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Average stress-average strain tension-stiffening relationships based on provisions of design codes*

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Abstract: This research was aimed at deriving average stress-average strain tension-stiffening relationships in accordance with the provisions of design codes for reinforced concrete (RC) members. Using a proposed inverse technique, the tension-stiffening relationships were derived from moment-curvature diagrams of RC beams calculated by different code methods, namely Eurocode 2, ACI 318, and the Chinese standard GB 50010-2002. The derived tension-stiffening laws were applied in a numerical study using the nonlinear finite element software ATENA. The curvatures calculated by ATENA and the code methods were in good agreement.

Key words: Reinforced concrete (RC), Code technique, Tension-stiffening, Numerical modeling
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1 Introduction

Worldwide, the strength analysis of reinforced concrete (RC) members subjected to bending moment is based on a unified approach. In contrast, for cracking and stiffness analysis, the design codes of different countries use various approaches and models, often resulting in conflicting predictions (Kaklauskas, 2004; Borosnyói and Balázs, 2005; Juozapaitis *et al.*, 2010).

The main disadvantage of the design code methods is their limited application regarding the structural shape and the loading cases. An alternative to these methods is numerical techniques. These techniques can evaluate irregular geometrical shapes of structures and specific loading conditions and nonlinear properties of materials (Wu and Gilbert, 2009; Gribniak *et al.*, 2010a; 2010b; Wang *et al.*, 2010). But results of numerical analysis have become dependent on the applied material models. In serviceability problems, modeling of reinforcing steel and concrete in compression is simple. No significant difference was observed in deflection predictions of RC beams assuming alternative constitutive laws for concrete in compression (Stramandinoli and Rovere, 2008). However, modeling the behavior of cracked tensile concrete is a much more complicated issue. Due to bonding with reinforcement, the concrete between cracks adheres to the reinforcement bars and contributes to the overall stiffness of the structure. This phenomenon, called tension-stiffening, has a significant influence on the numerical results of short-term deformation analysis. In the present study, the behavior of an RC member is modeled by a stress-strain tension-stiffening relationship assumed to be uniform over the tension area of concrete. Stress in the concrete is taken as the combined stress due to tension-stiffening and tension-softening, collectively called tension-stiffening. A number of stress-strain

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tension-stiffening relationships have been proposed (Torres *et al.*, 2004; Stramandinoli and Rovere, 2008; Dede and Ayvaz, 2009; Wu and Gilbert, 2009; Ng *et al.*, 2010; Bacinskas *et al.*, 2011). Note that most tension-stiffening relationships were derived using experimental data on the tension (Hsu, 1993; Fields and Bischoff, 2004) or shear (Vecchio and Collins, 1986; Collins and Mitchell, 1991) of RC members. However, application of such laws in bending members may not be justified and frequently results in inaccurate deformation predictions. Kaklauskas and Ghaboussi (2001) proposed an alternative approach for deriving tension-stiffening relationships from test data (moment-curvature diagrams) of flexural RC members.

The present study aimed to derive average stress-average strain tension-stiffening relationships conforming to deformation analysis of RC beams using well-known design codes. The European (CEN, 2004), American (ACI Committee 318, 2008), and Chinese (MCPRC, 2004) design code techniques were considered. Using the algorithm proposed by Kaklauskas and Ghaboussi (2001) and modified by Kaklauskas and Gribniak (2011), the tension-stiffening relationships were derived from the moment-curvature diagrams of RC beams calculated by the codes. The obtained stress-strain relationships, as the material models for tensile concrete, were applied in curvature analysis using the nonlinear finite element software ATENA.

2 Solution of the inverse problem

Our investigation was aimed at deriving tension-stiffening models using an inverse technique. The models obtained allow the simulation of the same moment-curvature responses as predicted by the code methods. This section sketches a solution of the inverse problem, discussing major aspects only. The inverse procedure uses a simple iterative technique of deformation analysis of composite members based on the layer section model and material diagrams (Kaklauskas, 2004). The following assumptions have been adopted: (1) average strain hypothesis, also called the smeared crack concept; (2) linear strain distribution within the depth of the section; (3) perfect bonding between layers.

Let us consider a doubly RC member subjected to pure bending. A cross-section for such a member is shown in Fig. 1a. b and h are the width and height of the section, respectively; A_{s1} and A_{s2} are the areas of tensile and compressive reinforcements, respectively; d is the effective depth; a_{s2} is the distance from the compressive edge of the section to the centroid of the compressive reinforcement. The cross-section is divided into n horizontal layers of thickness t_i corresponding to either concrete or reinforcement. Thickness of the reinforcement layer t_{s1} (Fig. 1b) is taken from the condition of the equivalent area. The analysis needs to assume stress-strain (σ-ε) material laws for the reinforcement and the concrete in compression and in tension (Fig. 1c). σ_{s1} and σ_{s2} are the stresses in tensile and compressive reinforcements, respectively. Curvature κ and strain ε_i at any layer i (Fig. 1d) can be calculated by

$$\begin{gathered}\kappa=\frac{M_{\mathrm{ext}}}{\mathrm{IE}},\ \varepsilon_i=\frac{M_{\mathrm{ext}}}{\mathrm{IE}}(y_i-y_{\mathrm{c}}),\ y_{\mathrm{c}}=\frac{\mathrm{SE}}{\mathrm{AE}},\\ \mathrm{AE}=\sum_{i=1}^{n}b_it_iE_{i,\mathrm{sec}},\quad \mathrm{SE}=\sum_{i=1}^{n}b_it_iy_iE_{i,\mathrm{sec}},\\ \mathrm{IE}=\sum_{i=1}^{n}\left[\frac{b_it_i^3}{12}+b_it_i(y_i-y_{\mathrm{c}})^2\right]E_{i,\mathrm{sec}},\end{gathered}\tag{1}$$

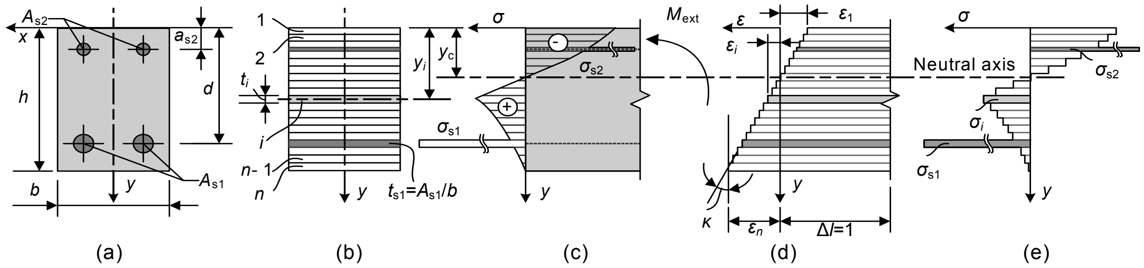

Fig. 1 Layer section model

(a) Reinforced concrete section; (b) Layered section; (c) Stress distribution across the section; (d) and (e) Strain and stress distributions in layered section where M_{ext} is the external bending moment; y_c and y_i are the centroid coordinates of the section and the ith layer, respectively; b_i and t_i are the width and height (thickness) of the ith layer, respectively; AE, SE, and IE are the area, the first and the second moments of the area, respectively, multiplied by the secant modulus $E_{i,\text{sec}}$.

For the given strains ε_i and the assumed constitutive laws (Figs. 1d and 1c), the stresses and corresponding secant modulus are calculated. In Fig. 1d, Δl represents the unit length of the member. The analysis is performed iteratively until convergence of the secant modulus at each layer is reached. Figs. 1d and 1e illustrate the strain and the stress distributions respectively, within the layer section model performing direct deformation analysis.

Unlike the direct analysis, which results in prediction of structural response using a specified constitutive model, the inverse analysis aims to determine parameters of the model based on the response of the structure. In the present study, the inverse problem is solved iteratively with incrementally increasing bending moment, using the tension-stiffening law obtained at previous loading stages.

Fig. 2 shows a flow chart of the inverse technique. Based on geometrical parameters of the cross-section, the layer section model is composed. Stress-strain material laws for steel and compressive concrete are assumed. As noted, computations are performed iteratively for an incrementally increasing bending moment from the initial M_1 to the maximum $M_{\max}$ values.

At each moment increment M_i, an initial value of the secant deformation modulus of stress-strain relationship under derivation is assumed to be equal to zero ($E_{i,0}$=0). The curvature $\kappa_{\text{calc},i}$ is calculated by the direct procedure. If the agreement between the calculated curvature and the experimental curvature $\kappa_{\text{obs},i}$ is not within the assumed tolerance Δ, i.e., Condition 1 is not fulfilled (Fig. 2), the analysis is repeated using the hybrid Newton-Raphson and bisection procedure (Gribniak, 2009) until Condition 2 is satisfied. At each iteration k, the secant deformation modulus $E_{i,k}$ is determined as the ratio of the obtained stress $\sigma_{n,i}$ and strain $\varepsilon_{n,i}$ in the nth layer. If the solution is found, i.e., Condition 1 is satisfied, the obtained value of $E_{i.k}$ is fixed and used for the next load increments. If the limit iteration number is exceeded (k>N=30), the calculated $E_{i,30}$ is rejected, meaning that the secant deformation modulus E_i is not defined at the moment increment i, and the analysis moves to the next load step. The calculation is terminated when the ultimate loading step is reached (Condition 3). The analysis results in the derived tension-stiffening relationship. Note that the assumed number of layers n might have an influence on the calculation results. The recommended number, n=200 (Gribniak, 2009), most effectively secures the computational efficiency in terms of convergence and accuracy.

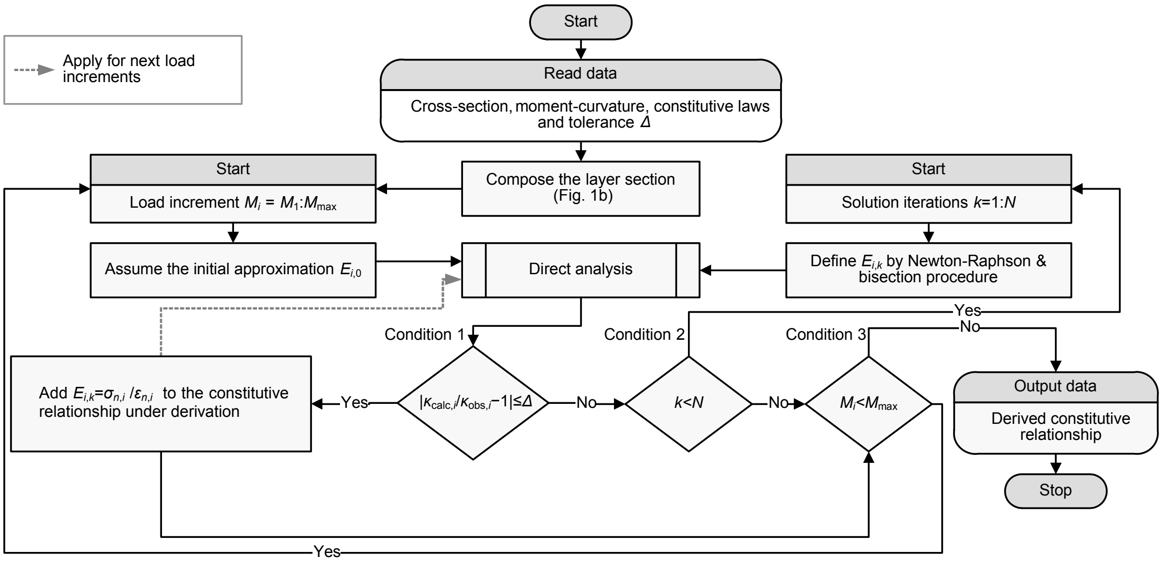

Fig. 2 Flow chart for solving the inverse problem

3 Tension-stiffening analysis

The tension-stiffening analysis was carried out for the rectangular RC section shown in Fig. 1a, assuming h=400 mm, b=200 mm, d=370 mm, and a_{s2}=30 mm. The inverse technique was applied for deriving the stress-strain relationships using the moment-curvature diagrams calculated by the Eurocode 2 (CEN, 2004), the ACI 318 code (ACI Committee 318, 2008) and the Chinese code GB 50010-2002 (MCPRC, 2004). These diagrams were calculated for a number of RC sections having a uniform grade of concrete C30/37 (C47.5 according to the Chinese code), a modulus of elasticity of steel E_s=200 GPa and a variable amount of tensile reinforcement: $p=A_{s1}/(b \cdot d)$=0.3%, 0.6%, 1.0%, and 2.0%. The ratio of the area of the compressive reinforcement A_{s2} and the tensile reinforcement A_{s1} was taken to be 0.25.

The calculated moment-curvature diagrams are shown in Fig. 3a by grey solid lines, whereas Fig. 3b shows the derived tension-stiffening relations. The obtained relations may be approximated by three lines: linear ascending, sudden linear drop, and a descending branch. The latter was practically linear for the ACI 318 and GB 50010-2002 codes and curved for the Eurocode 2. Other differences between the obtained stress-strain diagrams were as follows:

1. The tension-stiffening effect expressed in terms of the ultimate strain was far more pronounced in the lightly reinforced members. Note that different scales were applied for strain in Fig. 3b.

2. Differences in the tension-stiffening diagrams obtained for different codes were most significant for the lightly reinforced members (p=0.3%). The relations of the Eurocode 2 and the ACI 318 code were particularly contrasting. With an increase in reinforcement ratio, the tension-stiffening diagrams approached each other. Due to the sensitivity of the inverse technique (Kaklauskas and Gribniak, 2011), some of the diagrams had oscillations.

3. For the ACI 318 code, the maximum stresses were found to be dependent on the reinforcement ratio p. Reduction in the maximum stresses with increasing p could possibly be due to indirect evaluation of the restrained shrinkage effect on the cracking resistance (Kaklauskas *et al.*, 2009; Kaklauskas and Gribniak, 2011).

Fig. 4 gives a few well-known tension-stiffening laws along with the relationships derived from the GB 50010-2002 code. Note that most of the relationships,

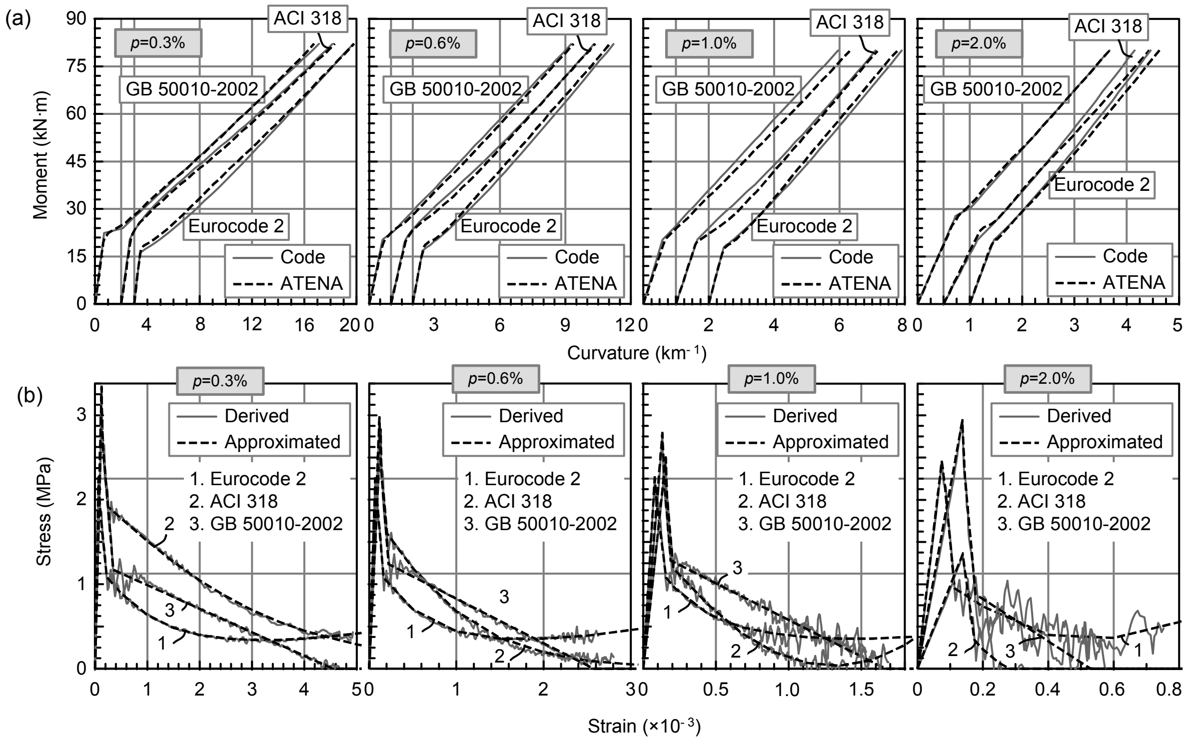

Fig. 3 Moment-curvature diagrams (a) and the obtained tension-stiffening relationships (b)

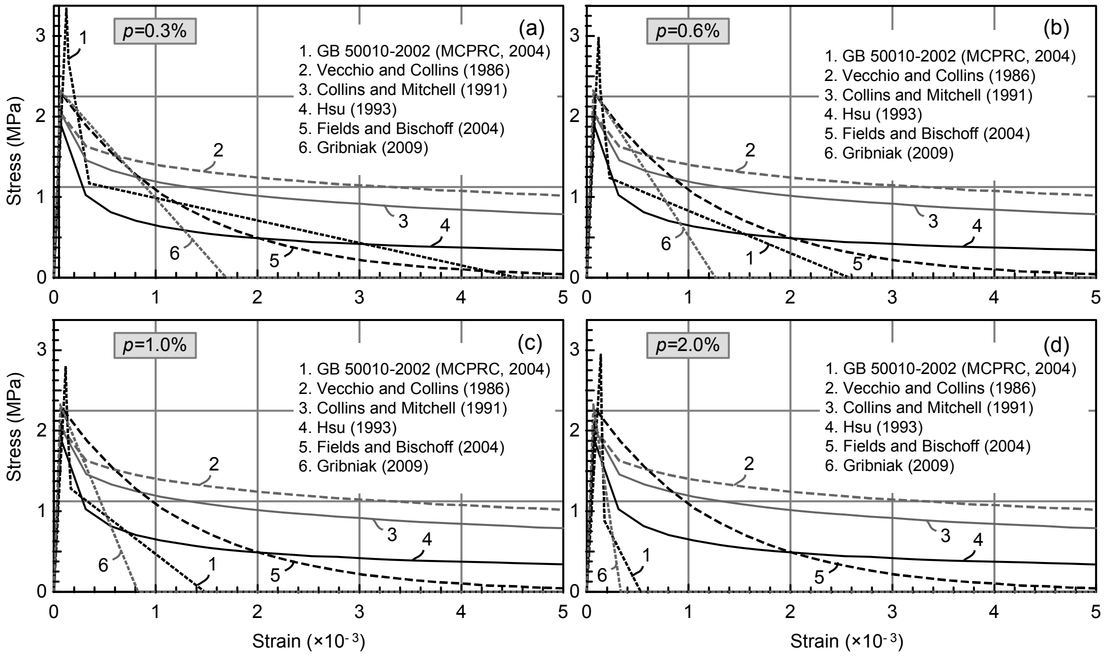

Fig. 4 Tension-stiffening relationships for different reinforcement ratios
(a) p=0.3%; (b) p=0.6%; (c) p=1.0%; (d) p=2.0%

except 1 and 6, were independent of the reinforcement ratio. The ultimate strain of the simple linear tension-stiffening relationship 6 was derived by Kaklauskas and Ghaboussi (2001) from test data of flexural RC members:

$$\varepsilon_{\text{ct,ult}}=\varepsilon_{\text{cr}}\times\begin{cases}32.8-27.6p+7.12p^2,\ p<2\%,\\ 5,\ \ p\geq 2\%,\end{cases}\tag{2}$$

where ε_{cr} represents the cracking strain of the tensile concrete. Note that the ultimate strain from Eq. (2) follows the tendency of reducing tension-stiffening with increasing reinforcement ratio (Fig. 3).

Applicability of the derived tension-stiffening relationships for deformation analysis of RC beams was verified using the commercial finite element (FE) software ATENA. Performing non-linear curvature analysis, the obtained stress-strain diagrams were approximated as shown in Fig. 3b and introduced into ATENA as the constitutive laws for tensile concrete. Isoparametric quadrilateral finite elements (15 mm in size) with 8 degrees of freedom and four integration points were used for modeling of the beams. The tension-stiffening effect is included in the FE model through the interaction of reinforcement and concrete between cracks using the principles of fracture mechanics (the crack band model). This requires the characteristic length l_{ch} of the crack localization zone to be specified (Gribniak *et al.*, 2010a; 2010b). In this study, l_{ch} was assumed to be 50 mm. The modeled responses of RC beams are shown in Fig. 3a by dashed lines.

In general, good agreement was obtained between the curvatures predicted by ATENA and the codes. Some differences could be due to approximation errors and shear effects which were neglected in the code techniques.

4 Conclusions

In the present study, based on an inverse technique proposed by the authors, average stress-average strain tension-stiffening relationships conforming to well-known design codes were derived. The tension-stiffening laws were derived using the moment-curvature diagrams of RC beams predicted by European, American, and Chinese codes. Significant differences were obvious in the tension-stiffening diagrams representing different codes and reinforcement ratios. Unlike earlier proposed tension-stiffening laws, the shapes of the stress-strain diagrams obtained were strongly dependent on the tensile reinforcement ratio. The tension-stiffening effect was far more pronounced in members with small reinforcement ratios, particularly in the case of the American code.

The obtained relationships were applied in a numerical study, using the nonlinear finite element software ATENA. Good agreement between the curvatures predicted by ATENA and the codes showed the validity of the proposed approach.

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