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Modified particle swarm optimization for optimum design of spread footing and retaining wall

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Abstract: This paper deals with the economically optimized design and sensitivity of two of the most widely used systems in geotechnical engineering: spread footing and retaining wall. Several recent advanced optimization methods have been developed, but very few of these methods have been applied to geotechnical problems. The current research develops a modified particle swarm optimization (MPSO) approach to obtain the optimum design of spread footing and retaining wall. The algorithm handles the problem-specific constraints using a penalty function approach. The optimization procedure controls all geotechnical and structural design constraints while reducing the overall cost of the structures. To verify the effectiveness and robustness of the proposed algorithm, three case studies of spread footing and retaining wall are illustrated. Comparison of the results of the present method, standard PSO, and other selected methods employed in previous studies shows the reliability and accuracy of the algorithm. Moreover, the parametric performance is investigated in order to examine the effect of relevant variables on the optimum design of the footing and the retaining structure utilizing the proposed method.

Key words: Particle swarm optimization (PSO), Spread footing, Retaining wall, Sensitivity analysis
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1 Introduction

In the field of geotechnical structure, the design of a safe and economical structure is one of the main concerns of geotechnical engineers. Economy in design together with the desire for safety can be achieved through an optimization procedure, which is an inherent part of all engineering practice. The optimization problem can be addressed using either deterministic or heuristic methods. In deterministic algorithms, the objective function must be differentiable or continuous or the reasonable region must be convex. Conversely, the heuristic method is not restricted in the aforementioned manner. The heuristic approach cannot always guarantee the best global solutions, but is often found to obtain a fast and near global optimal solution. Heuristic approaches contain several algorithms such as genetic algorithm, simulated annealing, ant colony optimization, and particle swarm optimization. These methods are particularly useful for complex optimization problems, for which deterministic approaches are often unable to find the solution within a reasonable amount of time. Accordingly, numerous studies have been undertaken in recent years to implement the heuristic algorithms to solve civil engineering problems. Lee and Geem (2004) implemented a harmony search algorithm for structural optimization problems. Cheng *et al.* (2007) applied six heuristic optimization algorithms to slope stability analysis. Application of four heuristic methods for design of reinforced concrete bridge frames was examined by Perea *et al.* (2008). Paya-Zaforteza *et al.* (2009) used simulated annealing for optimization of reinforced concrete frames.

This paper is concerned with the optimization of two common types of geotechnical structures: spread footing and retaining wall. Optimum design of these structures using both deterministic and heuristic approaches has been the subject of a number of studies (Saribas and Erbatur, 1996; Basudhar *et al.*, 2008; Wang and Kulhawy, 2008; Yepes *et al.*, 2008; Ahmadi-Nedushan and Varaee, 2009; Wang, 2009). However, the method presented here for the optimization of spread footing and retaining wall, is a new approach using the particle swarm optimization (PSO) algorithm.

PSO was introduced by Kennedy and Eberhart (1995), and is a kind of random search algorithm that simulates natural evolutionary processes, by mimicking the social behavior of flocks (swarms) of birds and insects (particles). Compared with other evolutionary computation algorithms, like genetic algorithms, PSO has some advantages including simple implementation, small computational load, and fast convergence. Therefore, it is efficient for solving many problems for which it is difficult to find accurate mathematical models. Despite these advantages, the PSO algorithm easily degrades into local minima when solving complex optimization problems. Recently, various studies have been undertaken to overcome this weakness and to improve the performance of the standard PSO. Trials for this approach include guaranteed convergence PSO by van den Bergh and Engelbrecht (2002), adaptive PSO by Xie *et al*. (2002), fully informed PSO by Mendes *et al*. (2004), PSO with disturbance term by He and Han (2006), θ-PSO by Zhong *et al.* (2008), etc.

In the present study, we propose a modified particle swarm optimization (MPSO) for optimum design of spread footing and retaining wall. Three numerical examples are presented to illustrate the effectiveness and robustness of the new method. The results show that the presented algorithm has a fast convergence rate with a high degree of accuracy. In addition, a sensitivity analysis is carried out using the new procedure to investigate the effect of the most relevant parameters on the optimum design of the tested structures. Although the proposed method is applied to spread footing and retaining wall, it is a general optimization procedure that can be easily adapted to other types of engineering optimization problems.

2 Particle swarm optimization

Particle swarm optimization is a population based stochastic optimization method. It seeks the optimal solution from a population of moving particles, based on a fitness function. Each particle represents a potential answer, and has a position (X_i^k) and a velocity (V_i^k) in the problem space. Each particle keeps a record of its individual best position (P_i^k), which is associated with the best fitness it has achieved thus far, at any step in the solution. This value is known as pbest. Moreover, the optimum position between all the particles obtained so far in the swarm is stored as the global best position (P_{g}^k). This location is called gbest. The velocity of each particle and its new position will be updated according to the following equations and Fig. 1 (Shi and Eberhart, 1998):

$$X_i^{k+1} = X_i^k + V_i^{k+1}, \tag{1}$$

$$V_i^{k+1} = wV_i^k + c_1 r_1 (P_i^k - X_i^k) + c_2 r_2 (P_{\mathrm{g}}^k - X_i^k), \quad i = 1, 2, 3, \ldots, N, \tag{2}$$

where w is an inertia weight that controls a particle's exploration during a search, c_1 and c_2 are positive numbers explaining the weight of the acceleration terms that guide each particle toward the individual best and the swarm best positions respectively, r_1 and r_2 are uniformly distributed random numbers in the range of 0 to 1, and N is the number of particles in the swarm. The inertia weighting function in Eq. (2) is usually calculated by

$$w = w_{\max} - (w_{\max} - w_{\min}) \times k / G, \tag{3}$$

where $w_{\max}$ and $w_{\min}$ are the maximum and minimum values of w, G is the maximum number of iterations, and k is the current iteration number.

The first term in Eq. (2), wV_i^k, enables each particle to perform a global search by exploring a new search space. The last two terms in Eq. (2), $c_1 r_1 (P_i^k - X_i^k)$ and $c_2 r_2 (P_{\mathrm{g}}^k - X_i^k)$, enable each particle to perform a local search around its individual best position (pbest) and the swarm best position (gbest).

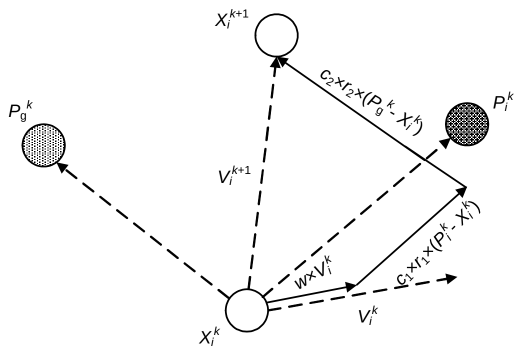

Fig. 1 Position update of particles in particle swarm optimization

3 Modified particle swarm optimization

This study proposes an MPSO based on PSO with passive congregation (PSOPC) introduced by He *et al.* (2004). The theory of PSOPC introduces an additional part at the end of the velocity update formula, Eq. (2), known as the passive congregation part. The basic idea is that individuals need to monitor both their environment and their surroundings. Thus, each group member receives a multitude of information from other members, which may decrease the possibility of a failed attempt at detection or a meaningless search. This kind of information exchange can be realized by a model called passive congregation. The updated velocity equation in PSOPC is defined (He *et al.*, 2004):

$$V_i^{k+1} = wV_i^k + c_1 r_1 (P_i^k - X_i^k) + c_2 r_2 (P_g^k - X_i^k) + c_3 r_3 (R_i^k - X_i^k), \quad (4)$$

where R_i^k is a particle selected randomly from the swarm, c_3 is the passive congregation coefficient, and r_3 is a uniform random sequence in the range of 0 to 1. It must be noted that each particle obtains passive additional information from another particle that is selected at random. This could increase the diversity of the swarm and lead to a better result.

To improve the search performance of the algorithm, this study introduces a new velocity update equation by applying a time varying restriction factor in Eq. (4):

$$V_i^{k+1} = \psi[wV_i^k + c_1 r_1 (P_i^k - X_i^k) + c_2 r_2 (P_g^k - X_i^k) + c_3 r_3 (R_i^k - X_i^k)], \quad (5)$$

where ψ is a restriction factor used to control and constrict velocities, defined as follows:

$$\psi = \psi_{\max} - (\psi_{\max} - \psi_{\min}) \times \exp[-(4k/G)^2], \quad (6)$$

where $\psi_{\max}$ and $\psi_{\min}$ are the maximum and minimum values of ψ. The restriction factor greatly elevates the abilities of exploration (expanding global investigation of the search space) and exploitation (finding the optima around a good solution) of the algorithm. Eqs. (5) and (6) facilitate an initial global search with a relatively large value of ψ during early iterations, which allows the particles to move around the search space instead of moving toward pbest. The gradual reduction of the restriction factor over the iterations, decreases the amplitude of a particle's oscillations and narrows down the algorithm exploration. It encourages the particles to concentrate the search effort on the best solutions found so far and to converge to the global optima in the latter part of the optimization.

4 Constraint optimization using MPSO

The general constrained nonlinear optimization problem can be defined as

$$\begin{aligned} &\text{minimize } f(\boldsymbol{X}) \\ &\text{subject to} \\ &g_i(\boldsymbol{X}) \le 0,\ i=1, 2, \ldots, p, \\ &h_j(\boldsymbol{X})=0,\ j=1, 2, \ldots, m, \\ &L_k \le X_k \le U_k,\ k=1, 2, \ldots, n, \end{aligned} \quad (7)$$

where $\boldsymbol{X}$ is the n-dimensional vector of design variables, $f(\boldsymbol{X})$ is the objective function, $g(\boldsymbol{X})$ and $h(\boldsymbol{X})$ are the inequality and equality constraints, and L_k and U_k are the lower and upper bound constraints.

A careful inspection of the MPSO algorithm reveals that only the objective function is used to check if the new particle position is more favorable than the previous one. A number of approaches have been taken in the evolutionary computing field to execute the constraint handling. These methods can be grouped into four categories: methods that preserve the feasibility of solutions, penalty-based methods, methods that clearly distinguish between feasible and unfeasible solutions, and hybrid methods. The most common approach is the penalty method, which adds a penalty to the objective function to decrease the quality of unfeasible solutions.

In this work, the penalty-based method proposed by Parsopoulos and Vrahatis (2002) was used. In this approach, the constraint optimization problem in Eq. (7) is replaced with the alternative unconstrained problem as follows:

$$F(\boldsymbol{X}) = f(\boldsymbol{X}) + r\sum_{i=1}^{p+m} q_i^l(\boldsymbol{X}), \quad (8)$$

where $F(\boldsymbol{X})$ is the penalized objective function, $f(\boldsymbol{X})$ is the original objective function of the problem in Eq. (7), r is a penalty factor, and l is the power of the penalty function. The function $q_i(\boldsymbol{X})$ is a relative violation function of the constraints, and is given as

$$q_i(\boldsymbol{X}) = \begin{cases} \max\{0, g_i(\boldsymbol{X})\}, & 1 \le i \le p, \\ \left|h_{i-p}(\boldsymbol{X})\right|, & p+1 \le i \le p+m. \end{cases} \quad (9)$$

The parameters r and l are problem dependent, and r should be a suitably large positive constant. In the present study, the values set for r and l were 1000 and 2, respectively.

5 Optimum cost design of spread footing

Spread footings are the most widely used type of foundations because they are usually more economical than the others. A minimum amount of equipments and skill are required for the construction of spread footings. Furthermore, the conditions of the footings and the supporting soil can be readily examined. Design requirements for these structures fall into three classes: structural design constraints, geotechnical design constraints, and economics. Traditional methods for the design of these structures are based on trial and error. In the traditional methods, a trial design is proposed and checked against the geotechnical and structural requirements, which is followed by revision of the trial design, if necessary. Moreover, there is no guarantee that the final design is an economically optimum design. However, in the case of optimum design all requirements are considered simultaneously, and it is guaranteed that the final design is optimized economically. In order to optimize footing design using the proposed MPSO, it is necessary to define the design variables, design constraints, and the objective function. A brief description of these parameters is presented in the following sections.

5.1 Design variables

The design variables chosen for the formulation are related to the cross-sectional dimensions of the footing and various reinforcing steel areas. Six design variables are considered, including the length of footing (Y_1), width of footing (Y_2), thickness of footing (Y_3), depth of embedment (Y_4), long direction reinforcement (Y_5), and short direction reinforcement (Y_6) (Fig. 2).

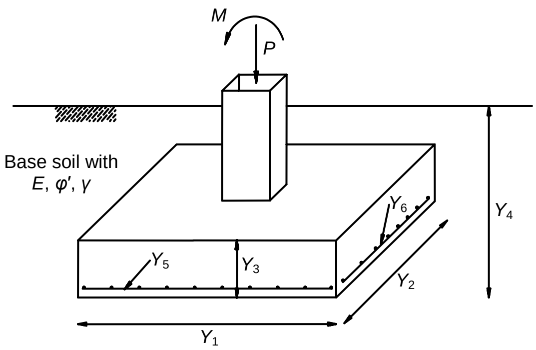

Fig. 2 Reinforced spread footing
E, φ', and γ: Young's modulus, effective friction angle, and unit weight of soil, respectively; P and M: axial load and bending moment applied on the footing

5.2 Design constraints

According to Bowles (1982), ACI 318-05 (2005), and Budhu (2006), the design constraints may be classified as geotechnical and structural requirements summarized in Table 1. These requirements represent the failure modes as a function of design variables.

5.3 Objective function

The total cost of the spread footing is considered as the objective function in the analysis. The cost function may be expressed in the following form:

$$f(\boldsymbol{Y}) = C_\mathrm{c}V_\mathrm{c} + C_\mathrm{e}V_\mathrm{e} + C_\mathrm{b}V_\mathrm{b} + C_\mathrm{f}A_\mathrm{f} + C_\mathrm{s}W_\mathrm{s}, \quad (10)$$

where C_c, C_e, C_b, C_f, and C_s show the unit price of concrete, excavation, backfill, formwork, and

Table 1 Failure modes of spread footing

IC	Failure mode	Constraint
$g_1(\boldsymbol{Y})$	Settlement of footing	$\delta \le \delta_{all}$
$g_2(\boldsymbol{Y})$	Bearing capacity	$q_{max} \le q_{ult}/FS$
$g_3(\boldsymbol{Y})$	Flexure failure	$M_u/\phi_M \le M_n$
$g_4(\boldsymbol{Y})$	Punching shear failure	$V_u / \phi_V \le \min \times \sqrt{f_c'} b_0 d$
$g_5(\boldsymbol{Y})$	One way shear failure	$V_u / \phi_V \le \sqrt{f_c'} bd / 6$
$g_6(\boldsymbol{Y})$	Minimum depth of embedment	$0.5 \le Y_4$
$g_7(\boldsymbol{Y})$	Maximum depth of embedment	$Y_4 \le 2$
$g_8(\boldsymbol{Y})$	Minimum steel area for long direction	$Y_5 \ge 0.002 Y_1 Y_3$
$g_9(\boldsymbol{Y})$	Minimum steel area for short direction	$Y_6 \ge 0.002 Y_2 Y_3$

IC: inequality constraint; δ: immediate settlement of footing; δ_{all}: allowable settlement of footing, 40 mm; q_{max}: maximum intensity of soil pressure; q_{ult}: ultimate bearing capacity; FS: factor of safety, 3.0; M_u and M_n: ultimate and nominal bending moments, respectively; ϕ_M: flexure strength reduction factor, 0.9; V_u: upward ultimate shear force; ϕ_V: shear strength reduction factor, 0.85; min: $\min\left\{(1+2/\beta_c)/6, \left(\alpha_s d / b_0 + 2\right)/12, 1/3\right\}$, where β_c is the ratio of the long side to the short side of the loaded area, and values of α_s are given as follows: 40 for interior, 30 for edge, and 20 for corner columns; f_c': compressive strength of concrete; b_0: perimeter of critical section of footing; d: effective depth of footing; b: width of resisting shear area

reinforcement, respectively. The unit prices considered here are presented in Table 2 (Wang and Kulhawy, 2008). In addition, V_c, V_e, and V_b denote the volumes of concrete, excavation, and backfill, A_f shows the area of formwork, and W_s indicates the weight of steel. These quantities are functions of design parameters (Y_1, Y_2, …, Y_6) and can be calculated explicitly.

Table 2 Spread footing assembly unit price (Wang and Kulhawy, 2008)

Work task	Price
Excavation	25.16 USD/m^3
Formwork	51.97 USD/m^2
Reinforcement	2.16 USD/kg
Concrete	173.96 USD/m^3
Compacted backfill	3.97 USD/m^3

By substituting the objective function of Eq. (10) and the inequality constraints of Table 1 into Eq. (8), the main objective function (fitness function) for cost optimization of spread footing using MPSO is defined as follows:

$$F(\boldsymbol{Y}) = f(\boldsymbol{Y}) + r\sum_{i=1}^{9}\max\left\{0, g_i(\boldsymbol{Y})\right\}^{l}. \qquad (11)$$

6 Optimum cost design of retaining wall

The retaining wall is a soil-structure system intended to support earth backfills. Retaining walls have traditionally been constructed with plain or reinforced concrete, with the purpose of sustaining the soil pressure arising from the backfill. This section is concerned with the economical design of a reinforced concrete cantilever (RCC) retaining wall. Design variables, constraints, and the objective function for optimization of RCC retaining walls using the proposed MPSO are illustrated in the following sections.

6.1 Design variables

As shown in Fig. 3, eight design variables are the width of heel (X_1), stem thickness at the top (X_2), stem thickness at the bottom (X_3), width of the toe (X_4), thickness of the base slab (X_5), vertical steel area of the stem (X_6), horizontal steel area of the toe (X_7), and horizontal steel area of the heel (X_8) per unit length of the wall.

In Fig. 3, φ_1' and γ_1 are the effective friction angle and unit weight of retained soil; φ_2', γ_2, and c_2 are the effective friction angle, unit weight, and cohesion of base soil; P_a is the active earth pressure; P_p is the passive earth pressure; β is the backfill slope angle;

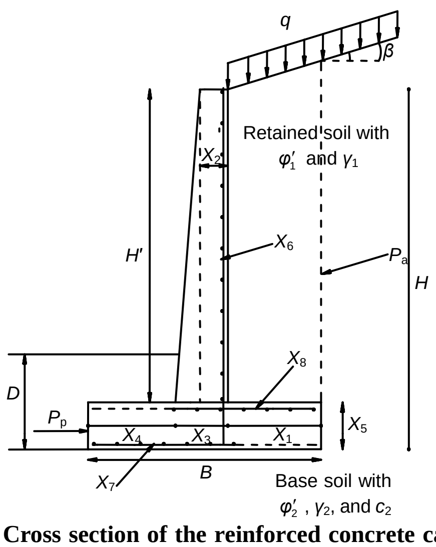

Fig. 3 Cross section of the reinforced concrete cantilever retaining wall

D is the depth of soil in front of the wall; B is the base width of the wall's foundation; H and H' are the heights of wall and stem, respectively; q is the surcharge load.

6.2 Design constraints

The various design constraints shall be considered in the optimization of the retaining structure. The constraints may be classified as geotechnical and structural constraints. According to Bowles (1982), these constraints are summarized in Table 3. Furthermore, as recommended by Yepes *et al.* (2008), the maximum deflection at the top of the stem should not exceed an acceptable threshold level (1/150 height of the stem).

Table 3 Failure modes of the retaining wall (Bowles, 1982; Yepes *et al.*, 2008)

IC	Failure mode	Constraints
$g_1(\boldsymbol{X})$	Sliding stability	$FS_{sl} \le \Sigma F_R / \Sigma F_d$
$g_2(\boldsymbol{X})$	Overturning stability	$FS_{ot} \le \Sigma M_R / \Sigma M_O$
$g_3(\boldsymbol{X})$	Bearing capacity	$FS_b \le q_{ult}/q_{max}$
$g_4(\boldsymbol{X})$	Eccentricity failure	$e \ge B/6$
$g_5(\boldsymbol{X})$	Toe shear	$\tau_c \ge \tau_{vtoe}$
$g_6(\boldsymbol{X})$	Toe moment	$M_{Rtoe} \ge M_{toe}$
$g_7(\boldsymbol{X})$	Heel shear	$\tau_c \ge \tau_{vheel}$
$g_8(\boldsymbol{X})$	Heel moment	$M_{Rheel} \ge M_{heel}$
$g_9(\boldsymbol{X})$	Shear at bottom of stem	$\tau_c \ge \tau_{vstem}$
$g_{10}(\boldsymbol{X})$	Moment at bottom of stem	$M_{Rstem} \ge M_{stem}$
$g_{11}(\boldsymbol{X})$	Deflection at top of stem	$\delta_{max} \le H'/150$

IC: inequality constraint; FS_{sl}: factor of safety against sliding, 1.5; FS_{ot}: factor of safety against overturning, 1.5; FS_b: factor of safety against bearing capacity, 3.0; F_R: horizontal resisting force; F_d: horizontal driving force; M_R: resisting moment; M_{Rtoe}, M_{Rheel}, and M_{Rstem}: resistant moments of toe slab, heel slab, and stem, respectively; M_{toe}, M_{heel}, and M_{stem}: maximum bending moments of toe slab, heel slab, and stem, respectively; M_O: overturning moment; q_{max}: maximum intensity of soil pressure; q_{ult}: ultimate bearing capacity; τ_c: shear strength of concrete; τ_{vtoe}: nominal shear stress of toe; τ_{vheel}: nominal shear stress of heel; τ_{vstem}: nominal shear stress of stem; e: eccentricity of the resultant force; δ_{max}: maximum deflection at the top of the stem

Moreover, as recommended in (Bowles, 1982) and (ACI 318-05, 2005), all design variables have practical minimum and maximum values. The upper and lower bound constraints are presented in Table 4. According to Tables 3 and 4, a total of 24 inequality constraints should be considered in the optimization of the retaining wall.

6.3 Objective function

The objective function is considered as the total cost of the retaining wall:

Table 4 Upper and lower bounds for design variables of retaining wall

Parameter	Lower bound	Upper bound
B (m)	$0.4H$	$0.7H$
X_2 (cm)	20	–
X_4 (m)	$0.4H/3$	$0.7H/3$
X_5 (m)	$H/12$	$H/10$
X_6 (m^2)	$0.0035(X_2-0.01)$	$0.016(X_3-0.01)$
X_7 (m^2)	$0.0035(X_5-0.01)$	$0.016(X_5-0.01)$
X_8 (m^2)	$0.0035(X_5-0.01)$	$0.016(X_5-0.01)$

B: width of the footing; X_2: stem thickness at the top; X_3: stem thickness at the bottom; X_4: width of the toe; X_5: thickness of the base slab; X_6: vertical steel area of the stem; X_7: horizontal steel area of the toe; X_8: horizontal steel area of the heel; H: height of the wall

$$f(\boldsymbol{X}) = C_c V_c + C_e V_e + C_b V_b + C_f A_f + C_s W_s. \quad (12)$$

The parameters are defined in Eq. (10) and the values of unit price are presented in Table 5 (Yepes *et al.*, 2008). Finally, the main objective function may be obtained by substituting the objective function of Eq. (12) and inequality constraints presented in Tables 3 and 4 into Eq. (8). Therefore, the final objective function (fitness function) for cost optimization of the retaining structure using MPSO can be formulated in the following form:

$$F(\boldsymbol{X}) = f(\boldsymbol{X}) + r\sum_{i=1}^{24} \max\{0, g_i(\boldsymbol{X})\}^l. \quad (13)$$

Table 5 Reinforced concrete cantilever retaining walls assembly unit price (Yepes *et al.*, 2008)

Work task	Price
Earth removal	3.01 USD/m^3
Foundation formwork	18.03 USD/m^2
Stem formwork	18.63 USD/m^2
Reinforcement	0.56 USD/kg
Concrete in foundations	50.65 USD/m^3
Concrete in stem	56.66 USD/m^3
Earth fill-in	4.81 USD/m^3

7 Model applications

The implementation procedure of the proposed MPSO for the economic design of spread footings and retaining walls is shown as a flowchart in Fig. 4. The flowchart is self-explanatory, as each single part of the algorithm has been already discussed in previous sections. To verify the effectiveness of the implemented algorithm, the formulation and solution are tested with designs of several cases of spread footings

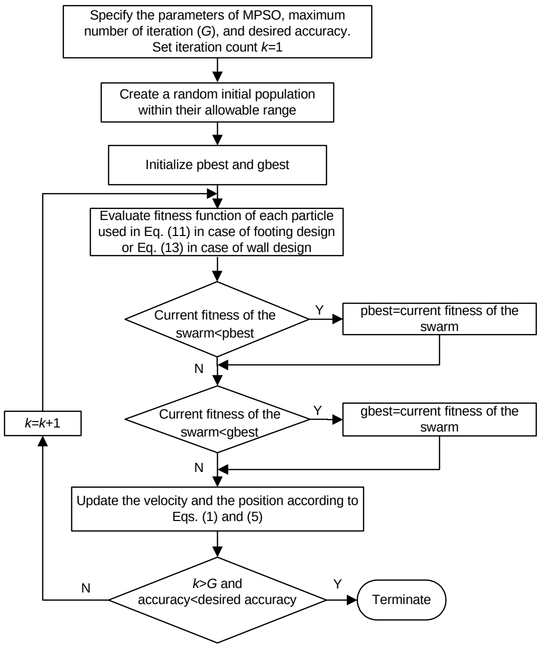

Fig. 4 Flowchart of the modified particle swarm optimization used for the optimization of the spread footing and the retaining wall

and retaining walls by changing the soil parameters, material properties, etc. However, three representative cases are reported in the following sections. The result of the proposed method is compared with the results of PSO, PSOPC, and other selected methods employed in previous studies. The optimization procedure following the methods described above was undertaken using a specially prepared computer program coded in MATLAB. All the programs were executed on a 2.10 GHz Pentium IV processor with 2 GB of random access memory (RAM).

To achieve optimum performance in the proposed methodology, the MPSO parameters need to be carefully adjusted. The parameters that may affect the performance of the algorithm include acceleration constants (c_1 and c_2), passive congregation coefficient (c_3), maximum and minimum values of inertia weight (w_{max} and w_{min}), maximum and minimum values of restriction factors (ψ_{max} and ψ_{min}), and swarm size (N). In our study, correct fine tuning of these parameters was obtained utilizing several experimental studies examining the effect of each parameter on the final solution and convergence of the algorithm. As a result, for all algorithms, a population of 40 individuals was used, w_{max} and w_{min} were chosen as 0.95 and 0.45 respectively, and the values of the acceleration constants (c_1 and c_2) were selected equal to 2. The passive congregation coefficient (c_3) was set to 0.4 for both PSOPC and MPSO. The maximum and minimum values of the restriction factors (ψ_{max} and ψ_{min}) were selected as 0.9 and 0.7 respectively. Finally, a fixed number for maximum iterations (G) of 3000 was applied. The optimization procedure was terminated when one of the following stopping criteria was met: (1) the maximum number of generations is reached; (2) after a given number of iterations, there is no significant improvement of the solution.

7.1 Example 1: spread footing under vertical loads

The first case has been previously analyzed by Wang and Kulhawy (2008) using a Microsoft Excel spread-sheet. The example is an interior spread footing in dry sand to carry a vertical load. Other input parameters for the optimum design are given in Table 6.

Table 6 Input parameters for optimum design of the spread footing

Input parameter	Input value	
	Example 1	Example 2
Effective friction angle of base soil (°)	35	30
Unit weight of base soil (kN/m^3)	18.5	18
Young's modulus (MPa)	50	35
Poisson's ratio	0.3	0.3
Vertical load (kN)	3000	3480
Bending moment (kN·m)	0.0	840
Concrete cover (cm)	7.0	7.0
Yield strength of reinforcing steel (MPa)	400	400
Compressive strength of concrete (MPa)	28	30

The optimization results of the design variables are tabulated using MPSO for this case in Table 7. As shown in the table, the best price achieved by the proposed method is 1065 USD.

As mentioned before, this problem was solved by Wang and Kulhawy (2008), but they considered only geotechnical constraints (bearing capacity and settlement of footing). The method used in their study was a generalized reduced gradient nonlinear optimization implemented with Excel software yielding a best price of 1086 USD. For the sake of comparison,

Table 7 Optimization result for spread footing

Design variable	Optimum value	
	Example 1	Example 2
Length of footing (m)	2.18	5.75
Width of footing (m)	1.70	1.70
Depth of footing (m)	0.56	0.67
Depth of embedment (m)	2.0	1.70
Long direction reinforcement (cm^2)	38	160
Short direction reinforcement (cm^2)	24	23
Best price (USD)	1065	2926

the problem was solved again with the same conditions using the proposed procedure, and the best price computed by MPSO was 1028 USD, somewhat lower than the value reported by Wang and Kulhawy (2008).

Table 8 presents a comparison of the results obtained by MPSO, PSOPC, and PSO in terms of the number of iterations, elapse time, and optimum cost achieved by different methods. In order to make a fair comparison among all the three algorithms, the same number of iterations and same ranges of parameters are used. To compare the accuracies of the algorithms, a maximum number of iterations is considered as a stopping condition and the results obtained from the algorithms are compared. The values of the different variables relevant for each algorithm are discussed in the previous section. The best fitness value achieved by each algorithm is a measure of the strength of the algorithm. Each algorithm was run 50 times, and the average elapsed time is considered as a measure of the computational time. As can be seen in Table 8, the best price obtained by MPSO is 1065 USD and is slightly lower than those obtained by PSOPC (1073 USD) and PSO (1115 USD).

Fig. 5 shows the variation in best price obtained by different methods through the optimization procedure. It is obvious that the proposed algorithm requires far fewer iterations and less computational time when compared with other algorithms (Fig. 5, Table 8). Hence, it can be concluded that the MPSO is the best among the aforementioned algorithms in terms of accuracy and convergence speed.

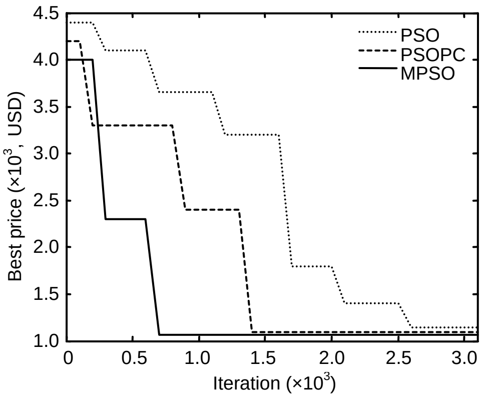

Fig. 5 Convergence rate of the algorithms for example 1

To further validate the reliability of the results, nonparametric statistical analysis of the data obtained from the 50 independent runs is undertaken using SPSS Release 11.5.0 statistic software. In this study, a significance level of 0.05 (P-value under 0.05) is considered. First, one sample Kolmogorov-Smirnov test (Kvam and Vidakovic, 2007) is undertaken to determine whether the data has the characteristics of normal distribution. The results are presented in Table 9. As derived from Table 9, the P-value (Asymp. Sig.) of each algorithm is less than 0.05, which implies that the values do not match normal (Gaussian) distribution.

Because of non-Gaussian distribution of the data, the Kruskal-Wallis nonparametric test (Kvam and Vidakovic, 2007) is applied to compare the medians of the algorithms (Table 10). The results show that the P-value is less than the significance level

Table 8 Comparisons of the results of different methods

Example	Method	Best result			Average result			Worst result		
		Iteration	Elapse time (s)	Optimum cost (USD)	Iteration	Elapse time (s)	Optimum cost (USD)	Iteration	Elapse time (s)	Optimum cost (USD)
1	PSO	2650	54	1115	2980	61	1128	2230	43	1200
	PSOPC	1390	24	1073	1610	28	1086	1150	21	1137
	MPSO	780	15	1065	880	22	1080	810	16	1118
2	PSO	2910	69	3125	2900	65	3154	2710	57	3308
	PSOPC	1760	38	3001	1700	34	3016	1740	36	3095
	MPSO	1430	26	2926	1400	23	2954	1250	29	3121
3	PSO	1820	92	261	2100	103	265	1520	75	274
	PSOPC	1210	61	257	1400	71	261	1010	46	279
	MPSO	930	42	255	1010	46	260	1240	53	281

Table 9 Single sample Kolmogorov-Smirnov test

Example	Method	Normal parameter		Most extreme difference			Kolmogorov-Smirnov	Asymp. Sig. (2-tailed)
		Mean	Standard deviation	Absolute	Positive	Negative		
1	PSO	1128	18.14	0.303	0.303	−0.239	2.143	0
	PSOPC	1086	15.86	0.269	0.269	−0.202	1.905	0.001
	MPSO	1080	13.21	0.23	0.23	−0.108	1.629	0.01
2	PSO	3154	43.4	0.317	0.317	−0.252	2.242	0
	PSOPC	3016	27.54	0.416	0.416	−0.292	2.939	0
	MPSO	2954	30.06	0.22	0.22	−0.176	1.557	0.016
3	PSO	265	4.51	0.301	0.301	−0.181	2.127	0
	PSOPC	261	3.48	0.194	0.194	−0.17	1.369	0.047
	MPSO	260	4.91	0.265	0.265	−0.153	1.874	0.002

of 0.05, which verified that there is a significant difference in the medians.

Finally, to make a pair wise comparison between the algorithms to understand the significance of their results and to validate each algorithm separately, this study uses Mann-Whitney *U* test (Kvam and Vidakovic, 2007). As shown in Table 11, all pairs have a *P*-value less than 0.05, which indicated that the three pairs are significantly different. Furthermore, the mean rank differences of the MPSO test results are smaller and better than those of PSO and PSOPC algorithms, and the performance of the new method is superior.

Table 10 Kruskal-Wallis nonparametric test

Example	Method	Mean rank	Chi-square	*df*	Asymp. Sig. (2-tailed)
1	PSO	123.59	96.483	2	0
	PSOPC	60.67			
	MPSO	42.24			
2	PSO	125.5	128.19	2	0
	PSOPC	73.76			
	MPSO	27.24			
3	PSO	108.81	48.8	2	0
	PSOPC	67.5			
	MPSO	50.19			

Table 11 Mann-Whitney *U* test

Example	Pair	Mean rank deference	*Z*	Mann-Whitney *U*	Asymp. Sig. (2-tailed)
1	PSO-PSOPC	46.72	−8.057	82	0
	PSO-MPSO	49.46	−8.53	13.5	0
	PSOPC-MPSO	17.06	−2.944	823.5	0.003
2	PSO-PSOPC	50	−8.633	0	0
	PSO-MPSO	50	−8.624	0	0
	PSOPC-MPSO	46.52	−8.034	87	0
3	PSO-PSOPC	29.86	−5.201	503.5	0
	PSO-MPSO	36.76	−6.39	331	0
	PSOPC-MPSO	13.86	−2.408	903.5	0.016

7.2 Example 2: spread footing under eccentric loads

The second example considers a reinforced spread footing under an eccentric load in dry sand, input parameters for which are shown in Table 6.

The optimization results for this case are presented in Table 7 and the best price achieved by the MPSO is 2926 USD. A detailed comparison between the results obtained by the present methodology, PSOPC, and PSO is presented in Table 8. It can be observed that the best price achieved by MPSO (2926 USD) is lower than the values obtained by PSOPC (3001 USD) and PSO (3125 USD). Moreover, the result shows that using MPSO to determine the optimum design of spread footing has a faster convergence rate compared with those of PSOPC and PSO.

In this example, Kruskal-Wallis nonparametric test is undertaken because the data do not match normal distribution (Table 9). The results of Kruskal-Wallis test are presented in Table 10. As shown in Table 10, the *P*-value is less than 0.05, which shows a significant difference among the medians of the algorithms. Then, the pair wise comparison between the algorithms is carried out using the Mann-Whitney *U* test (Table 11). The three pairs are significantly different (*P*-value of each pair is less than 0.05), the mean rank difference of MPSO is less than those of the other algorithms, and as a whole, MPSO shows the best performance.

7.3 Example 3: optimum design of retaining wall

The objective of this example is to optimize an RCC retaining wall with a height of 3 m. Other input parameters for this example are given in Table 12.

Table 12 Input parameters for optimum design of retaining wall of example 3

Input parameter	Input value
Height of stem (m)	3.0
Internal friction angle of retained soil (°)	36
Effective friction angle of base soil (°)	0.0
Unit weight of retained soil (kN/m^3)	17.5
Unit weight of base soil (kN/m^3)	18.5
Unit weight of concrete (kN/m^3)	23.5
Cohesion of base soil (kPa)	125
Depth of soil in front of wall (m)	0.5
Surcharge load (kPa)	20
Backfill slop (°)	10
Concrete cover (cm)	7.0
Yield strength of reinforcing steel (MPa)	400
Compressive strength of concrete (MPa)	21
Shrinkage and temporary reinforcement percentage (%)	0.2

Results of the optimization are presented in Table 13. Saribas and Erbatur (1996) solved this example using nonlinear programming. When Saribas and Erbatur (1996) solved this problem, they only considered seven design variables. They used different unit prices without measuring the cost of excavation, formwork, and backfill, and the best price achieved was 82.47 USD/m. For comparison, this problem was solved under the same conditions using MPSO, and the best price computed was 72.20 USD/m. It must be emphasized that the optimum design is dependent on the unit price of construction, which varies from one area to another. Therefore, optimum design will change with variation of the unit prices.

Table 13 Optimization results for retaining wall of example 3

Design variable	Optimum value
Width of heel (m)	0.76
Stem thickness at the top (m)	0.20
Stem thickness at the bottom (m)	0.35
Width of toe (m)	0.60
Thickness of base slab (m)	0.27
Vertical steel area of the stem (cm^2)	14
Horizontal steel area of the toe (cm^2)	6.3
Horizontal steel area of the heel (cm^2)	6.0
Best price (USD/m)	255

Fig. 6 and Table 8 present a performance comparison of three algorithms for the optimum design of the retaining wall. As can be seen in Table 8, the best price obtained by MPSO (255 USD/m) is slightly lower than those achieved by PSOPC (257 USD/m) or PSO (261 USD/m). In addition to generating superior results, the MPSO had a very fast convergence rate in the early iterations and performed significantly better than other methods.

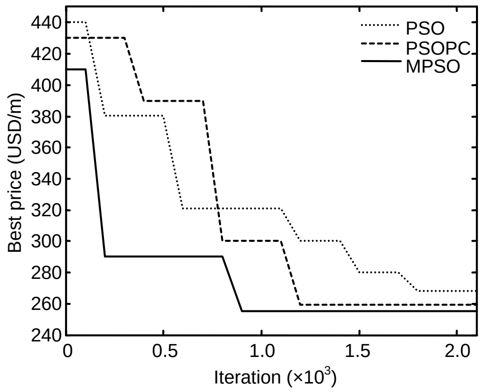

Fig. 6 Convergence rate of the algorithms for example 3

Moreover, to verify the reliability of the results, a statistical test of the data obtained from the 50 independent runs is conducted. According to the results of Kolmogorov-Smirnov test in Table 9, the data do not have a normal distribution. Hence, Kruskal-Wallis test is undertaken, and the results are presented in Table 10. The results indicate a difference between the medians of the algorithms. Finally, Mann-Whitney *U* test is performed to make a pair wise comparison amongst the algorithms (Table 11). The *P*-value of each pair is less than 0.05, which implies that the three pairs are significantly different. Furthermore, the mean rank difference of MPSO is lower than those obtained by PSO and PSOPC, and indicates a better performance of MPSO.

8 Sensitivity analysis

The first part of this section is concerned with sensitivity analysis to explore the effects of soil properties on optimum design of spread footing. Ground conditions and soil properties are key factors that control geotechnical engineering designs. Therefore, proper site investigation is necessary to define the ground conditions and to establish design input parameters. Furthermore, site investigation should be conducted in a cost-effective manner. An economically optimized design permits quantitative assessment of the benefits of improving soil property characterization by means of a sensitivity study.

In order to investigate the effect of soil properties on the final design and sensitivity analysis of the footing, example 2 is considered in which the total cost of the footing is computed by different values of Young's modulus (E), effective friction angle (φ'), Poisson's ratio (v), and unit weight of soil (γ), and the results are graphically presented in Fig. 7. In the first stage, the total cost of the footing is computed using different values of E while other parameters are kept fixed. The value of E is gradually increased from 25 to 50 MPa through 10 time steps (equal to 2.5 MPa for each time step). Consequently, the total price decreases from 5060 to 2228 USD. In the second stage, the total price is obtained using different values of effective friction angle of soil while the other properties are kept fixed. The result shows that the total price of the footing is reduced by 2032 USD (from 3910 to 1878 USD) when φ' increased by 20° (from 25° to 45°) through 10 time steps. Similarly, Fig. 7 also displays the variations in the best price of the footing with different values of v and γ in each of the time steps, while the other parameters are constant. The results shows that variation of Young's modulus and effective friction angle have the greatest effects on total price so that these parameters play a key role in the optimum design of spread footing. In other words, these parameters should be measured as accurately as possible during the site investigation.

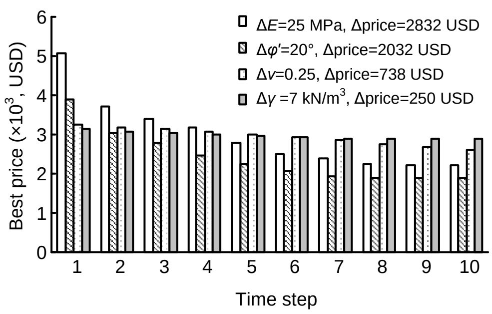

Fig. 7 Effects of soil properties on the total cost of spread footing

Since an economically optimized design incorporates construction cost estimates, it is possible to explore the effect of design requirements on the construction costs. Fig. 8 illustrates the effects of the safety factor on the construction cost of the spread footing in example 2. The total cost of spread footing versus the effective friction angle of soil with different values of factor of safety (FS) is plotted. It is obvious that by increasing the friction angle of the soil the effect of FS on the total cost of footing is reduced. These variations suggest that, in spread footing design, when the effective friction angle is rather small, FS becomes a sensitive issue and should be chosen with care.

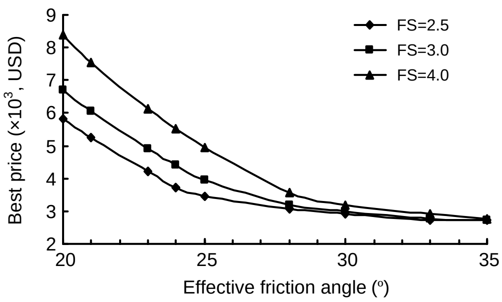

Fig. 8 Effects of factor of safety on the total cost of spread footing

Similar investigation is performed for different values of allowable settlement. The results show that, by increasing the effective friction angle (φ'), the effect of allowable settlement on the total cost of spread footing will be increased (Fig. 9). This demonstrates that allowable settlement controls the design of shallow foundations when φ' is relatively large, while it has no effect on the final design for soils with low values of φ'. In this case FS is a significant parameter and it controls the optimum design.

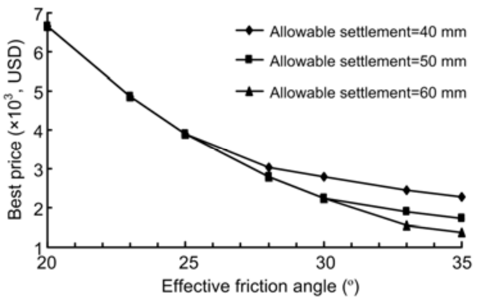

Fig. 9 Effects of allowable settlement on the total cost of spread footing

In the second part of this section, sensitivity analysis is carried out to investigate the effect of relevant parameters on the optimization of retaining structures using the proposed method. Accordingly, in example 3, the total cost of the retaining wall is computed using different values of the friction angle of retained soil (φ_1'), unit weight of retained soil (γ_1), unit weight of base soil (γ_2), and cohesion of base soil (c_2). The height of the wall is taken as 4.5 m. In the first stage, the total price of the wall is computed with different values of φ_1' and the other parameters are kept fixed. The results show that, when φ_1' changes from 25° to 45° during five time steps (equal to 5° for each time step), the total price of the wall decreases by 185 USD/m (596 to 411 USD/m). Similarly, to determine the effect of the unit weight of retained soil, the total price of the wall is calculated using different values of γ_1. The result indicates that when γ_1 increases from 15 to 20 kN/m^3 through five time steps, the total price of the wall increases by 18 USD/m. Similar investigation is carried out for γ_2 and c_2, and the results are graphically presented in Fig. 10. It is obvious that the effect of φ_1' on the total cost of the wall is dominant over the other variables.

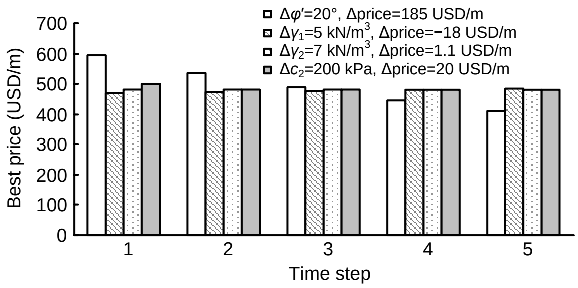

Fig. 10 Effects of soil properties on the total cost of retaining wall

As was mentioned in the previous section, the friction angle of retained soil is the most effective design parameter for RCC retaining walls. Fig. 11 shows the varying construction cost of the retaining wall in example 3 as a function of both the friction angle of retained soil (φ') and height of the wall (H). For a given H, the construction costs decrease as φ' increases. This decline will be intensified when the height of the wall increases. In other words, the friction angle (φ') significantly decreased the total cost of high RCC walls.

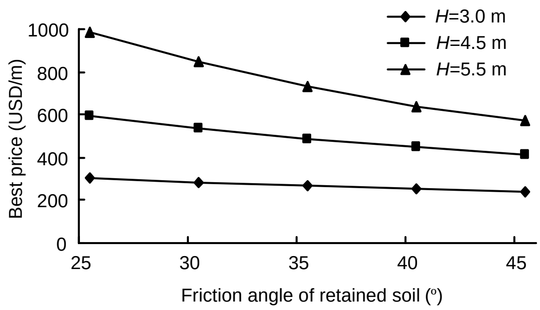

Fig. 11 Effects of wall height on the total cost of retaining wall

9 Conclusions

In this paper, a modified particle swarm optimization algorithm is proposed for the optimum design of spread footings and retaining walls. Attractive advantages of the presented method are easy to implement, rapidly converging toward an optimum and more accurate solution. The proposed algorithm handles the problem specified constraints using a penalty function method. In the optimization formulation, the objective function was considered as the total cost of structures. All design variables are treated, which vary within the ranges of geotechnical and structural requirements. The performance of the proposed MPSO has been examined over a variety of spread footing and retaining wall problems that three illustrative examples present. The results show that, compared with two other similar methods, PSO and PSOPC, the proposed MPSO calculates smaller values of the objective function in a lower number of iterations, thus demonstrating its reliability and robustness. Furthermore, the result of nonparametric statistical analysis verified the reliability of the new method. Moreover, comparison of the result of the current study with those of previous studies shows the efficiency of MPSO.

Finally, the effects of soil parameters and design requirements on the total cost of spread footing and retaining wall have been investigated using the proposed methodology. The following points were obtained using sensitivity analysis. First, the Young's modulus and effective friction angle of the base soil are the main parameters for optimum design of a spread footing. Second, factor of safety controls the optimum design of the footing when the effective friction angle is relatively small, while allowable settlement controls the final design for high values of the effective friction angle. Third, the friction angle of retained soil is the main parameter in the optimization of RCC retaining walls, especially when the height of wall increases.

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