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Car-following theory of steady-state traffic flow using time-to-collision*

Sheng JIN[†1], Zhi-yi HUANG[1], Peng-fei TAO[2], Dian-hai WANG[†‡1,2]

([1]College of Civil Engineering and Architecture, Zhejiang University, Hangzhou 310058, China)

([2]College of Traffic and Transportation, Jilin University, Changchun 130022, China)

[†]E-mail: jinsheng@zju.edu.cn; wangdianhai@zju.edu.cn

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Abstract: The conventional car-following theory is based on the assumption that vehicles will travel along the center line of lanes. However, according to the field survey data, in complex traffic conditions, a lateral separation between the leader and the follower frequently occurs. Accordingly, by taking lateral separation into account, we redefined the equation of time-to-collision (TTC) using visual angle information. Based on the stimulus-response framework, TTC was introduced into the basic General Motors (GM) model as a stimulus, and a non-lane-based car-following model of steady-state traffic flow was developed. The property of flow-density relationship was further investigated after integrating the proposed car-following model with different parameters. The results imply that the property of steady-state traffic flow and the capacity of each lane are highly relevant to the microscopic staggered car-following behavior, and the proposed model significantly enhances the practicality of the human driving behavior model.

Key words: Non-lane-based, Visual angle, Time-to-collision (TTC), Flow-density curve
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1 Introduction

The concept of car-following describes the detailed movement of vehicles proceeding close together in a single lane. This theory is based on the assumption that each driver reacts in some specific fashion to a stimulus from the vehicle ahead of him. The car-following theory is of significance in microscopic traffic flow theory and has been widely applied in traffic safety analysis and traffic simulation (Luo *et al.*, 2010; Tordeux *et al.*, 2010). There have been many car-following models in the past 60 years, and the models can be divided into two categories. One is developed from the viewpoint of traffic engineering and the other is based on statistical physics. From the perspective of traffic engineers (Brackstone and McDonald, 1999), car-following models can be classified as stimulus-response models (Gazis *et al.*, 1961; Newell, 1961), safety distance models (Gipps, 1981), psycho-physical models (Wiedemann, 1974), and artificial intelligence models (Kikuchi and Chakroborty, 1992; Wu *et al.*, 2000). From the perspective of statistical physicists, wherein vehicles are considered as the self-driven particle (Chowdhury *et al.*, 2000), car-following models can be classified as optimal velocity models (Bando *et al.*, 1995; Li and Shi, 2006; Batista and Twrdyo, 2010), intelligent driver models (Treiber *et al.*, 2000; Treiber and Helbing, 2003), and cellular automata (CA) models (Nagel and Schreckenberg, 1992; Fukui and Ishibashi, 1996). For further reviews of these models, please refer to Weng and Wu (2002) and Toledo (2007).

The car-following theory is based on a key assumption that vehicles will travel in the center line of

‡ Corresponding author
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a lane, which is unrealistic, especially in developing countries. In these countries, poor road conditions, irregular driving discipline, unclear road markings, and different lane widths typically lead to non-lane-based car-following driving (Gunay, 2007). Heterogeneous traffic, characterized by diverse vehicles, changing composition, lack of lane discipline, etc., results in a very complex behavior and a non-lane-based driving in most Asian countries (Mathew and Radhakrishnan, 2010). Therefore, it is difficult for every vehicle to be moving in the middle of the lane. Vehicles are positioned laterally within their lanes, and the off central-line effect results in lateral separations. As shown in Fig. 1, video cameras were used to record the lateral position of vehicles in Weixing Road, Changchun City, China. Each lane was divided into 10 equal parts laterally. There were 583 samples collected between 9:00 to 10:00 am on April 5, 2009. The mean of the lateral separation distance is 0.75 m and the standard deviation is 0.63 m. Fig. 2 shows the distribution of the lateral separation distance between the leader and the follower. Indeed, lateral separation frequently occurs in real conditions.

However, to the limit of our knowledge, the effect of lateral separation in the car-following process has been ignored by the vast majority of models. A few researchers have contributed efforts on this matter. Gunay (2007) first developed a car-following model with lateral discomfort. He improved a stopping-distance based approach that was proposed by Gipps (1981), and presented a new car-following model, taking into account lateral friction between vehicles. Jin *et al.* (2010) proposed a non-lane-based car-following model using a modified full-velocity difference model. All the above models have assumed that drivers are able to perceive distances, speeds, and accelerations. However, car-following behavior is a human process. It is difficult for a driver of the following vehicle to perceive minor lateral separation distances, and drivers may not have precise perception of speeds and distances, not to mention accelerations. To overcome these shortcomings, a non-lane-based car-following model using time-to-collision (TTC) information is proposed. The TTC, explaining how the following drivers found the change of the lateral separation distance, was used as the stimulus of the driver. Based on the stimulus-response framework, a new car-following model is developed to be more realistic in modeling driver behavior, especially in complex conditions.

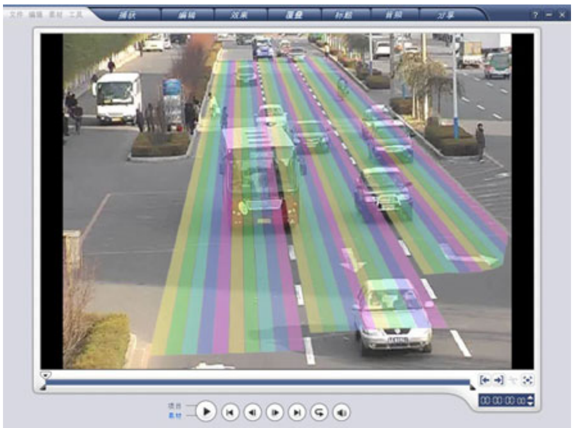

Fig. 1 Video image of lateral separation distance survey in Weixing Road, Changchun City, China

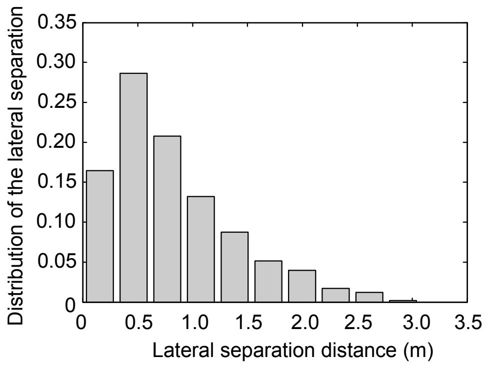

Fig. 2 Distribution of the lateral separation distance

This paper aims to illustrate that a car-following theory with a sensitivity proportional to the TTC gives a law of vehicle interaction that leads to a flow-density relationship, with the desired macroscopic traffic flow characteristics. Through a microscopic approach of modeling, a macroscopic phenomenon, which emerges from the interaction of a formation of vehicles, can be explained by the behavior of human drivers.

2 Time-to-collision of the following vehicle

The TTC is a common vision feature used to avoid obstacles and is widely used in analysis of driving behavior and evaluation of potential conflicts. TTC refers to the following driver's perception of the safety of the leading vehicle in a driving condition. Based on some psychological principles, facts and experiments, deceleration and acceleration are strongly related to the TTC. On one hand, a smaller TTC results in a faster movement of the foot to the brake pedal and a larger deceleration. On the other hand, a negative TTC results in the following driver's accelerating behavior to maintain the same speed as the leader. The correlation between the movement of the following vehicle and the TTC information was presented by Lee (1976), Van Winsum and Heino (1996), and Van Winsum and Brouwer (1997). Van Winsum (1999) showed that the acceleration initiated by the driver was a function of the TTC, and proposed a new car-following model based on psychological knowledge using TTC information. Therefore, TTC is very important information perceived by drivers in car-following situations.

There are two principal cognitive strategies for perceiving TTC. One strategy is from separate perceptions of distance and relative velocity. TTC can be defined as the distance between a following and a leading vehicle divided by the relative velocity between two vehicles. Thus, the formulation is

$$\mathrm{TTC}_n(t) = -\frac{\Delta x_n(t)}{\Delta v_n(t)}, \tag{1}$$

where $\mathrm{TTC}_n(t)$ is the TTC of the following vehicle at time t, $\Delta x_n(t)=x_n(t)-x_{n-1}(t)$ is the distance between two vehicles at time t ignoring the length of leading vehicle, and $\Delta v_n(t)=v_n(t)-v_{n-1}(t)$ is the relative velocity between two vehicles that can be approximated by the derivative $\Delta x_n(t)$.

However, data from psychological experiments (Lee, 1976; Schiff and Detwiler, 1979; Van der Horst, 1990) showed that drivers of following vehicles could not make accurate estimates about TTC based on Eq. (1). Lee (1976) reported that the inverse rate of expansion of an approaching vehicle was a visual variable that could be used to estimate TTC by the follower. Therefore, the other strategy is a computation from optic flow fields, the apparent dilation of the approaching vehicle from a distant focus of expansion. TTC is perceptually defined as the angular size of the approaching vehicle divided by its angular speed. The formulations of TTC that use visual angle information were incorporated into the basic and staggered car-following situations in the following.

2.1 Time-to-collision for the basic car-following behavior

Fig. 3 depicts the basic car-following behavior. The follower and the leader are both in the center line of the lane and there is no lateral separation. The visual angle can be formulated as

$$\varphi_n(t) = 2\arctan\left[\frac{W_{n-1}/2}{Z_n(t)}\right] \approx \frac{W_{n-1}}{Z_n(t)}, \tag{2}$$

where $\varphi_n(t)$ is the visual angle which is observed by the driver of the nth car at time t, $Z_n(t)$ is the distance between the front of the nth car (following vehicle) and the rear of the $(n-1)$th car (leading vehicle), and W_{n-1} is the width of the $(n-1)$th car (Jin *et al.* 2011).

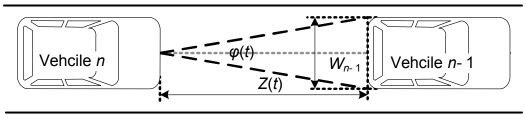

Fig. 3 Basic car-following behavior

The derivative of Eq. (2) is give as

$$\varphi_n'(t) = -\frac{W_{n-1}}{Z_n^2(t)} Z_n'(t). \tag{3}$$

Substituting Eq. (2) into Eq. (3) and eliminating W_{n-1}, TTC can be obtained:

$$\mathrm{TTC}_n(t) = \frac{\varphi_n(t)}{\varphi_n'(t)} = -\frac{Z_n(t)}{Z_n'(t)}. \tag{4}$$

Comparing with Eq. (1), TTC can be perceived by the following driver using the approaching leader's instantaneous visual angle divided by its rate of change.

2.2 Time-to-collision for the staggered car-following

Unlike the basic car-following behavior, the staggered car-following behavior widely exists in the real world (Gunay, 2007; Jin *et al.*, 2010). The key assumption of the basic car-following behavior that vehicles travel in the center line of the lane, does not hold. Thus, Eq. (2) cannot be used to calculate TTC in the non-lane-based car-following situation in which the lateral separation between the leader and the follower widely exists.

Fig. 4 shows the non-lane-based car-following behavior. In this situation, there is a lateral separation distance between the leader and the follower and they do not travel in the same lateral position. In order to calculate TTC in this situation, it is assumed that the $(n-1)$th car has a constant lateral speed and adjusts its lateral position. And then, the nth car and the $(n-1)$th car would collide at the collision point C (Fig. 4). Consequently, the middle point of the front bumper of the following vehicle A, the middle point of the rear bumper of the leading vehicle B, and the collision point C form a triangle.

Based on the geometrical analysis (Bootsma and Oudejans, 1993), the sine law is used and it is obtained as

$$\frac{Z(t)}{\sin\beta}=\frac{D(t)}{\sin\theta}, \quad (5)$$

where $Z(t)$ is the distance between the middle point of the front bumper of the following vehicle and the middle point of the rear bumper of the leading vehicle, $D(t)$ is the distance between the middle point of the rear bumper of the leading vehicle and the collision point, and θ is the visual gap angle separating the leading vehicle from the collision position.

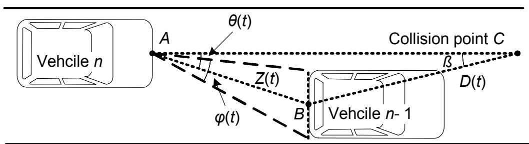

Fig. 4 Staggered car-following behavior

It is easily assumed that the leading car has constant horizontal and vertical velocities, and the angle β is constant all the time. Thus, the time derivative of Eq. (5) is given as

$$\frac{\mathrm{d}D(t)}{\mathrm{d}t}=\left(\frac{\mathrm{d}Z(t)}{\mathrm{d}t}\sin\theta+Z(t)\cos\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)\Big/\sin\beta. \quad (6)$$

Then substituting Eq. (5) into Eq. (6) and eliminating $\sin\beta$:

$$\frac{D'(t)}{D(t)}=\frac{Z'(t)}{Z(t)}+\frac{\theta'(t)}{\tan\theta}. \quad (7)$$

According to Eq. (1), the TTC can also be defined as $-D(t)/D'(t)$. Thus, substituting Eq. (4) into Eq. (7) and using θ instead of $\tan\theta$ for θ is small, TTC perceived by the following driver in non-lane-based car-following is presented as

$$\frac{1}{\mathrm{TTC}}=\frac{\varphi'(t)}{\varphi(t)}-\frac{\theta'(t)}{\theta(t)}. \quad (8)$$

The above geometrical analysis demonstrates that visual angle information is available for perceiving TTC. Eq. (8) shows the general form of TTC which is the combination of the relative rate of dilation/constriction of the optical contour of the leading vehicle and the relative rate of dilation/ constriction of the optical gap angle separating the leading vehicle from the collision point. This equation shows that the following driver is also sensitive to the visual information contained in the relative rate of dilation/constriction of the optical gap angle if no leading vehicle contour dilation component is present. Once the lateral separation between the follower and the leader equals to zero, Eq. (8) can be simplified as Eq. (4). Thus, Eq. (4) is a special case of Eq. (8). This section proposed a general equation of TTC in a non-lane-based car-following situation. TTC will be used as the inputs of car-following models in the next section.

3 Model

Most of car-following models have assumed that the inputs of models are distances, speeds, and relative speeds. However, since car-following behavior is a human process, it should be noted that drivers do not have direct access to parameter information such as distances and relative speeds. Michaels (1963) firstly suggested that the visual size of the leading vehicle should be used in car-following models. This study introduced TTC into the famous General Motors (GM) model and proposed a non-lane-based car-following model.

The GM model (Gazis *et al.*, 1961) is perhaps the best-recognized car-following model. According to the stimulus-response framework, these models use the relative speed between the leader and the follower as the stimulus to describe the motion of the nth vehicle following the (n−1)th vehicle. The basic expression of the GM model is rewritten as

$$a_n(t+T)=\frac{\lambda v_n^m(t+T)}{\left[x_{n-1}(t)-x_n(t)\right]^{l-1}}\cdot\frac{v_{n-1}(t)-v_n(t)}{x_{n-1}(t)-x_n(t)},\quad(9)$$

where $a_n(t+T)$ is the acceleration of vehicle n at time $t+T$, $v_n(t+T)$ is the velocity of vehicle n at time $t+T$, $v_{n-1}(t)$ and $v_n(t)$ are the velocities of the (n−1)th and the nth vehicles, respectively, $x_{n-1}(t)$ and $x_n(t)$ are the positions of the (n−1)th and the nth vehicles, respectively, T is the driver reaction time, and λ, m, and l are parameters that should be determined.

Unlike the GM model, the formulation of acceleration is composed of two terms: the former is the sensitivity coefficient of stimulus and the latter is the stimulus that is defined as the relative velocity divided by the relative distance. In other words, if ignoring the length of leading vehicle, the stimulus of Eq. (9) can be seen as the negative reciprocal of the TTC.

We introduce TTC in a non-lane-based car-following situation expressed by Eq. (8) as the following driver's stimulus. Thus, Eq. (9) could be modified as follows:

$$a_n(t+T)=\frac{\lambda v_n^m(t+T)}{\left[x_{n-1}(t)-x_n(t)\right]^{l-1}}\cdot\left[\frac{\theta'(t)}{\theta(t)}-\frac{\varphi'(t)}{\varphi(t)}\right].\quad(10)$$

A non-lane-based car-following model using TTC was proposed by taking into account the lateral separation effect between the follower and the leader on a single lane highway. The proposed model is similar to the GM model and is important for the analysis of characteristics of the steady-state traffic flow. This may be for three reasons. Firstly, similar to the GM model, the proposed model is also based on the stimulus-response framework and a hypothesis that the acceleration of the following vehicle is proportional to TTC. Secondly, a great deal of work has been performed on the calibration and validation of the GM model, and the results can provide us a favorable reference. Thirdly, several extensions of the GM model, such as acceleration and deceleration asymmetry, memory functions, and multiple car-following, have been proposed over nearly 60 years. These extensions can also offer assistance to the rapid development of the proposed model.

4 Characteristics of the steady-state flow

One important feature of GM models is that these models give a law of vehicle interaction that leads to a relationship between density and volume with the desired traffic flow characteristics (Rakha and Arafeh, 2010). Gazis *et al.* (1959) first described characteristics of the steady-state flow based on the GM model and compared with experimental data. They obtained the Greenberg macroscopic relationship from a car-following model by integrating the GM model equation. Using Eq. (9), some famous macroscopic relations for the steady-state flow and density were derived. The case for m=0, l=2 can be identified with a model developed from photographic observations of traffic flow by Greenshields (1935). The case for m=0, l=1 generated a steady-state relation that can be developed by a fluid flow analogy to traffic (Greenberg, 1959). Consideration of a free speed near low density for non-congested traffic led to the proposal of the noted case m=1, l=2 (Edie, 1961).

The purpose of this section is to investigate properties of the steady-state traffic flow based on the proposed model and to analyze how the non-lane-based car-following behavior affects the macroscopic relationship between flow and density. To derive the steady-state traffic flow model from Eq. (10), the visual angle and the optical gap angle separating the leading vehicle from the collision point should be expressed using distance parameters. Thus, two angles are approximately expressed as

$$\theta_n(t)=\frac{W_{n-1}}{\Delta x_n(t)},\quad(11)$$

$$\varphi_n(t)=\frac{b}{\sqrt{\Delta x_n^2(t)+b^2}},\quad(12)$$

where b is the lateral separation distance between the follower and the leader. Substituting Eqs. (11) and (12) into Eq. (10), we have

$$\ddot{x}_n(t+T)=\frac{\lambda x^m}{(x_{n-1}-x_n)^{l-1}}\times\left[\frac{\dot{x}_{n-1}-\dot{x}_n}{x_{n-1}-x_n}-\frac{(x_{n-1}-x_n)(\dot{x}_{n-1}-\dot{x}_n)}{(x_{n-1}-x_n)^2+b^2}\right],\quad(13)$$

where $\ddot{x}_n(t+T)$ is the acceleration of the nth car at time $t+T$, $\dot{x}_{n-1}-\dot{x}_n$ is the relative velocity, $x_{n-1}-x_n$ is the spacing between the leader and the follower at time t. Assuming that l=2 and setting T=0, since for the steady-state the lag time is neglected, the speed-density relationship for the steady-state may be derived from Eq. (13) by integration.

For different values of the parameter m, there are different forms of flow-density relationship. Firstly, when $m\neq1$, the parameters are such that the traffic stream is stable, and Eq. (13) can be integrated and the steady-state relation for speed and density is shown:

$$\frac{1}{1-m}\dot{x}_n^{1-m}=\lambda_1\left(-\frac{1}{x_{n-1}-x_n}-\frac{1}{b}\arctan\frac{x_{n-1}-x_n}{b}\right)+C_1, \tag{14}$$

where C_1 is the constant of integration. For the steady-state flow, $\dot{x}_n$ can be replaced by the macroscopic speed of traffic flow, u, and $x_{n-1}-x_n$ can be replaced by the inverse of the vehicle density, k.

To determine parameters C_1 and λ_1, one can assume two initial conditions. One is $u=u_\text{f}$ (free velocity) when k=0. The other is u=0 when $k=k_j$, where k_j is the density for stopped traffic and frequently referred to as the jam density. Using the two initial conditions above, it is shown that

$$\begin{cases}\lambda_1=\dfrac{1}{1-m}\cdot\dfrac{u_\text{f}^{1-m}}{k_j+\dfrac{1}{b}\left(\arctan\dfrac{1}{b\cdot k_j}-\dfrac{\pi}{2}\right)},\\ C_1=\dfrac{1}{1-m}u_\text{f}^{1-m}+\dfrac{\pi}{2b}\lambda_1.\end{cases} \tag{15}$$

Thus, substituting Eq. (15) into Eq. (14), the final equation of the steady-state relationship for speed and density is obtained:

$$u^{1-m}=u_\text{f}^{1-m}\left[1-\frac{k+\dfrac{1}{b}\left(\arctan\dfrac{1}{b\cdot k}-\dfrac{\pi}{2}\right)}{k_j+\dfrac{1}{b}\left(\arctan\dfrac{1}{b\cdot k_j}-\dfrac{\pi}{2}\right)}\right]. \tag{16}$$

Secondly, when m=1, Eq. (13) can also be integrated as

$$\ln u=\lambda_2\left(-\frac{1}{x_{n-1}-x_n}-\frac{1}{b}\arctan\frac{x_{n-1}-x_n}{b}\right)+C_2, \tag{17}$$

where C_2 is the constant of integration. To determine parameters C_2 and λ_2, one initial condition is $u=u_\text{f}$ (free velocity) when k=0. The other initial condition, however, is loss of realism, since as $u\to0$, $k\to\infty$. Thus, the initial condition that is $k=k_\text{m}$ at the maximum flow is given. The parameter, k_m, can be called the optimum density corresponding to the density at the maximum flow. The physical nature of the parameter u_f is clear by definition, but it is shown that why k_m should necessarily be the optimum density at the maximum flow. This can be done by derivation of the equation:

$$q=ku, \tag{18}$$

with respect to u, and setting the derivative equal to zero in order to find the value of k when q is the maximum. The result is $\frac{\mathrm{d}q}{\mathrm{d}u}=u\frac{\mathrm{d}k}{\mathrm{d}u}+k=0$, and solving for k:

$$k_\text{m}=-u\frac{\mathrm{d}k}{\mathrm{d}u}. \tag{19}$$

The second initial condition is given in Eq. (19). Thus, when m=1, parameters C_2 and λ_2 are

$$\begin{cases}\lambda_2=-\dfrac{1}{k_\text{m}\left[1/(b^2\cdot k_\text{m}^2+1)-1\right]},\\ C_2=\ln v_\text{f}+\dfrac{\pi}{2b}\lambda_2.\end{cases} \tag{20}$$

The steady-state flow for m=1 is shown as

$$v=v_\text{f}\exp\left(\frac{-k-\dfrac{1}{b}\arctan\dfrac{1}{b\cdot k}+\dfrac{\pi}{2b}}{k_\text{m}\left[1-1/(b^2\cdot k_\text{m}^2+1)\right]}\right). \tag{21}$$

It is evident that Eqs. (16) and (21) are the integral of Eq. (13) provided that T is neglected. In the steady-state flow, Eq. (16) and Eq. (21) show the speed-density relationship. The parameter b, which describes the staggered car-following behavior, is very important for the proposed model. If b=0, the non-lane-based car-following model considering lateral separation could be reduced to the conventional lane-based GM model, and Eq. (16) and Eq. (21) could be reduced to the Greenshields model and the Underwood model, respectively. Thus, the Greenshields model and the Underwood model are special cases of the proposed macroscopic models of speed-density relationship when b=0. The parameter b describes the lateral offset distance between the leader and the follower and is strongly related to the lane width.

In order to verify the proposed model and to show the macroscopic characteristics of traffic from a microscopic modeling approach, flow-density diagrams are presented in Fig. 5, where the free speed v_f equals to 100 km/h, the density of stopped traffic k_j equals to 120 vehicle/km, and the density at the maximum flow k_m equals to 120/e vehicle/km. The flow-density curves in the parameter space (q, k) are shown in Fig. 5. The apex of each curve indicates the critical point that can be called as the capacity of each lane. Fig. 5 shows that although the shape and peak values of curves are different for different m, the trend of curves' peak values with different b is the same. By introducing the TTC and the lateral separation effect into the conventional GM model, the critical points and the capacity are higher than those without the lateral separation effect. Thus, the capacity of steady-state traffic flow increases. There may be several reasons to explain the increase of the capacity. Firstly, with an increase of the lane width, the lateral separation distance between the leader and the follower is larger, and the driver will follow closer. Secondly, the staggered car following behavior leads to a visual gap angle separating the leading vehicle from the collision position that may result in a larger TTC. Thus, the driver of the following vehicle will perceive less collision risk and move at a smaller time headway.

The properties of steady-state traffic flow based on the proposed car-following model of single-lane traffic flow are similar with the experimental macroscopic traffic data. The same results can be found in the HCM2000 that proposed a capacity adjustment

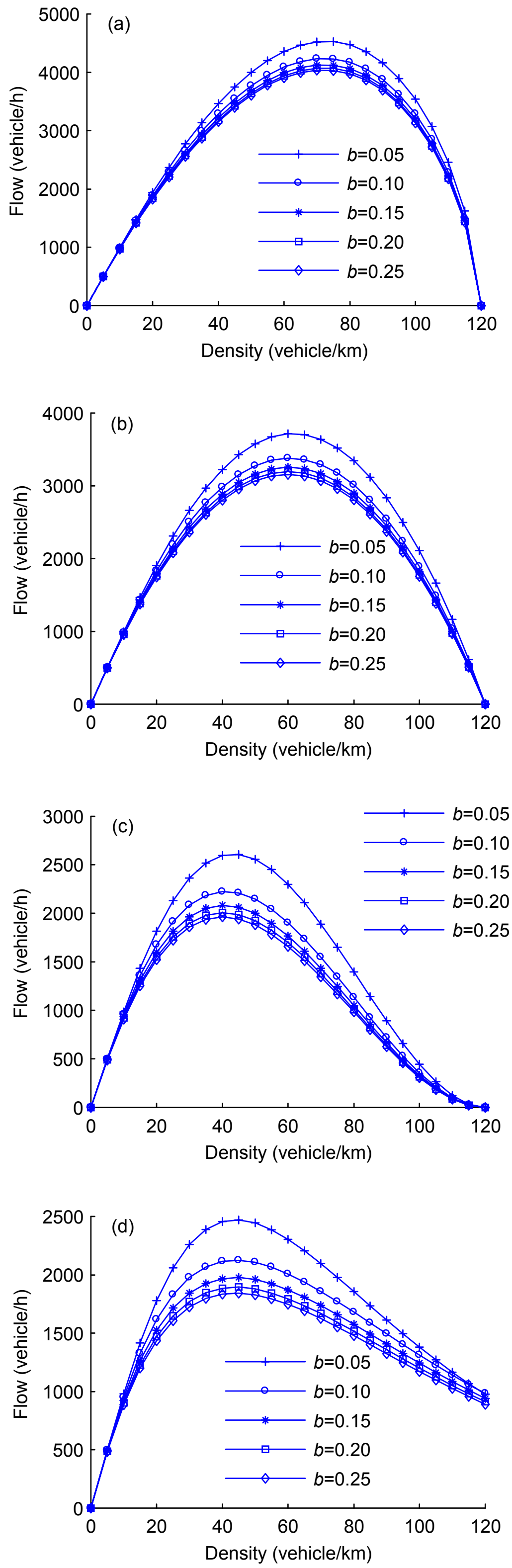

Fig. 5 Relationship between flow and density for different parameter *b*

(a) m=−0.5; (b) m=0; (c) m=0.5; (d) m=1.0

factor for lane width from 2.4 to 4.8 m (Transportation Research Board, 2001). The adjustment factor is expressed as

$$f_W = 1 + \frac{W - 3.6}{9}. \tag{22}$$

The lane width adjustment factor, f_W, accounts for the negative impact of narrow lanes on capacity and allows for an increased capacity on wide lanes. Eq. (22) gives a linear relationship between the capacity adjustment factor and the lane width. Fig. 6 shows the capacity of a single lane for different *b*, from which the results are similar with those in HCM2000. This provides a method to obtain a macroscopic parameter from an approach of microscopic modeling.

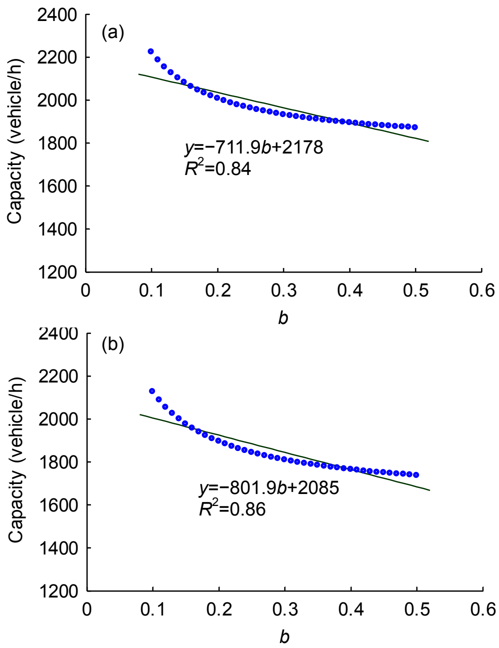

Fig. 6 Relationship between capacity and parameter *b*
(a) m=0.5; (b) m=1.0

This section discusses the properties of steady-state traffic flow based on the non-lane-based car-following model of single-lane traffic flow. In particular, the associated equivalent speed density relationship, as well as the flow-density relationship with different lane widths is developed. The analysis of properties of the steady-state traffic flow builds a bridge between the microscopic car following behavior of drivers and the macroscopic phenomenon of traffic flow.

5 Conclusions

In this paper, a staggered car-following model is presented by introducing the TTC as a stimulus. The TTC is obtained by using visual angle information that can be perceived by drivers directly. It is unrealistic to assume that vehicles travel in the center line of a lane. Therefore, the equation of TTC for the non-lane-based car-following was modified and extended into 2D space. The general form of TTC is the combination of the relative rate of dilation/constriction of the optical contour of the leading vehicle and the relative rate of dilation/constriction of the optical gap angle separating the leading vehicle from the collision point. This equation shows that the following driver is also sensitive to the visual information contained in the relative rate of dilation/constriction of the optical gap angle if no leading vehicle contour dilation component is present. The TTC, which substitutes relative velocity as a stimulus, was introduced into the conventional GM model, and a new car-following theory considering lateral separation was presented. The introduction of visual information into the GM model has more consideration for characteristics of drivers and makes the proposed model more realistic. The flow-density relationship for the steady-state flow is shown by integrating the proposed equation, as are the flow-density curves with different lateral separation parameters. It is shown that the critical points and the capacity of flow-density curves are higher than that without the lateral separation effect, and the capacity of steady-state traffic flow increases with the lane width. The same results can be found in the HCM2000 that proposed a capacity adjustment factor for different lane widths. It is implied that the macroscopic traffic flow model can be obtained from some microscopic car-following models and the staggered car-following behavior has an impact on the capacity of a single lane. Thus, compared to the previous car-following theories, the present work establishes 2D interactions for better practicality and reality in modeling human driver's behavior and describes the characteristics of the steady-state flow. Further research will examine calibrating model parameters and comparisons with experimental data. It is expected that this theory, once calibrated and validated, could provide a more realistic and powerful traffic model and lead to better understanding of complex traffic problems.

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