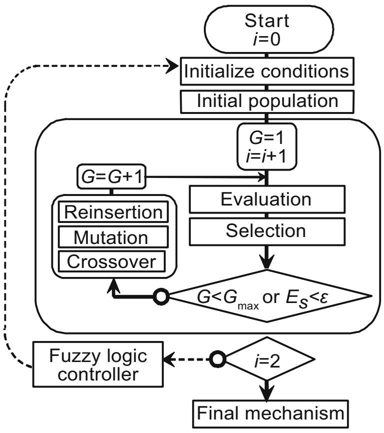

Fig. 5 Modified GA-FL optimization scheme (G: the number of generation; $G_{\max}$: the maximum number of generation; E_s: error)

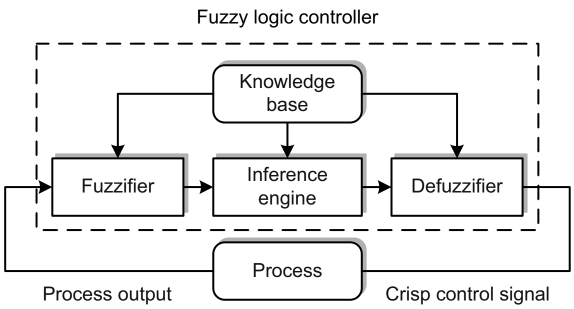

Fig. 6 Architecture of the fuzzy logic controller

4.3 Results

All the results are obtained on a processor of 1600 MHz and the programs are developed under MATLAB. The two problems of function synthesis and path synthesis are solved for the two proposed mechanisms, i.e., the RSCR and RSPC mechanisms. The calculation time varies between 15 and 45 s, depending on the complexity of the problem. All the angles are expressed in degrees. A small increment is used during the optimization, so that the branching problem is not encountered (Prentis, 1991). The function generation problem is independent of the unit used to express the length of different links. Whereas, the dimensions of the mechanism generating a specified path are expressed in the same unit as the one used to specify the different points of the path. Each problem is solved using either the GA method or the GA-FL method, and the well-known gradient method. Ananthasuresh and Kramer (1994) noted that in the optimization procedures involving the multi-variable constraint functions, the search for a feasible solution is a problem in itself. This problem is worse in optimal synthesis of mechanisms, due to the complexity of the problem in this case. Ananthasuresh and Kramer (1994) presented a method based on a mobility chart to choose a feasible solution to start the gradient method. In our simulation using the gradient method, the successful initial guess was taken from an intermediate solution given by the GA.

4.3.1 Simulation results for the RSCR mechanism

1. Function generation

This problem was solved by Ananthasuresh and Kramer (1994) using the gradient method. Their results are reproduced here and compared to those found by the GA method. The range of the design variables is given in Table 8.

Table 8 Bounding interval for the design variables

I	d_1	α_1	a_1	a_2	a_3	d_0	a_0	α_0
$x_{\min}$	0	0	0	0	0	0	0	0
$x_{\max}$	0	0	8	10	8	7	10	360

The two variables d_1 and α_1 are chosen to be equal to zero, to have the special case presented in Ananthasuresh and Kramer (1994). This case is called cylindrical because the axes of the revolute joint and cylindrical joint are parallel (Ananthasuresh and Kramer, 1994). The function to generate, $\theta_1 = f(\theta_0)$, is given by Eq. (13).

Table 9 contains the user-defined function, θ_1^{d}. θ_1^{grad} is the function generated by the optimal mechanism using the gradient method (Ananthasuresh and Kramer, 1994). θ_1^{GA} is the function generated by the optimal mechanism using the GA method. In this case again, the GA method yielded more accurate results than the gradient method.

Table 10 presents the optimal dimensions of the RSCR mechanism using the GA and the gradient method (Ananthasuresh and Kramer, 1994).

Table 9 Desired function to generate and the obtained solutions using the GA method and the gradient method

θ_0	θ_1^{d}	θ_1^{grad}	θ_1^{GA}
0	0	0	0
10	15	13.96	14.22
20	25	24.25	25.37
40	45	38.42	42.98
60	57	50.30	57.42
85	70	68.42	73.82
110	90	91.48	90.69
125	105	106.43	102.37
140	120	120.50	117.28
150	130	128.69	131.90
E		2.13	1.95

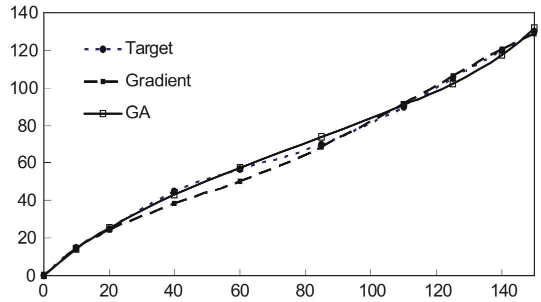

Fig. 7 Optimal solutions found by the GA method and the gradient method, $\boldsymbol{\theta_1 = f(\theta_0)}$

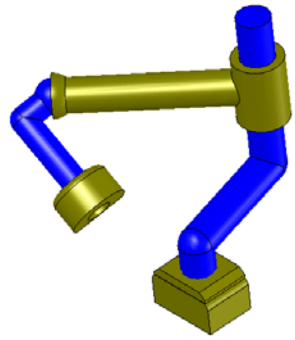

Fig. 8 Optimal RSCR mechanism for function generation problem

Note that the two mechanisms are completely different. This result could be explained that the GA method searches for the best solution within an interval whereas the gradient method is a direct search method and the final solution is conditioned by the initial guess. The input-output function in this case is relatively simple, which is why the GA yielded a good solution and we felt no need to use the GA-FL method (Fig. 7). Fig. 8 shows a solid model of the obtained mechanism.

2. Path generation

The present example is also taken from Ananthasuresh and Kramer (1994). Due to the complexity of the problem, only the special case ($a_1 = 0$) was considered in this reference. This case is called the conical case because the two axes Z_0 and Z_1 intersect. In our work, thanks to the geometric description presented earlier, we were able to solve the problem in the general case. The design variables are the same as that in the previous case (Table 8) to which we have to add the local position of the tracer point. These variables and ranges are given in Table 11.

Table 11 also shows the results using the GA method and the GA-FL method. One can see the effectiveness of the GA-FL method in reducing the final error from 1.01 to 0.25.

The gradient method was used and the results are shown in Table 11. Two sets of starting points

Table 10 Otpimal solutions found by the GA method and the gradient method

Method	a_0	d_0	α_0	d_1	a_1	α_1	a_2	a_3	E
Gradient*	2.62	4.89	0	5.02	0	6.97	8.20	8.20	2.13
GA	0.50	6.97	0	5.01	0	7.99	0.55	0.55	1.95

*Ananthasuresh and Kramer, 1994

Table 11 Bounding intervals, the optimal solution by the GA-FL method and gradient method for the path generation for the RSCR mechanism

Method		d_1	α_1	a_1	a_2	a_3	d_0	a_0	α_0	x_P	y_P	z_P	X_0	Y_0	Z_0	E
GA	$\boldsymbol{x}_{\min}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	$\boldsymbol{x}_{\max}$	10	360	10	10	10	10	10	360	10	10	10	10	10	10	
	X	4.1	103.8	9.08	8.81	1.56	7.05	9.07	264.68	2.18	6.27	2.28	5.2	7.54	3.22	1.01
GA-FL	$\boldsymbol{x}_{\min}^*$	0	0	0	0	0	0	0	0	-8.10	-7.19	-1.55	-7.24	-9.67	-1.97	
	$\boldsymbol{x}_{\max}^*$	21.93	360	17.92	20.28	11.76	21.44	20.27	360	15.26	16.17	21.47	16.12	13.82	21.63	
	X^*	5.68	105.89	6.68	9.08	1.21	3.99	9.96	229.17	5.04	2.74	3.47	1.83	3.98	9.79	0.25
Gradient	X^{grad}	0	271.97	10	8.21	1	10	10	101.73	0.08	10	2.61	9.94	2.57	0	0.76

are used for this method. The first one did not yield a solution, whereas the second one, chosen as an intermediate solution of the GA, yielded a solution (Table 11) with an error $E = 0.76$, which is greater than that given by the GA-FL method ($E = 0.25$). Fig. 9 shows the desired path along with the path generated by the GA-FL method.

These two examples show clearly the efficiency of the GA method in solving both the function and the path generation problems. In both cases, the GA method proved to be more accurate than the gradient method. It is also worth mentioning that the gradient method has the problem of choosing the starting feasible solution to initiate the optimization process. This problem was solved here by choosing an intermediate solution of the GA method as a starting point for the gradient method. Moreover, the minimum number of parameters used to describe the geometry of the mechanism, proved to yield a simple geometric model, helping in the optimization process.

In the next section, another type of mechanisms, e.g., the RSPC mechanism, is considered. To the best of our knowledge, this type of mechanisms has not been used before in a synthesis problem. In this case the function and path generation problems were also solved. The GA method did not yield acceptable results, which is why we proposed the use of the modified GA-FL method. The obtained results are then compared to those found by the gradient method.

4.3.2 Simulation results for the RSPC mechanism

1. Function generation

Table 12 shows the dimensions of the optimal RSPC mechanism given by the design vector $\boldsymbol{X}$. The error found in the case of the GA method, i.e., $E = 1.84$ is not acceptable. The modified GA-FL method (Laribi *et al.*, 2004) was then applied to improve the final results. Table 12 also shows the new bounding intervals and the dimensions of the final result, $\boldsymbol{X}^*$. The error obtained for this mechanism, i.e., $E = 1.193$, is improved compared to that given by the simple GA method. Fig. 10 represents the desired function to generate and the obtained function using the optimal mechanisms.

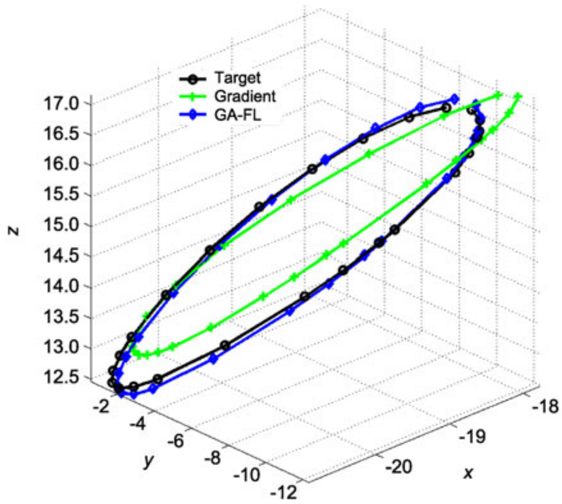

Fig. 9 Target path along with obtained paths using the GA-FL method and the gradient method for RSCR mechanism

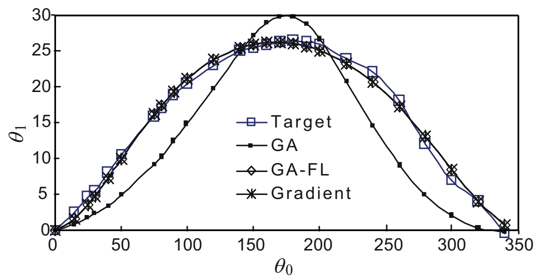

Fig. 10 Target function along with the generated functions using the GA method, the GA-FL method, and the gradient method

Table 12 Bounding intervals, the optimal solution by the GA-FL method and gradient method for the function generation for the RSPC mechanism

Method		a_0	d_0	α_0	a_1	α_1	a_2	θ_2	a_3	E
	$\boldsymbol{x}_{\min}$	0	0	0	0	0	0	0	0	
GA	$\boldsymbol{x}_{\max}$	10	10	360	10	360	7	360	10	
	X	5.30	1.27	4.33	0.07	304.14	2.70	179.39	1.98	1.84
	$\boldsymbol{x}^*_{\min}$	0	0	0	0	0	0	0	0	
GA-FL	$\boldsymbol{x}^*_{\max}$	17.97	12.67	360	11.98	360	9.11	360	12.56	
	X^*	7.27	1.38	359.87	0	317.57	5.71	171.11	1.37	1.193
Gradient	X^{grad}	8.41	1.43	349.06	0.32	359.77	6.92	172.39	1.57	1.194

Fig. 11 shows a solid model of the obtained mechanism. When compared to the error found by the gradient method, the obtained mechanism generates less error. Moreover, the gradient method suffers from the choice of the initial guess. Indeed, due to the “bad choice” of the initial guess, the gradient method does not even converge. In our work, the initial guess is given by an intermediate solution of the GA method and the corresponding optimal mechanism is given by Table 13.

Although very different in dimensions, the two mechanisms found by the GA-FL method and the gradient method yield the same input-output relation. This fact could be explained by the multiple possible solutions for the synthesis problem. Note that the GA-FL yielded a solution with the same precision as that given by the gradient method. This result is encouraging because most of the previous works on GAs in mechanism synthesis (Kunjur and Krishnnamuty, 1995; Cabrera *et al.*, 2002) came to the conclusion that GAs have several advantages but they always fall short from getting the same precision as deterministic methods. Actually, the GA-FL yields a whole set of candidate solutions and that presented in Table 12 is the best one among them. The other obtained solutions in the final population could be used if the best one is not adequate for any other reason. Although very different in dimensions, the two mechanisms found by the GA-FL and gradient method, yield the same input-output relation. This fact could be explained by the multiple possible solutions for the synthesis problems.

2. Path generation

The design variables are the same as the previous case (Table 12) to which we add the local position of the tracer point. These variables and ranges are given in Table 13.

The path to generate is given by a set of 49 points (Fig. 12). Fig. 13 shows the desired path along with the path generated by the GA-FL method and Fig. 14 shows a solid model of the obtained mechanism.

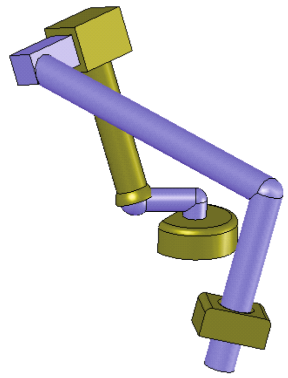

Fig. 11 Optimal RSPC mechanism for function generation problem

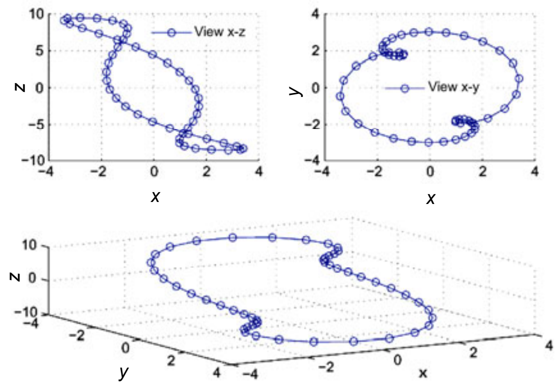

Fig. 12 3D desired path for the RSPC mechanism

The results obtained by the two methods (Table 13) show clearly that the GA-FL method is a viable alternative in solving synthesis problems for spatial mechanisms. Moreover, the GA-FL method does not need a starting point and is very simple to implement. It is interesting to note that the solution presented in Table 13 is actually one

Table 13 Bounding intervals, the optimal solution by the GA method and the GA-FL method for the path generation for the RSPC mechanism

Method		a_0	d_0	α_0	a_1	α_1	a_2	θ_2	a_3	x_p	y_p	z_p	X_0	Y_0	Z_0	E
GA	$\boldsymbol{x}_{\min}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	$\boldsymbol{x}_{\max}$	10	10	360	10	360	7	360	10	10	10	10	10	10	10	
	X	1.61	4.42	297.9	0.62	31.55	2.88	92.42	8.42	7.48	2.77	1.1	2.84	1.61	3.89	2.05
GA-FL	$\boldsymbol{x}^*_{\min}$	0	0	141.74	0	139.76	0	20.96	0	0.62	-2.28	3.74	0.26	0.64	-3.51	
	$\boldsymbol{x}^*_{\max}$	3.99	11.73	441.6	7.45	447.36	3.99	301.53	13.54	8.58	5.33	12.07	8.05	8.07	4.9	
	X^*	0.335	5.83	288.33	1.73	255.55	2.19	214.49	9.51	5.27	3.13	6.69	7.3	4.92	2.93	0.263

individual of a whole population. Each individual of this population could be considered as a potential solution for this problem and the choice of the final solution is left to the designer.

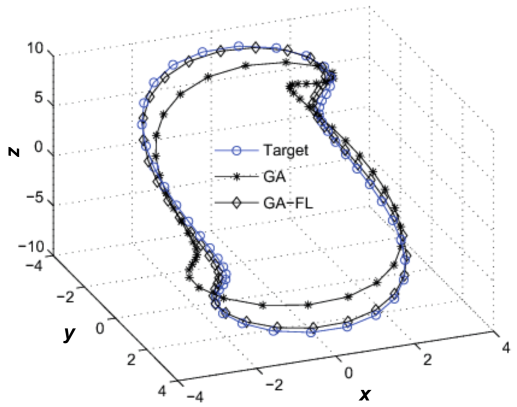

Fig. 13 Desired path and the obtained path using the GA-FL method for the RSPC mechanism

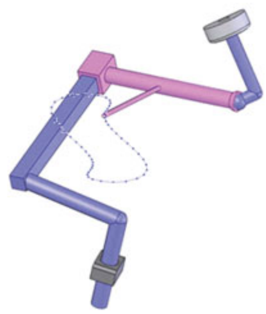

Fig. 14 Optimal RSPC mechanism for path synthesis

5 Conclusions

In this work, a systematic approach is presented to obtain the input-output equations of a single loop 4-bar spatial mechanisms. We used the scheme of dialytic elimination along with Denavit-Hartenberg parameters to obtain the algebraic formulation of the input-output function. A GA was applied to solve the problem of the spatial mechanisms synthesis. Two types of mechanisms, e.g., RSCR and RSPC, illustrated the application of the GA to solve the problem of function generation and the path generation. We also showed that the number of design variables is greatly reduced with respect to other geometrical description found in the literature, hence improving the optimization process. Indeed, we were able to solve the general case for the RSCR mechanism, whereas only special cases were treated in the literature. The results obtained in this work show that GAs, combined with an adequate description of the mechanism, are well suited for spatial mechanism synthesis problems and do not have any difficulties inherent to the choice of the initial feasible guess nor a problem of convergence as it is the case for deterministic methods.

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Appendix

The input-output equation of the RSCR mechanism is given by

$$\cos^2\theta_1 \begin{pmatrix} a_3^2\cos\alpha_0^2 + a_3^2\cos^2\alpha_1\cos^2\theta_0\cos^2\alpha_0 \\ +a_0^2\cos^2\alpha_1 - d_0^2\cos^2\alpha_1 \\ -2a_3d_0\ \sin\theta_0\cos\alpha_0\sin\alpha_0 - a_0^2 \\ -a_3^2\cos^2\alpha_0\cos^2\theta_0 - a_3^2\cos\alpha_1^2\cos^2\alpha_0 \\ +2a_3d_0\ \cos^2\alpha_1\sin\theta_0\sin\alpha_0\cos\alpha_0 \\ -2a_3a_0\ \cos\theta_0 + d_0^2 - a_3^2\cos^2\theta_0 \\ +2a_3a_0\ \cos\theta_0\cos^2\alpha_1 \\ +d_0^2\cos^2\alpha_1\cos^2\alpha_0 \\ +a_3^2\cos^2\theta_0\cos^2\alpha_1 - d_0^2\cos^2\alpha_0 \end{pmatrix}$$

$$+\sin\theta_1\cos\theta_1 \begin{pmatrix} -2a_3^2\ \cos\theta_0\sin\theta_0\cos\alpha_0\cos^2\alpha_1 \\ -2\,a_3\cos\theta_0\sin\alpha_0 d_0 - 2\,a_0d_0\sin\alpha_0 \\ +2d_0\,a_0\sin\alpha_0\cos^2\alpha_1 \\ +2\,a_3d_0\cos\theta_0\sin\alpha_0\cos^2\alpha_1 \\ +2a_3^2\ \cos\theta_0\sin\theta_0\cos\alpha_0 \\ +2\,a_0a_3\sin\theta_0\cos\alpha_0 \\ -2\,a_0a_3\sin\theta_0\cos\alpha_0\cos^2\alpha_1 \end{pmatrix}$$

$$+\sin\theta_1 \begin{pmatrix} 2a_3\ \sin\alpha_1\cos^2\theta_0\cos\alpha_1\sin\theta_0\sin\alpha_0 \\ +2a_3d_0\ \sin\alpha_1\cos\theta_0\cos\alpha_1\cos\alpha_0 \\ +2a_3\ \sin\alpha_1\cos\theta_0\cos\alpha_1 d_1 \\ +2a_0d_0\ \sin\alpha_1\cos\alpha_1\cos\alpha_0 \\ +2a_0d_1\ \sin\alpha_1\cos\alpha_1 \\ +2a_0a_3\ \sin\alpha_1\cos\alpha_1\sin\theta_0\sin\alpha_0 \end{pmatrix}$$

$$+\cos\theta_1 \begin{pmatrix} 2\ a_3^2\ \sin\alpha_1\cos\alpha_0\cos\alpha_1\sin\alpha_0 \\ -2d_0a_3\ \sin\alpha_1\cos\alpha_1\sin\theta_0 \\ +4a_3d_0\ \sin\alpha_1\sin\theta_0\cos^2\alpha_0\cos\alpha_1 \\ +2d_1a_3\ \sin\alpha_1\sin\theta_0\cos\alpha_0\cos\alpha_1 \\ -2d_1d_0\ \sin\alpha_1\sin\alpha_0\cos\alpha_{\ 1} \\ -2a_3^2\ \sin\alpha_1\cos\alpha_0\cos\alpha_1\sin\alpha_0\cos^2\theta_0 \\ -2d_0^2\ \sin\alpha_1\sin\alpha_0\cos\alpha_1\cos\alpha_0 \end{pmatrix}$$

$$
\begin{aligned}
&- a_0^2 \cos^2\alpha_1 - 2a_3a_0 \cos\theta_0 \cos^2\alpha_1 \\
&+ a_3^2 \cos^2\alpha_1 \cos^2\theta_0 \cos^2\alpha_0 \\
&- d_1^2 + 2\, d_1 d_0 \cos^2\alpha_1 \cos\alpha_0 + a_3^2 \cos^2\theta_0 \\
&+ 2a_3 d_0 \cos^2\alpha_1 \sin\theta_0 \sin\alpha_0 \cos\alpha_0 \\
&- 2a_3^2 \cos^2\theta_0 \cos^2\alpha_1 - d_0^2 + d_0^2 \cos^2\alpha_1 \cos^2\alpha_0 \\
&+ d_1^2 \cos^2\alpha_1 + a_3^2 \cos^2\alpha_1 + a_2 \\
&+ 2d_1\, a_3 \cos^2\alpha_1 \sin\theta_0 \sin\alpha_0 \\
&- 2\, d_1 a_3 \sin\theta_0 \sin\alpha_0 - a_3^2 \cos^2\alpha_1 \cos^2\alpha_0 \\
&- 2\, d_0 d_1 \cos\alpha_0 - a_3^2 = 0,
\end{aligned}
$$

where θ_0 is the input angle and θ_1 is the output angle.

$$
\begin{aligned}
A\cos^2\theta_1 + &B\cos\theta_1 + C\sin\theta_1 \\
&+ D\cos\theta_1 \sin\theta_1 + E = 0.
\end{aligned}
$$

Note that $t_1 = \tan(\theta_1/2)$, the above equation yields a 4th degree polynomial in t_1:

$$A_1 t_1^4 + A_2 t_1^3 + A_3 t_1^2 + A_4 t_1 + A_5 = 0.$$

This equation was solved using the MATLAB© command "root".